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Informational feedback between voting and speculative trading ☆

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This paper develops a model to investigate the interaction between collective decision making in voting and financial speculation. Protesting voters demand policy reforms by voting against the incumbent, but too many opposing votes result in an unfavorable outcome: a political regime change. Traders speculate on the change of the political regime. The size of the speculation informs voters about the electorate's composition, thereby influencing the outcome of the election. We find that, in equilibrium, the strategic substitutability of protest voting makes speculations strategic substitutes via informational feedback, thereby incentivizing speculators to trade less on the correlated public signal. This strengthens the role of financial markets in providing information and amplifies the impact of the financial market's information on ex post political outcomes. We relate our theory to the Brexit referendum, and further discuss the robustness and limitations of our findings by considering more general information environments and voter preferences.

1. Introduction

Political elections influence public policy and, thus, can have a significant impact on financial markets. Financial markets, on the other hand, aggregate individual traders' dispersed information and may efficiently predict election outcomes.[1] In addition, ample evidence demonstrates that voters are paying close attention to financial market information.[2] This paper studies the informational linkage between the financial market and political decision making, as well as how that connection could determine the informational role played by the financial market and shape the political outcome.

To fix ideas, take the Brexit referendum as an example. The leave campaign won the majority vote, but it was a narrow victory (52% vs. 48%). Shortly after the referendum, the GBP/USD exchange rate fell more than 10% to a 31-year low, while

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[1] The literature on prediction markets (e.g., Wolfers and Zitzewitz, 2004) shows that markets provide the most efficient mechanism for predicting election outcomes.

[2] For example, based on a Morning Consult nationwide poll in the U.S., Rainey (2018) found that stock market volatility dominates voters' attention over political headlines.

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the GBP/EUR rate fell more than 7%.[3] Despite the dramatic market response, no significant currency shorting occurred prior to the Brexit vote, and exchange rates remained stable. Why was the financial market unable to forecast the outcome of the election? Is it possible that the financial market affected the referendum and contributed to Brexit's surprise victory?

Motivated by the example of Brexit, we develop a theoretical model to investigate how the informativeness and effectiveness of the financial market are shaped by the informational feedback between speculative trading and voting. In our model, traders speculate on a particular voting outcome — a political regime change — prior to an election by shorting the domestic currency, and voters extract information about the electorate's composition from the size of the speculation. This generates informational feedback between voting and speculative attacks. That is, on the one hand, speculators' informational choices will change the information aggregation in the financial market and, thus, determine voters' responses to that information; on the other hand, voters' responses will shape speculators' informational choices through their cumulative impact on a political regime change.

Our theoretical analysis demonstrates, perhaps surprisingly, that when speculators are more optimistic about a political regime change, the increased size of the speculation discourages voters from voting in favor of the change. As a result, speculative attacks are *strategic substitutes*, and speculators tend to differentiate their informational choices by trading less (more) on the correlated public signal (conditionally independent private signal). This facilitates information aggregation in the financial market. As such, voters become more responsive to financial market information, thereby magnifying the real impact of the financial market on voting; that is, financial market information becomes more influential, ex post, in terms of changing the political outcome.

More specifically, we consider *protest voting* according to the motivating example of Brexit.[4] Protesting voters are dissatisfied with current policies. However, they want the incumbent to stay in power so that necessary policy reforms can be implemented. These voters signal their dissatisfaction and demand necessary policy reform by casting their protest votes against the incumbent party if they are confident that the incumbent party will win. This "protest" will succeed, and the subsequent policy reform will occur only if a sufficient number of opposing votes are cast (say, more than 20%). However, too many opposing votes (say, more than 50%) will overthrow the political regime and result in political extremism (e.g., a party with an opposing ideology or extremist views will come into power). The expectation is that the political regime change will be followed by a radical policy shift, lowering voters' living standards and resulting in a significant devaluation of the domestic currency.

Protesting voters prefer policy reform to alternative voting outcomes — namely, incumbent wins in the absence of policy reform and a political regime change. Voters have idiosyncratic preferences that can be characterized by the extent to which they would suffer from a political regime change. If voters suffer less from this voting outcome, then we say that their preference for political regime change is (relatively) higher. Voters decide simultaneously and independently whether or not to cast a vote.[5] We consider equilibrium under *monotone voting strategies*, whereby voters cast a vote if and only if their preference for a regime change is sufficiently high.

Protesting voters strictly prefer to vote only when the percentage of opposing votes (or the vote share) is 20%, at which point their vote is pivotal and will result in policy reform. Conversely, they strictly prefer not to vote when the vote share is 50%, at which point their vote is also pivotal but will result in a political regime change — an outcome they wish to avoid. Voters are unsure whether their votes will be pivotal. Thus, the distribution of voters' preferences is critical for their strategic decisions, as it determines the pivotality of their votes. The higher the mean of the preference distribution (or the extent to which the average voter prefers the political regime change), the greater the vote share that will result from any monotone voting strategy and, therefore, the higher the likelihood of a political regime change. In this sense, the mean of the preference distribution defines the weakness of the political regime, which is referred to as the *fundamental* of the economy.

In the financial market, speculators (e.g., foreign investors and financial institutions) do not have the right to vote. They do, however, have some relevant information about the electorate's composition, which they use to guide their speculation decisions. More precisely, speculators have one private signal that is conditionally independent and another signal that is conditionally correlated. Both signals are informative about the average voter's preference. In the benchmark model, we consider the conditionally correlated signal to be a public signal, and we extend this result to an alternative information environment without public signals in Section 4.3. The sources of the noisy public signal could be financial analysts' reports or expert opinions accessible only to professional traders and not to voters.

Prior to the election, speculators bet on the outcome of the vote. They choose whether or not to "attack" — shorting the domestic currency. Attacking is optimal only if their information indicates a sufficiently high probability of a political regime change. Voters do not have direct access to the information that speculators have. They are, however, able to observe the size of the speculation, from which they extract information for more informed decision making. A distinctive feature of this

[3] See Katie Allen, Jill Treanor, and Simon Goodley, "Pound slumps to 31-year low following Brexit vote," *Guardian*, June 24, 2016, https://www.theguardian.com/business/2016/jun/23/british-pound-given-boost-by-projected-remain-win-in-eu-referendum.

[4] For anecdotal evidence that the Brexit vote was a protest vote, see Dorian Lynskey, "I thought I'd put in a protest vote': the people who regret voting leave," *Guardian*, November 25, 2017, https://www.theguardian.com/politics/2017/nov/25/protest-vote-regret-voting-leave-brexit. In addition, see Alvarez et al. (2018) and Louis et al. (2020) for a discussion of why Brexit could be considered a good example of protest voting.

[5] To avoid confusion, we will refer to "vote" or "cast a vote" throughout the paper as shorthand for "casting an opposing vote," which means voting against the incumbent party. Accordingly, "not vote" or "not casting a vote" means the opposite: voting to support the incumbent party.

learning channel is that a larger speculative attack would discourage protesting voters from voting against the incumbent regime.

This discouragement effect is rooted in the *strategic substitutability* among protesting voters. To illustrate, if a larger vote share is expected from other voters, then one's vote is less likely to be pivotal to the desired outcome of policy reform but more likely to contribute to the undesirable outcome of political regime change. As a result, the strategic voter has a diminished incentive to vote, which explains why protest voting is a game of strategic substitutes.

From a larger-sized speculation, each protesting voter understands that the financial market is more optimistic about the political regime change. Based on this updated belief, voters anticipate that the electorate, on average, will suffer less from a political regime change, implying that a larger vote share is likely to materialize. Because of strategic substitutability, this observation discourages voting. This discouragement effect means that, everything else being equal, a larger-sized speculation reduces the likelihood of a political regime change, thereby making speculation less profitable. Consequently, the game among speculators is a game of *strategic substitutes*.

Given that both public and private signals provide information about the fundamental, speculators would treat these two signals equally and assign Bayesian weights to each signal based on its precision if there were no informational feedback between voting and speculative trading.[6] Unlike conditionally independent private signals, the public signal is common to all speculators, and, therefore, it provides extra information about the size of the speculation. Because speculators' actions are strategic substitutes, speculators now have an incentive to trade against this common signal. Hence, in equilibrium, the optimal speculation strategy gives the public signal a positive weight that is strictly lower than the Bayesian weight.

Information aggregation in the financial market can efficiently cancel out the conditionally independent noises in speculators' private signals. However, financial market information is always subject to the common noise that is embedded in the public signal. As a result, reducing the weight assigned to the public signal makes the financial market more informative. Compared with the case in which there is no informational feedback and speculators use Bayesian weights, the size of the speculation now becomes a more precise piece of information for voters.

The financial market, however, is not a sideshow. Rather than merely serving as a source of information, the financial market has a real impact on political outcomes via informational feedback. Since speculators trade on the public signal, the realization of the common noise can change the size of the speculation, ex post. Because of voters' inability to observe this noise directly, they cannot tell whether the change in the size of the speculation is due to this noise or the fundamental. Therefore, the realization of this noise determines voters' posterior beliefs, which, in turn, influences their decision making and the ultimate election outcome.

Ex post, the common noise that results in a larger- (smaller-) sized speculation will discourage (encourage) protesting voters from voting (to vote), potentially shifting the election outcome from one involving a political regime change to one with no change (and vice versa). In this sense, this common noise is likely to "mislead" speculators, which is consistent with the fact that speculators would trade less on the public signal. Finally, we show that, when the weight assigned to the public signal is lower, as the size of the speculation becomes a more precise piece of information, voters become more responsive to that information. This, in turn, amplifies the discouragement effect and makes the financial market's information more influential in affecting the political outcome, ex post.

Our theoretical findings correspond well to observations that the financial market moves in the opposite direction of a voting outcome that is difficult to explain solely on the basis of economic or political fundamentals. Going back to the example of Brexit, our theory offers a *possible* mechanism that could reconcile the aforementioned observations about the voting outcome as well as the financial market fluctuations around the referendum. According to our model, the financial markets' stability prior to Brexit reassured voters that their vote would be a protest vote rather than a vote to leave the EU. Protesting voters' decision to vote for Brexit was aided by this belief. If this were the rationale behind the Brexit vote, then protesting voters would have been discouraged, and the referendum could have resulted in the opposite outcome if the financial market's information had induced speculators to short the GBP more aggressively prior to the election.

Related literature Our theory is based on the seminal framework of protest voting built by Myatt (2016), which we extend to include information feedback between financial speculators and protesting voters. In this way, our paper contributes to the growing literature on protest voting (see, for example, Kselman and Niou, 2011, Alvarez et al., 2018, Birch and Dennison, 2019, Louis et al., 2020, and Van der Brug et al., 2000). In contrast to these papers, we focus on how financial market information shapes voters' decision making and how this interaction may explain why protesting voters receive an unfavorable electoral outcome.

At the core of our result is that public information indicating optimism for a political regime change discourages protest voting, thereby making this political outcome improbable. In a related study, Taylor and Yildirim (2010) find that perfect disclosure of the electorate's composition discourages (encourages) individuals who expect to be in the majority (minority) from voting (to vote). Therefore, with such information, the chance that the majority will win is reduced, and elections are more likely to be close. It is important to note that the underlying mechanism in our paper is different from that in Taylor and Yildirim (2010) in terms of how public information affects voters' decision making. The information in our

[6] Notably, there is no informational feedback when traders are unable to observe financial market information, even though speculators trade on the voting outcome. In this case, as shown in Lemma 4, adopting Bayesian weights is the optimal choice for speculators.

model changes protesting voters' estimated chances that their vote will be pivotal in two distinct pivotal states. In contrast, in the costly strategic voting model of Taylor and Yildirim (2010), information about other voters' preferences affects one's incentives to abstain (or free ride) and, consequently, affects voter turnout. In addition, the public information in our model is endogenously generated through financial market speculation, and it reveals only partial information about other voters' preferences.

Our paper also contributes to the literature on speculative currency attacks. This literature assumes direct payoff complementarity between speculators and highlights the self-fulfilling prophecy of speculative attacks (e.g., Obstfeld, 1996 and Morris and Song Shin, 1998).[7] A notable exception is Goldstein et al. (2011), which assumes away this direct payoff complementarity and considers the case in which a central bank learns from the speculative attack and endogenously decides to abandon a fixed exchange rate. Goldstein et al. (2011) establish the informational complementarity among speculators and show that, in equilibrium, speculators overweight the public signal. Similarly, our model does not include any direct payoff externalities between speculators; however, we examine a situation in which speculative currency attacks can provide useful information to a large number of voters. In contrast to previous research, we find that speculators' actions feature strategic substitutability in such an environment. According to Hellwig and Veldkamp (2009), under strategic substitutability, agents will differentiate their informational choices. Consistent with their findings, our model demonstrates that speculators will do so by underweighting the correlated public signal and trading more on the conditionally independent private signals.

Our paper is also related to the global game literature, beginning with Carlsson and Van Damme (1993), who introduce conditionally independent private signals for equilibrium selection in games with *strategic complementarity*. The analysis of global games with public information was pioneered by Hellwig (2002) and Morris and Song Shin (2002), and the role of *endogenous* public information has been investigated by Angeletos and Werning (2006), Angeletos et al. (2006), Hellwig et al. (2006), and Ozdenoren and Yuan (2008). Notably, despite their similar information structures, protest voting is a game of *strategic substitutes*, whereas global games feature *strategic complementarity*. In global games, public information homogenizes agents' beliefs and thus may restore the multiplicity of equilibria. Public information, however, does not induce multiplicity of equilibrium in the protest voting game. Depending on its realization, it either discourages or encourages all voters to vote, thereby influencing the outcome of the vote.

Finally, our paper belongs to the literature on the interaction between financial markets and political elections. While the majority of studies (e.g., Snowberg et al., 2007 and Wagner et al., 2018) investigate how elections and the associated policy uncertainties affect asset prices in financial markets, our focus is different.[8] We study the informational feedback between voters and speculators and highlight the real impact of financial markets on political outcomes. In this way, our study also relates to the literature on feedback effects in financial economics; that is, financial markets not only reflect but also affect the cash flows generated by traded assets.[9] To the best of our knowledge, this paper is the first to extend the feedback effect to voters' decision making and to use it to gain a better understanding of the political economy.

Outline The remainder of the paper is organized as follows. Section 2 presents the model setup and the solution concepts we adopt. We solve the model and discuss the theoretical implications in Section 3. Section 4 discusses the comparative statics results and the robustness and limitations of our results by considering more general environments regarding voter preferences and alternative information structures. Finally, Section 5 concludes. The omitted proofs are relegated to the appendix.

2. Model setup

The game lasts for two periods, $t = 1, 2$, and there are two separate groups of risk-neutral players: voters and speculative traders. At $t = 1$, traders speculate on the occurrence of political regime change. At the beginning of $t = 2$, voters receive information from the financial market and then decide whether or not to cast a vote. At the end of $t = 2$, the status of the political regime is determined by the voting game, and, accordingly, the payoffs to the voters and speculators are realized.

Below, we spell out the details about voting, financial market speculation, and the interaction between the two.

2.1. The voting game

There are N voters. Each voter i decides whether to cast a vote that opposes the incumbent political regime ($a_i = 1$) or not to vote ($a_i = 0$). We consider protest voting à la Myatt (2016), in which voters are not satisfied with the current policies. They would like to use their votes to voice this dissatisfaction and force the incumbent to conduct necessary policy

[7] The direct payoff complementarity is motivated by the fact that a central bank has limited foreign reserves to withstand speculative attacks. As such, a greater amount of speculation can increase the chance that a central bank will abandon its fixed change rate regime, and, thus, this can increase the incentive for these attacks.

[8] An exception is Musto and Yilmaz (2003). They investigate the role of financial markets in insuring the risk associated with elections and the subsequent redistribution of wealth. They find that because complete financial markets induce complete insurance, all voters, regardless of wealth, are indifferent toward alternative redistribution policies. As a result, wealth has no bearing on electoral outcomes.

[9] For example, see Goldstein et al. (2013) for the feedback effect on real investment; see Edmans et al. (2015) for the feedback effect on the limits of arbitrage, and see Bond and Goldstein (2015) for the feedback effect on government policy. Also, see Bond et al. (2012) for a survey of this literature.

reforms. However, if too many opposing votes are cast, then the political regime will change and will be replaced by political extremism (e.g., an ideological party will come into power).

The outcome of voting, denoted by $e(a_i, a_{-i})$, is determined by voter i's choice as well as by that of other voters, $a_{-i} \equiv (a_l)_{l \neq i}$. We denote the total number of votes as $M \equiv \#\{i | a_i = 1\}$, and, accordingly, the vote share is $m \equiv \frac{M}{N}$. There are three possible outcomes $e(a_i, a_{-i}) \in \{\mathcal{D}, \mathcal{R}, \mathcal{E}\}$ depending on m:

1. $\mathcal{D}$efault (status quo — no policy reform and no political regime change) if $m < p_l$;
2. $\mathcal{R}$eform (necessary policy reform but no political regime change) if $m \in [p_l, p_h)$;
3. $\mathcal{E}$xtremism (political regime change and radical policy shift) if $m \geq p_h$.

The parameters p_l and p_h, determined by the difficulty of calling for policy reform and voting rules, respectively, satisfy that $0 < p_l < p_h < 1$.

The voting outcome e yields U_i^e to voter i, and, accordingly, we can write voter i's payoff as

$$v_i(a_i, a_{-i}) = \sum_{e' = \mathcal{D}, \mathcal{R}, \mathcal{E}} \mathbb{1}\{e(a_i, a_{-i}) = e'\} U_i^{e'}. \tag{1}$$

We assume that policy reform (outcome $\mathcal{R}$) is the most favorable outcome to all voters; that is, $U_i^{\mathcal{R}} > \max\{U_i^{\mathcal{D}}, U_i^{\mathcal{E}}\}$ holds for all i.[10] To simplify our analysis, we introduce u_i to capture the preference of voter i,

$$u_i \equiv \ln \left[\frac{U_i^{\mathcal{R}} - U_i^{\mathcal{D}}}{U_i^{\mathcal{R}} - U_i^{\mathcal{E}}} \right]. \tag{2}$$

Note that u_i increases with $U_i^{\mathcal{E}}$, and, in this sense, it measures the relative preference for a political regime change. Moreover, we assume that u_i is distributed following a commonly known normal distribution $\mathcal{N}(\theta, \sigma_u^2)$, in which θ captures the average voter preference and σ_u scales the dispersion of voters' preferences. The mean of the voter's preference distribution θ is a key parameter in our analysis. In general, the higher the θ, the more acceptable the political regime change to the electorate is. As a result, voters are more willing to protest by casting an opposing vote. In this sense, we can interpret θ as the "fundamental" of the economy, and it measures the weakness of the political regime.

As in many strategic voting models, a voter's choice a_i matters only when they are a pivotal voter. However, since each voter only privately knows u_i but does not perfectly know other voters' preferences, they never know for sure whether their vote will be pivotal. It is worth noting that in our setting, since there are two tipping points (p_l and p_h), a protesting voter can be pivotal in two different ways: their vote may change the political outcome from the status quo ($e = \mathcal{D}$) to policy reform ($e = \mathcal{R}$), and it may also enable the regime change ($e = \mathcal{E}$). As such, what matters for voters' decision making is the relative ratio between these two possibilities of being pivotal, which is well defined and obtains a strictly positive value even when $N \to \infty$.[11] Therefore, for tractability, we will focus our attention on the case with infinite voters throughout the paper.

Apart from being a preference parameter, u_i serves an informational role to help voters gauge the average voter's preference θ. For simplicity, we assume an improper prior of θ; that is, θ is distributed uniformly over the real line.[12] More specifically, voter i with u_i has a subjective distribution on θ with cumulative distribution function (CDF) $\Phi(\frac{\theta - u_i}{\sigma_u})$, and the informativeness of u_i is denoted as $\tau_u \equiv \frac{1}{\sigma_u^2}$.[13] In addition, as we will soon introduce, voters can better learn θ from a noisy public signal μ from the financial market. This information helps voter i better predict their chance of being a pivotal voter. Therefore, the voter's strategy can be represented by $a_i(u_i, \mu) \in \{0, 1\}$ for all u_i and μ.

2.2. Financial market speculation

There is a unit mass of speculators in the financial market, indexed by $j \in [0, 1]$. They can be considered foreign investors or institutions that do not have the right to vote. The speculators trade on a political regime change ($e = \mathcal{E}$) and aim to profit from the significant depreciation of the domestic currency after the change in the political regime and the subsequent implementation of radical policies. For example, speculators understand that Brexit will result in a devaluation of the British pound sterling, so if they anticipate that the UK will vote to leave the EU, then they will short this currency prior to the Brexit vote.

[10] In Section 4.2, we discuss the robustness and limitation of our results if other types of voters are present in this voting game.

[11] This probability is derived in (A.2) in the appendix. For the property of the pivotal probability when $N \to \infty$, see Lemma 1 in Myatt (2016) for details.

[12] This assumption is a standard one in the existing literature that considers an information environment with dispersed private signals. Alternatively, one can assume that the prior distribution is $\theta \sim \mathcal{N}(\theta_0, \sigma_\theta^2)$ with $\sigma_\theta \to \infty$.

[13] We use $\Phi(.)$ and $\phi(.)$ to denote the cumulative distribution function (CDF) and probability density function (PDF) of the standard normal distribution, respectively.

Each speculator j at $t = 1$ chooses whether to short the currency ($d_j = 1$) or not ($d_j = 0$). We normalize the cost of speculation to $c \in (0, 1)$ and the return from a successful short position to 1. Therefore, for any political outcome e, the payoff of speculator j can be written as $\pi_j(d_j, e) = d_j (\mathbb{1}\{e = \mathcal{E}\} - c)$.

Speculators share with voters the improper common prior about the fundamental θ. Each speculator observes two noisy signals about θ: (1) a private signal, $s_j = \theta + \sigma_s \varepsilon_j$, and (2) a public signal, $s_p = \theta + \sigma_p \varepsilon_p$. We assume that all idiosyncratic noises $\{\varepsilon_j\}_j$ and the common noise ε_p are independently drawn from the standard normal distribution $\mathcal{N}(0, 1)$. The noisiness of private signal s_j and that of public signal s_p are scaled by σ_s and σ_p, respectively. Accordingly, the precision of s_i (s_p) is denoted as $\tau_s \equiv \frac{1}{\sigma_s^2}$ ($\tau_p \equiv \frac{1}{\sigma_p^2}$). Therefore, the strategy of speculator j can be represented by $d_j(s_j, s_p) \in \{0, 1\}$ for all s_j and s_p.

Furthermore, the size of the speculation is denoted by $A \equiv \int_0^1 \mathbb{1}\{d_j = 1\} \mathrm{d}j$. We assume that, at the beginning of $t = 2$, voters can observe the size of speculation A. As will be clear in Section 2.4, all relevant information regarding the fundamental θ revealed by the speculative size A can be summarized in a public signal μ.

2.3. Solution concept and equilibrium definition

We adopt a *perfect Bayesian equilibrium* (PBE) *in symmetric pure and linear monotone strategies* as our solution concept. More specifically, we restrict our attention to symmetric pure strategies — that is, in equilibrium, the voting strategy $a_i(u_i, \mu) = a(u_i, \mu) \in \{0, 1\}$ and the strategy of speculation $d_j(s_j, s_p) = d(s_j, s_p) \in \{0, 1\}$. Moreover, following Myatt (2016), we restrict the equilibrium strategy in voting to be *monotonic*; that is, for any u' and μ, if $a(u', \mu) = 1$, then $a(u'', \mu) = 1$ for any $u'' > u'$. As such, we can write the voting strategy as $a(u_i, \mu) = \mathbb{1}\{u_i \geq \hat{u}(\mu)\}$ for some mapping $\hat{u}(\cdot)$ that will be solved in equilibrium. Lastly, in the game of speculation, we consider *linear monotone strategies*; that is, $d(s_j, s_p) = \mathbb{1}\{s_j + ks_p \geq s_0\}$ for constants k and s_0, which will be solved in equilibrium.[14]

Definition 1 *(Equilibrium).* The voting strategy profile $\{a_i = a^*(u_i, \mu)\}_i$, the speculation strategy profile $\{d_j = d^*(s_j, s_p))\}_j$, and a posterior belief distribution $G(\cdot|u_i, \mu)$ constitute an equilibrium if

1. based on the strategy profile of speculation $\{d_j = d^*\}_j$, the observation of A can be mapped to a noisy public signal $\mu = \mu_k(A)$, and voter i's posterior belief can be captured by $G(\theta|u_i, \mu)$, which is obtained via Bayes' rule[15];
2. given the strategy profile of other voters $a_{-i} = \{a_l = a^*(u_i, \mu)\}_{l \neq i}$, the speculation strategy profile, $\{d_j = d^*(s_j, s_p)\}_j$, and the posterior belief $G(\cdot|u_i, \mu)$, the monotone strategy is
$$a^*(u_i, \mu) \in \arg\max_{a_i} \mathbb{E}_G[v_i(a_i, a_{-i})|u_i, \mu];$$
3. given voting strategy profile $\{a_i = a(u_i, \mu)\}_i$, and the posterior belief $G(\cdot|u_i, \mu)$, the linear monotone strategy is
$$d^*(s_j, s_p) \in \arg\max_{d_j} \mathbb{E}\left[\pi_j(d_j, e)|s_j, s_p\right].$$

2.4. Interaction between trading and voting

Before solving for the equilibrium, we want to first understand the interaction between the voting game and speculative trading.

2.4.1. Learning from the financial market

Financial market speculators trade on information (s_j and s_p) that voters cannot observe. Therefore, the size of speculation A reveals useful information about the fundamental θ. Consider any equilibrium strategy of speculation $d(s_j, s_p) = \mathbb{1}\{s_j + ks_p \geq s_0\}$. We first solve for the public signal $\mu(A)$ and the posterior distribution $G(\theta|u_i, \mu(A))$. Under this strategy, the size of speculation A, conditional on the fundamental θ and the common noise ε_p, is[16]

$$A(\theta, \varepsilon_p) = \int_0^1 \mathbb{1}\{s_j + ks_p \geq s_0\} \mathrm{d}j = \Phi\left(\sqrt{\tau_s}\left[(1+k)\theta + k\sigma_p\varepsilon_p - s_0\right]\right). \tag{3}$$

[14] Given that the noises are all Gaussian and the speculators are risk-neutral, restricting our attention to a linear strategy is standard in the literature (e.g., Goldstein et al., 2011).

[15] Note that, in our setting, any restriction on the off-path beliefs is not needed because any $A \in [0, 1]$ is possible under a linear monotone strategy $d = \mathbb{1}\{s_i + ks_q \geq s_0\}$, provided that the realization of θ can be any number over the real line.

[16] It is worth noting that if the speculators choose $k = -1$, then A is not informative at all about θ but only reflects the noise term ε_p (see (3)). Therefore, the learning channel is muted. We will proceed and focus our attention on the cases in which $k \neq -1$ and, as will soon be clear, $k = -1$ cannot be part of any equilibrium.

Next, we define a new mapping $\mu_k(\cdot)$ based on the speculator's choice k as follows[17]:

$$\mu_k(A) \equiv \frac{1}{1+k}\left(\frac{1}{\sqrt{\tau_s}}\Phi^{-1}(A) + s_0\right). \tag{4}$$

Based on expression (3), all relevant information about the fundamental θ revealed by the size of speculation A is summarized in the noisy public signal, $\mu = \mu_k(A)$; that is,

$$\mu = \mu_k(A(\theta, \varepsilon_p)) = \theta + \frac{k}{1+k}\sigma_p \varepsilon_p. \tag{5}$$

We denote the precision of this public signal as τ_μ. This precision essentially measures the efficiency with which voters learn from financial speculation or, equivalently, the informativeness of the financial market. It depends on equilibrium weight k and the precision of public signal τ_p as follows:

$$\tau_\mu = \tau_\mu(k) \equiv (\frac{1+k}{k})^2 \tau_p. \tag{6}$$

The following lemma summarizes how voters update their beliefs about θ after observing the size of speculation A.

Lemma 1 *(Learning and Belief Updating). Given any strategy of speculation $d = \mathbb{1}\{s_j + ks_q \geq s_0\}$, for any u_i and A, the posterior belief of θ is $\theta|u_i, \mu \sim \mathcal{N}(\frac{\tau_u u_i + \tau_\mu \mu}{\tau_u + \tau_\mu}, \frac{1}{\tau_u + \tau_\mu})$. Therefore,*

$$G(\theta|u_i, \mu) = \Phi\left(\sqrt{\tau_u + \tau_\mu}\left[\theta - \frac{\tau_u u_i + \tau_\mu \mu}{\tau_u + \tau_\mu}\right]\right), \tag{7}$$

in which $\mu = \mu_k(A)$ is given in (4) and $\tau_\mu = \tau_\mu(k)$ is given in (6).

2.4.2. Informational feedback

Thus far, we have understood that voters' learning relies on how speculators trade on their information; that is, the posterior belief G depends on the speculator's choice of k through the mapping of $\mu_k(\cdot)$. However, the interaction between voting and speculative trading is more than the flow of information from the financial market to the voters. Indeed, because speculators trade in order to profit from a political regime change, which is dependent on voters' collective decision making, the way voters learn (i.e., $\mu = \mu_k(A)$) and respond (i.e., the strategy $a(\cdot, \mu)$) to financial market information shapes the nature of speculators' strategic interactions and ultimately determines their informational choices. This two-way interaction between voters' responses to financial market information and speculators' informational choices defines the *informational feedback* between voting and speculative trading.

Lemma 2. *In equilibrium with voting strategy $a(u_i, \mu) = \mathbb{1}\{u_i \geq \hat{u}(\mu)\}$ and speculation strategy $d(s_j, s_p) = \mathbb{1}\{s_j + ks_p \geq s_0\}$, if $\hat{u}(\mu_k(A))$ increases (decreases) with A, then financial market speculation features strategic substitutability (complementarity).*

Lemma 2 demonstrates how this informational feedback determines whether the game of speculative attack is a game of strategic substitutes or a game of strategic complements. To understand the intuition behind this result, consider the case in which $\hat{u}(\mu_k(A))$ increases with A in equilibrium. Given this, if speculators attack more aggressively, resulting in a higher A,[18] voter i will be less likely to vote and, thus, the political regime will be less likely to change. Consequently, the speculator's incentive to attack decreases with others' attacks, thus making speculative attacks strategic substitutes. After solving the equilibrium strategies, we will come back to discuss how informational feedback shapes the nature of strategic interaction among speculators *in equilibrium*.

3. Equilibrium analysis

To solve the model, we begin by determining the equilibrium voting strategy $a^*(u_i, \mu)$ for any given speculator's strategy $d(s_j, s_p)$. Based on that, the equilibrium strategy of speculation $d^*(s_j, s_p)$ is then solved. According to the entire equilibrium, in this section, we further discuss the ex ante informativeness of the financial market and its ex post impact on the voting outcome.

[17] We use the subscript k to highlight the dependence of public signal μ on the speculator's choice of k. It should be clear that this public signal also depends on the choice of s_0 in equilibrium (see (4)). However, unlike k, the choice of s_0 never changes the meaning of the public signal, $\mu = \mu_k(A)$ (see (5)). Also see footnote 20 for a further discussion about why other speculators' choice of s_0 does not change a speculator's optimal choice of this constant.

[18] It is worth noting that, because each speculator is infinitesimally small, their decision cannot affect the aggregate variable A on its own. As a result, the aggregate size of speculation A is determined by the actions of other speculators.

3.1. Voting strategy in equilibrium

Each voter i uses both the public signal μ and the private valuation u_i to infer the electorate's composition, captured by the unknown parameter θ. Given any monotone strategy $\hat{u}(\mu)$ adopted by other voters, voter i forms a belief about the likelihood that they will be the pivotal voter for a political regime change ($e = \mathcal{E}$) as well as for the reform ($e = \mathcal{R}$). They cast a vote $a_i(u_i, \mu) = 1$ if and only if the expected payoff from voting dominates that from not voting, or[19]

$$\left(1 - \frac{z_h - z_l}{\sigma_u}\right) u_i + \frac{\tau_\mu}{2\tau_u}(z_h^2 - z_l^2) - \sigma_u \left[\tau_\mu \mu - (\tau_u + \tau_\mu)\hat{u}(\mu)\right](z_h - z_l) \geq 0, \tag{8}$$

in which $z_l \equiv \Phi^{-1}(p_l) < z_h \equiv \Phi^{-1}(p_h)$. In a symmetric equilibrium, the expected payoff difference, represented by the left-hand side of (8), must increase with u_i and must be zero for $u_i = \hat{u}(\mu)$. This gives rise to a necessary condition $\sigma_u > z_h - z_l$, under which there exists a unique equilibrium strategy, characterized by

$$u^*(\mu) \equiv \frac{\tau_\mu \sigma_u (z_h - z_l)}{1 + \tau_\mu \sigma_u (z_h - z_l)} \left[\mu - \frac{\sigma_u}{2}(z_h + z_l)\right]. \tag{9}$$

For ease of exposition, denote $\Lambda \equiv \sigma_u(z_h - z_l) > 0$ and $\delta(\tau_\mu) \equiv \frac{\tau_\mu \Lambda}{1+\tau_\mu \Lambda} \in [0, 1]$.

Lemma 3 *(Equilibrium Voting Strategy). Under the condition $\sigma_u > z_h - z_l$, for any strategy of speculation $d(s_j, s_p) = \mathbb{1}\{s_j + ks_p \geq s_0\}$, there exists a unique voting strategy in equilibrium; that is, $a_i^*(u_i, \mu) = 1$ if and only if $u_i \geq u^*(\mu) = \delta(\tau_\mu)\left[\mu - \frac{\sigma_u}{2}(z_h + z_l)\right]$.*

By Lemma 1, when the realization of $\mu = \mu_k(A)$ is higher, voters expect θ to be greater. This means that an average voter would suffer less from a political regime change. If there were no strategic concerns in voting, then more agents would vote and the regime would be more likely to change. However, in a protest voting game, as Lemma 3 shows, a higher realization of μ discourages voters from casting a vote.

This property relies critically on the *strategic substitutability* in protest voting. If other voters are more willing to cast a vote (i.e., $\hat{u}(\mu)$ decreases), then the expected payoff from casting a vote will decrease (see the left-hand side of (8)). To understand this, first observe that if there is a larger vote share, then voter i's vote is more likely to contribute to a political regime change ($e = \mathcal{E}$) — an unfavorable outcome — rather than to policy reform ($e = \mathcal{R}$) — the most preferred outcome. Therefore, an increase in the voting share m reduces protesting voters' incentive to cast a vote. Note that, under any monotone strategy, a higher public signal μ and the resulting more optimistic posterior belief of θ can be translated to a higher expectation of the voting share m. Therefore, such an expectation would discourage voting. We refer to this impact of the public signal μ on the incentive to vote as the *discouragement effect*.

The magnitude of this discouragement effect is $\delta(\tau_\mu) \in [0, 1]$. It is intuitive that when the financial market information is more precise, learning becomes more effective and, therefore, the discouragement effect becomes more significant. For that reason, $\delta(\cdot)$ is an increasing function. Had there been no learning from the financial market, there would be no discouragement effect (i.e., $\delta(\tau_\mu = 0) = 0$). However, as long as the financial market can generate some information for voters (i.e., $\tau_\mu > 0$), the discouragement effect is in force (i.e., $\delta(\tau_\mu) > 0$).

Finally, the existence of the voting equilibrium at $t = 2$ relies on the parameter condition $\sigma_u > z_h - z_l$. To understand this, first recall that voter's type u_i is informative about the average preference θ. Because of this informational role, a higher u_i, similar to a greater μ, discourages voting. This impact is of size $\frac{z_h - z_l}{\sigma_u}$ (see (8)). Next, note that u_i is a preference parameter dictating voters' payoffs from different political outcomes. Therefore, the expected payoff difference (the left-hand side of (8)) strictly increases with u_i only when $\sigma_u > z_h - z_l$, under which the discouragement effect from u_i is strictly dominated by its direct impact on the payoff of voter i.

Based on the solved equilibrium strategy, the vote share m, conditional on θ and μ, is

$$m(\theta, \mu) = \int_i \mathbb{1}\{u_i \geq u^*(\mu)\} \mathrm{d}i = \Phi(\sqrt{\tau_u}(\theta - u^*(\mu))). \tag{10}$$

Accordingly, the following proposition establishes how the political outcome depends on the realization of the fundamental θ.

Proposition 1 *(Political Outcomes). Given any strategy of speculation $d(s_j, s_p) = \mathbb{1}\{s_j + ks_p \geq s_0\}$, under strategy $a_i = a^*(u_i, \mu_k(A))$, the political regime changes ($e = \mathcal{E}$) if and only if*

$$\theta \geq \theta^*(\mu_k(A), \tau_\mu) \equiv u^*(\mu_k(A)) + \sigma_u z_h = \delta(\tau_\mu)\left[\mu_k(A) - \frac{\sigma_u}{2}(z_h + z_l)\right] + \sigma_u z_h. \tag{11}$$

Accordingly, the reform ($e = \mathcal{R}$) takes place if and only if $\theta \in \left[u^*(\mu_k(A)) + \sigma_u z_l,\ \theta^*(\mu_k(A), \tau_\mu)\right)$.

[19] For the derivation of this condition, see the proof of Lemma 3 in the appendix for details.

Proposition 1 gives the political regime change condition (11). From that condition, it should be clear that a higher realization of μ discourages voting and makes a political regime change less likely to occur. For speculators, shorting the domestic currency is profitable only if condition (11) holds. Therefore, this condition plays a vital role in solving the equilibrium speculation strategies.

3.2. Speculator's strategy in equilibrium

Let us now turn to solving for the speculator's strategy in equilibrium. When deciding how to trade on the private signal s_j and public signal s_p, speculators understand that the size of speculation A will be observed by voters and will affect the political regime change (see (11)). More formally, for any given strategy $q_l = \mathbb{1}\{s_l + ks_p \geq s_0\}$ chosen by other speculators, speculator j understands that, from observing the size of speculation A, voters will learn

$$\mu = \mu_k(A) = \theta + \frac{k}{1+k}\sigma_p\varepsilon_p = \theta + \frac{k}{1+k}(s_p - \theta). \tag{12}$$

Compared with the private signal s_j, which only informs the speculators about the realization of θ, the correlated public signal s_p provides additional information about the beliefs of other speculators and, thus, better predicts the size of speculation A and the realization of public signal μ.

Based on their posterior beliefs about θ and the realization of μ, speculator j would choose to attack if and only if the perceived probability of a regime change is greater than c. This best response can be simplified to $d_j(s_j, s_p) = \mathbb{1}\{s_j + b(k)s_p \geq s_0(k)\}$,[20,21] in which the weight placed on the public signal is

$$b(k) = B(k, \tau_\mu(k)) \equiv \frac{\tau_p}{\tau_s} - \frac{\tau_p + \tau_s}{\tau_s}\frac{\delta(\tau_\mu(k))\frac{k}{1+k}}{1 - \delta(\tau_\mu(k))\frac{1}{1+k}}, \tag{13}$$

and the constant is

$$s_0(k) \equiv \frac{\sqrt{\tau_p + \tau_s}}{\tau_s}\Phi^{-1}(c) + \frac{\tau_s + \tau_p}{\tau_s}\frac{\sigma_u z_h - \frac{\sigma_u}{2}\delta(\tau_\mu(k))(z_h + z_l)}{1 - \delta(\tau_\mu(k))\frac{1}{1+k}}. \tag{14}$$

It is important to note that other speculators' choices of the weight placed on s_p (i.e., k) determine the correlation between this public signal and the aggregate size of the speculation (or, equivalently, the realization of public signal μ) that voters observe (see (12)). Furthermore, because of the discouragement effect of public signal μ, it negatively predicts the political regime change that the speculators trade on (see (11)). That explains speculator j's best response $b(k)$ to others' choices of k.

To better understand how informational feedback shapes the strategic interaction between speculators, we introduce a new function $B(k, \tau_\mu)$ to separate two independent channels through which one's choice is affected by the choices of others. The next lemma gives the key properties of this best-response function $B(k, \tau_\mu)$.

Lemma 4 *(Best-Response Function). Given the other speculators' choice k, speculator j's optimal choice of the weight assigned to public signal s_p is $b(k) = B(k, \tau_\mu(k))$, which is defined in (13). This best-response function has the following properties:*

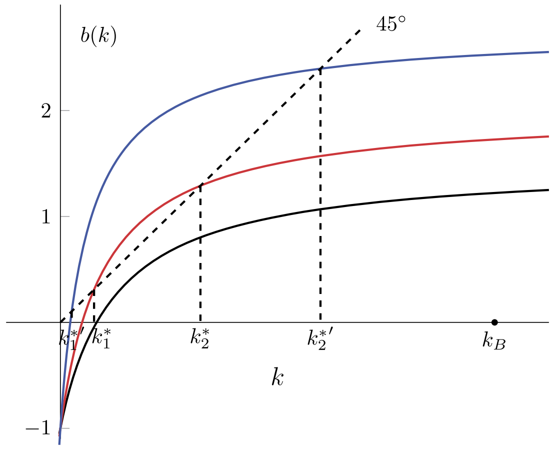
1. *(The effect from the magnitude of discouragement) For any given $k > 0$, $\frac{\partial B(k,\tau_\mu)}{\partial \tau_\mu} < 0$; in addition, $\lim_{\tau_\mu \to 0} B(k, \tau_\mu) = k_B \equiv \frac{\tau_p}{\tau_s}$ and $\lim_{\tau_\mu \to +\infty} B(k, \tau_\mu) = -1$.*
2. *(Informational substitutability) Given any $\tau_\mu > 0$, $\frac{\partial B(k,\tau_\mu)}{\partial k} < 0$ and $B(k, \tau_\mu) < k_B$ for any $k > 0$.*
3. *The aggregate impact of k on b(k) features $\frac{\mathrm{d}b(k)}{\mathrm{d}k} = \frac{\partial B(k,\tau_\mu)}{\partial k} + \frac{\partial B(k,\tau_\mu)}{\partial \tau_\mu}\frac{\mathrm{d}\tau_\mu(k)}{\mathrm{d}k} > 0$. Moreover, $b(0) = -1$ and $b(k)$ is concave for $k > 0$ with $\lim_{k \to +\infty} b'(k) = 0$.*[22]

The first property in Lemma 4 shows how the best response $b(k)$ depends on the efficiency of voters' learning τ_μ. Recall that the public signal μ affects the regime change condition through its discouragement effect, the magnitude of which,

[20] As will be clear in the proof of Lemma 4 in the appendix, the speculator's best response is independent of other speculators' choice of s_0. This is because both voters' posterior belief (see (7)) and the regime change condition do not depend on s_0.

[21] Note that if we consider that the linear monotone strategy of speculation is in the form of $d_j = d(s_j, s_p) = \mathbb{1}\{s_j + ks_p < s_0\}$, then the aggregated information about θ that is contained in the observation of A will remain the same. More formally, the information that can be extracted from the observation of A is $\mu = \mu'_k(A) \equiv \frac{1}{1+k}(s_0 - \frac{1}{\sqrt{\tau_s}}\Phi^{-1}(A)) = \theta + \frac{k}{1+k}\sigma_p\varepsilon_p$. As such, the condition for a regime change remains the same, and the best-response function will be in the form of $s_j + b(k)s_p \geq s_0$. Therefore, we cannot find any symmetric equilibrium in the form of $d(s_j, s_p) = \mathbb{1}\{s_j + ks_p < s_0\}$.

[22] More precisely, as we show in the appendix, $b(k)$ is strictly increasing and differentiable in the range of $(-\infty, k_0)$ and $(k_0, +\infty)$, in which $k_0 = -\frac{\tau_p\Lambda}{1+\tau_p\Lambda} \in (-1, 0)$. However, it is not continuous at $k = k_0$, whereby $\lim_{k\uparrow k_0} b(k) = +\infty$ and $\lim_{k\downarrow k_0} b(k) = -\infty$. In addition, when others choose $k \to 0$, $\tau_\mu(k) \to +\infty$, and based on Property 1, $\lim_{k\to 0} b(k) = -1$.



Note: This figure presents the best-response function $b(k)$ under the parameter values $\sigma_u = 3$, $\sigma_p = 3$, $\sigma_s = 6$, $z_h = 0$, $z_l = -1$ (blue curve), $z_l = -2$ (red curve), and $z_l = -3$ (black curve). The intersection of the blue (or red) curve with the 45 degree line presents the fixed points k^*. (For interpretation of the colors in the figure, the reader is referred to the web version of this article.)

Fig. 1. Function $b(k)$ and Fixed Points k^*.

$\delta(\tau_\mu)$, increases with the precision of this public signal. In a benchmark case in which no learning from the financial market takes place (i.e., $\delta(\tau_\mu = 0) = 0$), the regime change condition (11) is reduced to $\theta \geq \sigma_u z_h$. In the absence of informational feedback, there is no strategic interaction between speculators. Therefore, any speculator will choose a *Bayesian weight* $k_B = \frac{\tau_p}{\tau_s}$ on s_p, regardless of the other speculators' trading strategies. In another extreme case, in which learning is perfect (i.e., $\tau_\mu \to +\infty$), the magnitude of the discouragement effect is maximized at $\delta(\tau_\mu) \to 1$. In this limiting case, the best response is to trade against the public signal s_p to the maximum; that is, the assigned weight is -1.

Suppose that the precision of information τ_μ increases while other speculators' choices of k remain fixed. In this case, the discouragement effect is of a larger magnitude (i.e., $\delta(\tau_\mu)$ increases), and, therefore, μ is more negatively correlated with the political regime change (see (11)). As a result, the speculators who trade on the political regime change would place a lower weight on the public signal s_p, which is positively correlated with the realization of μ. That explains why $B(k, \tau_\mu)$ decreases with τ_μ.

Note that when speculators trade more on the public signal, the public signal μ becomes less informative about θ; that is, $\tau_\mu(k)$ decreases with k (see (6)). Following the logic discussed above, this would induce speculator j to put a higher weight on s_p; that is, $\frac{\partial B(k,\tau_\mu)}{\partial \tau_\mu} \frac{d\tau_\mu(k)}{dk} > 0$. This demonstrates the first channel through which one's optimal choice of weight $b(k)$ is affected by the choices of others. This channel establishes the complementarity in speculators' choices of k, and it involves voters' learning efficiency and the magnitude of the discouragement effect.[23]

The second channel hinges on the substitutability in speculators' informational choices. Assume that τ_μ is fixed, as well as the magnitude of discouragement $\delta(\tau_\mu)$. Suppose that other speculators trade more on the public signal s_p (i.e., k increases). In this case, the size of speculation A and, accordingly, the public signal μ become more positively correlated with s_p (see (12)). Therefore, the likelihood of a regime change becomes more negatively correlated with s_p as a result of the discouragement effect. As a result, any rational speculator would assign a lower weight to s_p; that is, $\frac{\partial B(k,\tau_\mu)}{\partial k} < 0$. This demonstrates the *informational substitutability* among speculators. Because of informational substitutability, as long as other speculators assign a positive weight to s_p, as speculator j has an incentive to trade against this public signal, the optimal weight must be strictly lower than the Bayesian weight k_B. This explains the underlying intuition of property 2 in Lemma 4.

Interestingly, as the third property of Lemma 4 shows, the first channel that involves the magnitude of the discouragement effect always dominates the second one, which relies on this discouragement effect. Hence, the best response $b(k)$ always increases with k, and there is complementarity between speculators' choices of the weight placed on the public signal s_p. When k increases from 0, the complementarity becomes weaker as the discouragement effect weakens. That explains the concavity of $b(k)$.

Fig. 1 depicts the shape of the best-response function $b(k)$. We already know from Lemma 4 that $b(k)$ is an increasing and concave function for $k > 0$ with $b(0) = -1$ and $\lim_{k \to +\infty} b'(k) = 0$. Then, from Fig. 1, it should be clear that, depending

[23] Note that the precision of information provided by the financial market τ_μ in addition to affecting voters' learning, influences voters' decision rules in the strategic voting game through its impact on the magnitude of the discouragement effect $\delta(\tau_\mu)$. This can be seen from (9) and (11), in which the magnitude of the discouragement effect $\delta(\tau_\mu)$ determines voters' strategy and the regime change condition, respectively. In this sense, this channel highlights the difference between the case in which the regime change is determined by a single decision maker (for example, a central bank, as considered in Goldstein et al., 2011) and the case in which it is determined by the collective decision making of multiple agents.

on the parameter values, this best-response function and the 45 degree line may not intersect, or they may interact at two different fixed points.

Next, we explicitly find the parameter condition for the existence of such fixed points and solve for them. Under the parameter condition

$$\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_p}\right) \geq 0, \tag{15}$$

we can find two closed-form solutions k_1^* and k_2^* such that[24]

$$k_1^* \equiv \frac{2\Lambda}{\frac{1}{\tau_s} - 2\Lambda + \sqrt{\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_p}\right)}}, \; k_2^* \equiv \frac{2\Lambda}{\frac{1}{\tau_s} - 2\Lambda - \sqrt{\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_p}\right)}}. \tag{16}$$

Lemma 5 *(Equilibrium Speculation Strategy). Under the parameter condition (15), the speculator's equilibrium strategy is $d_j = 1$ if and only if $s_j + k^* s_p \geq s_0(k^*)$ for $k^* = k_1^*, k_2^*$ given in (16) and $s_0(\cdot)$ given in (14). In addition, $0 < k_1^* < k_2^* < k_B$.*

Equilibrium weight k^* is the focus of our study. It demonstrates how speculators treat the public signal differently from the private signal. As shown in Lemma 5, in equilibrium, speculators put a positive weight on s_p, but this weight is strictly lower than the Bayesian weight k_B that speculators would use if there were no informational feedback.

3.3. Equilibrium of the entire game

With all these preparations, we are ready to present the equilibrium for the entire game, which follows directly from Lemmas 1, 3, and 5. The next proposition demonstrates the existence of two different pure-strategy symmetric equilibria with $k^* = k_1^*, k_2^*$.[25]

Proposition 2 *(Equilibrium). Under the parameter condition $\sigma_u > z_h - z_l$ and (15), there exist two equilibria in which $k^* = k_1^*, k_2^* \in (0, k_B)$ defined in (16). For any equilibrium k^*,*

1. *the speculation strategy at $t = 1$ is $d_j(s_j, s_p) = \mathbb{1}\{s_j + k^* s_p \geq s_0(k^*)\}$, where $s_0(\cdot)$ is given in (14);*
2. *the voting strategy at $t = 2$ is $a_i(u_i, \mu) = \mathbb{1}\{u_i \geq u^*(\mu)\}$, where $u^*(\cdot)$ is given in (9);*
3. *for any observation of A and u_i, the posterior belief of voters $G(\cdot|u_i, A)$ is given in (7), in which $\mu = \mu_{k^*}(A)$ and its precision τ_μ are characterized in (4) and (6).*

Based on the solved equilibrium, the next proposition shows that, with the presence of informational feedback, the financial market speculation is a game of strategic substitutes.

Proposition 3 *(Strategic Substitutability in Speculation). In any equilibrium $k^* = k_1^*, k_2^*$, the game of speculative trading features strategic substitutability.*

This result follows immediately from Lemma 2 and the fact that $k^* > 0$ in equilibrium. Intuitively, if other speculators attack more (i.e., A increases), the realization of $\mu = \mu_{k^*}(A)$ would be greater since $\mu_{k^*}(A)$ is strictly increasing in A for any $k^* > 0$ (see (4)). Therefore, upon observing such a public signal from the financial market, the voters would believe that the average preference θ is likely to be higher (Lemma 1). That discourages voting and reduces the likelihood of a regime change, which, in turn, disincentivizes speculation.

Our finding complements the existing studies on regime change games that emphasize the strategic complementarity among speculators (e.g., Morris and Song Shin, 1998 and Goldstein et al., 2011). We find that the strategic substitutability in protest voting, through informational feedback, leads to strategic substitutability among speculative attacks. This, in turn, incentivizes speculators to underweight the correlated public signal s_p and trade more on the conditionally independent private signal s_i.

Next, based on the solved equilibrium, we discuss how informational feedback changes the financial market's informativeness and why it may bring about an unfavorable political outcome for all protesting voters.

[24] Note that, if the parameter condition (15) does not hold, then there is no real solution; and if (15) holds with equality, then the unique solution is $k_1^* = k_2^* = \frac{2\Lambda}{\frac{1}{\tau_s} - 2\Lambda}$.

[25] It is important to note that since the voters' information updating rule is consistent with the speculation strategy, in each equilibrium, the voters understand the speculators' choice of k^*. Therefore, it is not possible for speculators to randomize over k_1^* and k_2^*.

3.4. Informativeness of the financial market

One critical component of informational feedback is that the voters learn from financial market speculation. The informativeness of the financial market, denoted as ρ_k, can be measured by the precision of the signal μ that summarizes all information voters learn from the financial market; that is,

$$\rho_{k^*} \equiv \tau_\mu(k^*) = (\frac{1+k^*}{k^*})^2 \tau_p. \tag{17}$$

Clearly, the financial market's informativeness is determined by how speculators trade on their information in equilibrium, or k^*.

The next proposition shows that informational feedback makes the financial market more informative, even though the information aggregation is not efficient.

Proposition 4 *(Financial Market's Informativeness). Compared with no learning, informational feedback makes the financial market more informative ex ante; that is, $\rho_{k^*} \in (\rho_{k_B}, +\infty)$ for $k^* = k_1^*, k_2^*$.*

Note that the information in the financial market, if efficiently aggregated, is able to perfectly reveal θ. To see this, regardless of the precision of both the public and private signals τ_p and τ_s if speculators trade only on private signal s_j (i.e., $k = 0$), then the size of the speculation $A(\theta) = \Phi(\sqrt{\tau_s}(\theta - s_0))$ would efficiently aggregate the dispersed private signals so that voters would perfectly learn θ from A (i.e., $\rho_0 = \tau_\mu(k=0) = +\infty$). However, this does not arise in equilibrium.

Moreover, if no learning from the financial market takes place, then each speculator would choose a Bayesian weight k_B on public signal s_p (see Property 1 in Lemma 4), and, therefore, the financial market's informativeness would be ρ_{k_B}. As shown in Lemma 5, with the presence of informational feedback, the weight assigned to the noisy public signal s_p in either equilibrium is lower than k_B. As a result, compared with no learning, information aggregation becomes more efficient in equilibrium.

3.5. Real impact of the financial market

Another critical component of informational feedback is that the financial market can influence the political outcome as voters learn and respond to the information generated by speculative trading. Next, we discuss how the financial market information — in particular, the realization of the common noise ε_p — can have an impact on the political outcome ex post.

Recall that the public signal s_p depends on the true realization of θ and the noise term ε_p. If information aggregation is efficient, then the noise term ε_p should not play any role in influencing the political outcome. However, in equilibrium, since all speculators trade on s_p, the aggregate size of speculation A is sensitive to ε_p. Because voters' decision making depends on the size of the speculation ex post, the realization of ε_p has an impact on the voting outcome. Therefore, with the presence of informational feedback, the financial market is more than just a sideshow as it has a real impact on the political outcome.

Proposition 5 *(Ex Post Regime Change Condition). In either equilibrium ($k^* = k_1^*, k_2^*$) ex post, the political regime changes if and only if*

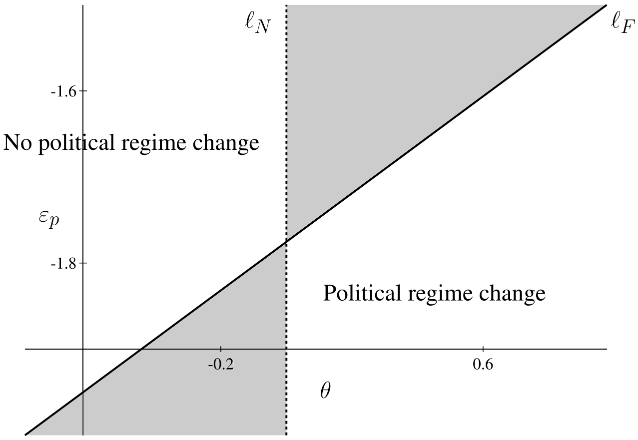
$$\theta \geq \Lambda\sqrt{\rho_{k^*}}\varepsilon_p - \Lambda\rho_{k^*}\frac{\sigma_u}{2}(z_h + z_l) + (1 + \Lambda\rho_{k^*})\sigma_u z_h. \tag{18}$$

Proposition 5 is a natural extension of Proposition 1. It shows that a higher realization of ε_p strengthens the status quo and makes a political regime change less likely. The next corollary can be helpful in understanding the real impact of the common noise ε_p.

Corollary 1 *(Real Impact of Financial Market Information). In either equilibrium ($k^* = k_1^*, k_2^*$), the common noise ε_p positively predicts the size of speculation A (i.e., $\frac{\partial A}{\partial \varepsilon_p} > 0$) and negatively predicts the vote share m (i.e., $\frac{\partial m}{\partial \varepsilon_p} < 0$). In addition, the ex post impact of ε_p on political regime change is of magnitude $\Lambda\sqrt{\rho_{k^*}} > \Lambda\sqrt{\rho_{k_B}}$.*

Everything else being equal, a higher realization of ε_p induces a higher public signal s_p. Upon observing a higher public signal, speculators cannot distinguish the common noise ε_p from the fundamental θ and will speculate more aggressively. Therefore, the materialized size of speculation A will be larger, which, in turn, discourages protest voting and reduces the vote share m. As such, a political regime change is less likely. This explains how the realization of the common noise ε_p, through the learning channel, affects the political outcome, ex post, as shown in Proposition 5.

By Corollary 1, it should be clear that the noise term ε_p, hidden in the public signal s_p, always misguides speculators. Under a higher (lower) realization of ε_p, the size of speculation A will be higher (lower), but political regime change will be less (more) likely. Because of this misguidance, speculators may miss out on a profitable opportunity of shorting the



Note: The solid line ℓ_F presents the ex post regime change condition (18) based on the larger equilibrium weight k_2^*. The dashed line ℓ_N represents this condition without learning (i.e., $\theta \geq \sigma_u z_h$). The shaded areas represent the set of realizations (θ, ε_p) where the common noise ε_p changes the status of the political regime through informational feedback. The parameter values used in this figure are $\sigma_u = 3$, $\sigma_p = 3$, $\sigma_s = 6$, $z_h = 0$, and $z_l = -1$.

Fig. 2. The dependence of political regime change on θ and ε_p.

currency when the political regime changes, or they may mistakenly short the currency when no political regime change occurs. Indeed, this misguidance accounts for why speculators underweight the public signal s_p in equilibrium.

Recall that the informational feedback makes the financial market more informative (i.e., $\rho_{k^*} > \rho_{k_B}$). From condition (18), the ex post impact of ε_p on the political outcome has a magnitude of $\Lambda\sqrt{\rho_{k^*}}$. Therefore, the presence of informational feedback makes the common noise ε_p (or the financial market information) more influential in determining the political outcome.

Fig. 2 depicts how the change of political regime is determined by the realization of fundamental θ and the noise ε_p in the financial market, ex post. The solid line (with a positive slope) ℓ_F in Fig. 2 represents the regime change condition (18) associated with the larger equilibrium weight k_2^*.[26] The political regime changes ($e = \mathcal{E}$) if and only if the realization of (θ, ε_p) lies below and to the right of the solid line ℓ_F. If no learning from the financial market occurs (i.e., $\rho_k = 0$), then the regime change condition reduces to $\theta \geq \sigma_u z_h$, which is represented by the dotted vertical line ℓ_N in Fig. 2.

Provided that the realization of θ is sufficiently high (low), ex post, the political regime will change (not change) if no learning from the financial market has taken place. However, with the presence of informational feedback, the political regime will not change (will change) if the realization of ε_p is sufficiently high (low). The shaded areas in Fig. 2 represent the set of realizations (θ, ε_p) under which the realization of the noise ε_p plays a dominant role in changing the voting outcome. These regions demonstrate the real impact of the financial market: the common noise ε_p contributes to an unfavorable political outcome that goes against the average voter's preference θ.

The following example illustrates the intuition of how financial market information (or, in particular, the realization of the noise ε_p) might influence the voting outcome through informational feedback. This example also demonstrates a *possible* mechanism that could help explain the unexpected success of Brexit and the resulting market volatility.

Example 1. Consider the case in which the average voter's preference $\theta < \sigma_u z_h$; that is, the realization of (θ, ε_p) lies to the left of line ℓ_N in Fig. 2. In this case, some protesting voters (with sufficiently high u_i) would choose to vote. However, if all voters voted purely based on their own preferences, a political regime change would never take place. Suppose that, prior to the vote, speculators in the financial market had a low propensity for shorting the domestic currency — not because they were informed but because the common noise ε_p has a low realization. This would lead to a stable financial market with a small-sized speculation A. The stability in the financial market, to some extent, would have reassured voters that a political regime change would be unlikely to gain enough support. In other words, their votes would have been unlikely to contribute to political regime change but would likely have been a call for necessary policy reforms. This, in turn, would have encouraged the protesting voters to cast a vote and may eventually have led to a political regime change. This can be seen from Fig. 2. As θ lies to the left of line ℓ_N, under a sufficiently low realization of the noise ε_p, (θ, ε) lies in the shaded area in the lower left corner, where the political regime change eventually occurs.

[26] The condition of regime change under the other equilibrium (captured by the smaller weight k_1^*) has the same qualitative patterns as the one considered here.

4. Discussion and extension

Thus far, we have constructed a model to study the informational feedback between financial market speculation and voting and have provided closed-form solutions to characterize the optimal strategy of voters and speculators. In this section, based on the solved equilibrium, we conduct comparative statics analysis and discuss the robustness and limitation of our theoretical results, taking into account more general setups regarding voter preferences and information structures.

4.1. Comparative statics

The tractability of the model enables clean comparative statics analyses. However, because of the multiplicity of equilibria, such analyses may not produce robust results. When parameters vary, it is possible that the economic agents in our model may switch from one equilibrium to another. In addition, different equilibria can have opposite comparative statics results. For example, as Fig. 1 shows, when $\Lambda = \sigma_u(z_h - z_l)$ varies, the equilibrium weights k_1^* and k_2^* change in opposite directions. Nonetheless, we would like to highlight one finding that, while inconclusive, can be useful in understanding how exogenous information affects financial market informativeness.

Suppose that the public signal s_p in the financial market becomes a more precise signal about the fundamental θ (i.e., τ_p increases). The conventional wisdom would be that such a change would make the financial market more informative. This intuition is correct if the weight k assigned to the public signal s_p is not dependent on τ_p. However, the following proposition shows that this conventional wisdom may not hold true as the speculators would respond to the change in precision τ_p by adjusting their equilibrium weight k^*.

Proposition 6. *The financial market informativeness ρ_{k^*} may decrease when public signal s_p becomes more precise. In particular, there exists $\tilde{\tau}_p > 0$ such that, for any $\tau_p < \tilde{\tau}_p$, $\rho_{k_2^*}$ decreases with τ_p.*

The proof of this proposition is relegated to the appendix.[27] The underlying intuition is simple. When the public signal s_p becomes more precise, speculators have an incentive to trade more on this signal. We formally prove that equilibrium weight k_2^* increases with precision τ_p. This, in turn, makes the information aggregation in the financial market less efficient and, thus, reduces the informativeness $\rho_{k_2^*}$. For a relatively small τ_p, this indirect impact through the change of k_2^* dominates the direct impact of an increased τ_p on $\rho_{k_2^*}$, thereby reducing the financial market's overall informativeness.

4.2. Voters' preferences

We follow the literature of protest voting to assume that all voters in our model have a specific form of ordinal preference. They all prefer policy reform over other possible voting outcomes, including a change in the political regime ($e = \mathcal{E}$) and no change ($e = \mathcal{D}$), albeit to varying degrees. As we have shown, for these protesting voters, their beliefs about the pivotality of their votes regarding two tipping points (i.e., p_l and p_h) matter for their strategic voting decisions. For that reason, the size of the speculative attack (A) in the financial market plays an informational role in influencing their choices, which is at the center of our theoretical analysis.

Notably, our model can be naturally extended to incorporate the two different types of voters: (i) type I voters who prefer a political regime change ($e = \mathcal{E}$) to policy reform ($e = \mathcal{R}$) to the status quo ($e = \mathcal{D}$); and (ii) type II voters who prefer the status quo ($e = \mathcal{D}$) to policy reform ($e = \mathcal{R}$) to a political regime change ($e = \mathcal{E}$). Unlike protesting voters, these types of voters always prefer to have a higher (or lower) voting share m. Therefore, voting (or not voting) is the dominant choice, and any information about the overall electorate's composition or about the pivotality of their votes becomes irrelevant. For instance, suppose that, in addition to the $(1 - \alpha - \gamma)$ share of protesting voters, there is an α share of type I voters and a γ share of type II voters in the population. For any given voting rule (i.e., p_l and p_h), as long as the condition $\alpha < p_l < p_h < 1 - \gamma$ holds true, all our results carry over if we replace the thresholds p_h and p_l in the benchmark model with the voting shares that protesting voters must meet in the augmented model $p'_h = \frac{p_h - \alpha}{1-\alpha-\gamma}$ and $p'_l = \frac{p_l - \alpha}{1-\alpha-\gamma}$, respectively.

However, it is important to note the following limitation of our theory. With the presence of protesting voters, our model cannot accommodate other voters with other non-single-peaked preferences — for example, those who prefer policy reform ($e = \mathcal{R}$) the least (i.e., $U_i^{\mathcal{R}} < \min\{U_i^{\mathcal{D}}, U_i^{\mathcal{E}}\}$). Incorporating voters of such types may introduce some potentially interesting strategic interaction between different types of voters; however, with the presence of protesting voters, it challenges our model's tractable equilibrium characterization.

4.3. Information structure and learning

In our benchmark model, we have made several assumptions on the information environment that are crucial to our theoretical results. Here, we discuss the robustness and limitations of our theory by considering those assumptions.

[27] More specifically, for any τ_s and Λ, parameter condition (15) gives rise to a unique $\underline{\tau}_p > 0$ such that the solution of k^* exists for all $\tau_p \in [\underline{\tau}_p, +\infty)$. We prove that $\tilde{\tau}_p > \underline{\tau}_p$ and $\rho_{k_2^*}$ strictly decreases with τ_p in the range of $\tau_p \in [\underline{\tau}_p, \tilde{\tau}_p)$. For details, see the proof of Proposition 6 in the appendix.

The first important assumption we have made is that voters cannot access financial market information except for the size of speculation. Note that, if all signals possessed by the speculators are accessible to the voters, then the voters do not learn any additional information from the size of the speculation. Alternatively, if the voters can access the public signal s_p but not the private signals s_j in the financial market, then, in equilibrium, the size of speculation A would perfectly reveal the fundamental θ no matter how speculators trade on their information.[28] Therefore, if voters were able to observe the speculators' information directly, no informational feedback between speculative trading and voting would take place. In this sense, the inaccessibility of speculators' information to voters is an essential feature of our model.[29]

Another crucial feature of the information structure is that speculators possess some conditionally correlated signals about fundamental θ. In the benchmark model, for simplicity, we assume that all speculators share a public signal s_p, and, thus, the size of the speculation is dependent on the common noise ε_p. To see why this is crucial to our results, suppose that speculators have only conditionally independent signal(s). In this case, as we have discussed, the size of speculative trading A would effectively cancel out all conditionally independent noises and perfectly reveal the fundamental θ. However, a reasonable concern could be that since signal s_p is public, then it might also be observable to (some) voters. Below, we consider an alternative information structure to show that our findings do not rely on the particular information setting with the public signal s_p.

Consider an alternative information structure in which two private signals are available to each speculator j: (1) $s_j = \theta + \sigma_s \varepsilon_j$ (the same as our benchmark setting); and (2) $x_j = \theta + \sigma_q \varepsilon_q + \sigma_\eta \eta_j$, in which the private noises $\{\eta_j\}_j$ and the common noise ε_q are independently and identically distributed following standard normal distribution $\mathcal{N}(0,1)$. We let $\tau_q \equiv \frac{1}{\sigma_q^2}$, $\tau_\eta \equiv \frac{1}{\sigma_\eta^2}$, and use $\tau_x \equiv \frac{\tau_q \tau_\eta}{\tau_q + \tau_\eta}$ to denote the precision of private signal x_i.[30] The following proposition presents the equilibria under this alternative information structure.

Proposition 7. *Under conditions $\sigma_u > z_h - z_l$ and $\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_x}\right) \geq 0$, there are two equilibria featured by $k_x^* = k_{x,1}^*, k_{x,2}^* \in (0, k_B' \equiv \frac{\tau_x}{\tau_s})$, in which $k_{x,1}^*$ and $k_{x,2}^*$ can be solved by replacing τ_p with τ_x in (16). In each equilibrium with k_x^*,*[31]

1. *the speculation strategy at $t = 1$ is $d_x^*(s_j, x_j) = \mathbb{1}\{s_j + k_x^* x_j \geq x_0(k_x^*)\}$,*
2. *the voting strategy at $t = 2$ is $a^*(u_i, \mu) = \mathbb{1}\{u_i \geq u^*(\mu)\}$ in which $u^*(\cdot)$ is given in (9), and*

$$\mu = \mu_{k_x^*}(A(\theta, \varepsilon_q)) \equiv \frac{1}{1 + k_x^*}\left(\sqrt{\frac{\tau_s + k_x^{*2}\tau_\eta}{\tau_s \tau_\eta}}\Phi^{-1}(A(\theta, \varepsilon_q)) + x_0(k_x^*)\right) = \theta + \frac{k_x^*}{1 + k_x^*}\sigma_q \varepsilon_q; \tag{19}$$

3. *and voters' posterior belief is given in (7) with $\mu = \mu_{k_x^*}(A)$ and $\tau_\mu = \left(\frac{k_x^*}{1+k_x^*}\right)^2 \tau_q$.*

It should be clear that, despite the absence of a public signal, this alternative information structure satisfies the two essential features we mentioned above. Since all private signals x_j contain the common noise ε_q, they are correlated across all speculators conditional on θ. As such, the common noise ε_q would affect each speculator's trading and thus the size of the speculation, whereas conditionally independent noises (i.e., ε_j and η_j) would be canceled out. For that reason, the informativeness of the financial market is dependent only on τ_q but not on the precision of the private signal x_i (i.e., τ_x). Other than this difference, as shown in Proposition 7, the equilibrium is qualitatively the same as in the baseline model if we replace the precision of the public signal s_p in the baseline model (i.e., τ_p) with the precision of the private signal x_j (i.e., τ_x). Therefore, we can draw the conclusion that all of our results carry over to this alternative information environment.

5. Conclusion

We develop a dynamic model in this paper to investigate the informational feedback between political decision making and financial speculation. The strategies of voting and speculation are jointly determined in our model. In equilibrium, the size of financial market speculation provides voters with information about the electorate's preferences. We demonstrate that strategic substitutability in protest voting leads to strategic substitutability in speculative attacks in equilibrium. As a result, speculators underweight the conditionally correlated signal but trade more on the conditionally independent signal. This strengthens the financial market's role in providing information; yet, it also increases the ex post impact of the financial market's noise, which can mislead speculators and contribute to an unfavorable electoral outcome. In such an environment,

[28] Note that, in any equilibrium, the voters understand how speculators trade on the private signal s_j and public signal s_p. If voters can also observe the realization of s_p, then they can perfectly infer the realization of θ from the size of speculation A.

[29] However, the fact that speculators have some information that is not accessible to voters does not necessarily mean that speculators are better informed than voters. As the voter's preference u_i is also informative about θ, it is possible that voters have more precise information than speculators. Nevertheless, even in that case, voters still extract information from the financial market since they do not have direct access to the speculators' information.

[30] Note that if $\sigma_\eta \to 0$, then this alternative information structure reduces to the one we studied in the benchmark model.

[31] The expression of the constant $x_0(k^*)$ is given in the appendix.

we find that more precise information does not always make the financial market more informative. We also discuss the essential features of the information structure upon which our results are based, as well as the robustness and the limitations of our results if we incorporate other types of voters who do not have protest demands.

Motivated by Brexit and for tractability concerns, we consider an economic environment in which the political outcome is determined by protest voting and voters extract information from the size of the speculation. We believe, however, that key insights gained from our study can be applied to other voting games (e.g., costly voting) or other learning technologies (e.g., learning from market prices). Nonetheless, there could be other forms of informational linkages between political decision making and financial markets beyond the one studied in this paper. For example, voting outcomes may provide traders with useful information about the future cash flows of certain financial securities, affecting the financial market's information aggregation; and financial market participants can trade as well as vote on corporate policies. These extensions, we believe, can be promising avenues for future research.

Declaration of competing interest

The authors hereby declare that they have nothing to disclose regarding funding sources, IRB approval or the any conflict of interests.

Appendix A. Omitted proofs

Proof of Lemma 1. The proof follows immediately that $u_i|\theta \sim \mathcal{N}(\theta, \frac{1}{\tau_u})$ and $\mu|\theta \sim \mathcal{N}(\theta, \frac{1}{\tau_\mu})$ based on the definition of $\mu = \mu_k(A)$ in (4) and the definition of $\tau_\mu = \tau_\mu(k)$ in (6), and the fact that the signals s_j and s_p are independent with u_i conditional on θ. □

Proof of Lemma 2. First, note that the expected payoff difference between attacking ($d_j = 1$) and not attacking ($d_j = 0$) is $\mathbb{P}(e = \mathcal{E}|s_j, s_p) - c$. To show strategic substitutability, it suffices to prove that $\mathbb{P}(e = \mathcal{E}|s_j, s_p; A)$ decreases with A.

Fix any choice of $\hat{u}(\cdot)$ and k, and consider any $A' > A''$. If $\hat{u}(\mu_k(A')) > \hat{u}(\mu_k(A''))$, then the vote share $m(\theta, \mu_k(A')) < m(\theta, \mu_k(A''))$ for any θ (see (10)). Therefore, for any s_j and s_p,

$$\mathbb{P}(e = \mathcal{E}|s_j, s_p; A') = \mathbb{P}(m(\theta, \mu_k(A')) \geq p_h|s_j, s_p) < \mathbb{P}(m(\theta, \mu_k(A'')) \geq p_h|s_j, s_p) = \mathbb{P}(e = \mathcal{E}|s_j, s_p; A'').$$

The same arguments can be used to prove strategic complementarity. □

Proof of Lemma 3. This proof is the same as Proposition 1 in Myatt (2016). Here, we reproduce the key steps for this result. Voter i chooses $a_i = 1$ if and only if

$$\begin{aligned}&\mathbb{P}(\text{pivotal at } p_l|u_i, \mu, \hat{u}(\mu))U_i^{\mathcal{R}} + \mathbb{P}(\text{pivotal at } p_h|u_i, \mu, \hat{u}(\mu))U_i^{\mathcal{E}}\\ \geq &\mathbb{P}(\text{pivotal at } p_l|u_i, \mu, \hat{u}(\mu))U_i^{\mathcal{D}} + \mathbb{P}(\text{pivotal at } p_h|u_i, \mu, \hat{u}(\mu))U_i^{\mathcal{R}}.\end{aligned}$$

This is equivalent to

$$u_i = \ln\left[\frac{U_i^{\mathcal{R}} - U_i^{\mathcal{D}}}{U_i^{\mathcal{R}} - U_i^{\mathcal{E}}}\right] \geq \ln\left(\frac{\mathbb{P}(\text{pivotal at } p_h|u_i, \mu, \hat{u}(\mu))}{\mathbb{P}(\text{pivotal at } p_l|u_i, \mu, \hat{u}(\mu))}\right). \tag{A.1}$$

The relative pivotal probability on the right-hand side of (A.1), for the case of $N \to +\infty$, can be solved as[32]

$$\ln\left(\frac{\mathbb{P}(\text{pivotal at } p_l|u_i, \mu, \hat{u}(\mu))}{\mathbb{P}(\text{pivotal at } p_h|u_i, \mu, \hat{u}(\mu))}\right) = \frac{z_l^2 - z_h^2}{2} + \ln\left[\frac{g(\hat{u}(\mu) + \sigma_u z_l|u_i, \mu)}{g(\hat{u}(\mu) + \sigma_u z_h|u_i, \mu)}\right], \tag{A.2}$$

in which $z_l = \Phi^{-1}(p_l) < z_h = \Phi^{-1}(p_h)$, and $g(\cdot|u_i, \mu)$ is the probability density function (PDF) of posterior belief $G(\cdot|u_i, \mu)$ in (7). Therefore, we can simplify condition (A.1) to condition (8). Under the condition $\sigma_u > z_h - z_l$, condition (8) is equivalent to $u_i \geq u^*(\mu)$. This completes the proof. □

Proof of Lemma 4. We first prove that speculator j's best response to other speculators' choice of k can be featured by $B(k, \tau_\mu)$ and $s_0(k)$, as shown in (13) and (14), respectively. Replacing μ with θ and s_p (see (12)) in the expression of θ^* in (11), the condition for a political regime change ($e = \mathcal{E}$) can be written as

$$\theta \geq \delta(\tau_\mu)\left[\frac{1}{1+k}\theta + \frac{k}{1+k}s_p - \frac{\sigma_u}{2}(z_h + z_l)\right] + \sigma_u z_h. \tag{A.3}$$

[32] This follows Lemma 1 of Myatt (2016).

Based on the payoff specification, speculator j would choose $d_j = 1$ if and only if the probability of a regime change is[33]

$$\mathbb{P}(e=\mathcal{E}|s_j,s_p)=\mathbb{P}\left(\theta \geq \frac{\delta(\tau_\mu)\left[\frac{k}{1+k}s_p-\frac{\sigma_u}{2}(z_h+z_l)\right]+\sigma_u z_h}{1-\delta(\tau_\mu)\frac{1}{1+k}}\middle| s_j,s_p\right)\geq c. \tag{A.4}$$

Note that, after observing s_j and s_p, speculator j forms a posterior belief: $\theta|s_j,s_p \sim \mathcal{N}(\frac{\tau_s s_j+\tau_p s_p}{\tau_s+\tau_p}, \frac{1}{\tau_s+\tau_p})$. Based on this belief, condition (A.4) is equivalent to

$$\sqrt{\tau_s+\tau_p}\left(\frac{\tau_s s_j+\tau_p s_p}{\tau_s+\tau_p}-\frac{\delta(\tau_\mu)\left[\frac{k}{1+k}s_p-\frac{\sigma_u}{2}(z_h+z_l)\right]+\sigma_u z_h}{1-\delta(\tau_\mu)\frac{1}{1+k}}\right)\geq \Phi^{-1}(c).$$

Based on the as definition of $B(k,\tau_\mu)$ and $s_0(k)$ (see (13) and (14), respectively), this condition can be written as $s_j + B(k,\tau_\mu)s_p \geq s_0(k)$.

Next, we prove the properties of the best response function $B(k,\tau_\mu)$. Recall that

$$B(k,\tau_\mu)=k_B-\frac{\tau_p+\tau_s}{\tau_s}\frac{\delta(\tau_\mu)\frac{k}{1+k}}{1-\delta(\tau_\mu)\frac{1}{1+k}}.$$

Obviously, it is a differentiable function. Taking the derivative of B with respect to τ_μ, we have

$$\frac{\partial B(k,\tau_\mu)}{\partial \tau_\mu}=-\frac{\tau_p+\tau_s}{\tau_s}\frac{k(1+k)}{(1+k-\delta)^2}\frac{\Lambda}{(1+\tau_\mu\Lambda)^2}, \tag{A.5}$$

which is strictly negative for any $k>0$. Moreover, since $\delta(\tau_\mu=0)=0$, we have $B(k,\tau_\mu=0)=\frac{\tau_p}{\tau_s}=k_B$. Similarly, as $\lim_{\tau_\mu\to+\infty}\delta(\tau_\mu)=1$, we have $\lim_{\tau_\mu\to+\infty}B(k,\tau_\mu)=-1$, thereby proving the first property.

Taking the derivative of B with respect to other speculators' choice k, we have

$$\frac{\partial B(k,\tau_\mu)}{\partial k}=-\frac{\tau_p+\tau_s}{\tau_s}\frac{\delta(1-\delta)}{(1+k-\delta)^2}. \tag{A.6}$$

As long as $\tau_\mu\neq 0$, we have $\delta(\tau_\mu)=\frac{\tau_\mu\Lambda}{1+\tau_\mu\Lambda}\in(0,1)$ and, therefore, the value of $\frac{\partial B(k,\tau_\mu)}{\partial k}$ is negative. Furthermore, it is easy to check that, for any $k>0$, $\frac{\delta(\tau_\mu)\frac{k}{1+k}}{1-\delta(\tau_\mu)\frac{1}{1+k}}>0$, and, thus, $B(k,\tau_\mu)<k_B$. This proves the second property.

Finally, we incorporate $\tau_\mu=\tau_\mu(k)=(\frac{1+k}{k})^2\tau_p$ into the function $B(k,\tau_\mu)$; that is,

$$b(k)=B(k,\tau_\mu(k))=\frac{\tau_p}{\tau_s}-\frac{\tau_s+\tau_p}{\tau_s}\frac{(k+1)\tau_p\Lambda}{\left[1+\tau_p\Lambda\right]k+\tau_p\Lambda}. \tag{A.7}$$

Note that $b(k)$ is not continuous at the point of $k_0\equiv-\frac{\tau_p\Lambda}{1+\tau_p\Lambda}\in(-1,0)$, whereby $\lim_{k\uparrow k_0}b(k)=+\infty$ and $\lim_{k\downarrow k_0}b(k)=-\infty$. However, we can show that in the range of $(-\infty,k_0)$ and $(k_0,+\infty)$, $b(k)$ is continuously increasing in k; that is,

$$\frac{\mathrm{d}b(k)}{\mathrm{d}k}=\frac{\partial B(k,\tau_\mu)}{\partial k}+\frac{\partial B(k,\tau_\mu)}{\partial \tau_\mu}\frac{\mathrm{d}\tau_\mu(k)}{\mathrm{d}k}=\frac{\tau_p+\tau_s}{\tau_s}\frac{\delta(1-\delta)}{(1+k-\delta)^2}>0.$$

Additionally, it is easy to check that $b(0)=-1$, $\lim_{k\to+\infty}\frac{\mathrm{d}b(k)}{\mathrm{d}k}=0$, and that $b(k)$ is concave for $k>0$ since, for $k\in(0,+\infty)$, $\frac{\mathrm{d}^2b(k)}{\mathrm{d}k^2}=-2\frac{\tau_p+\tau_s}{\tau_s}\frac{\delta(1-\delta)}{(1+k-\delta)^3}<0$. □

Proof of Lemma 5. Any k^* that satisfies

$$k^*=b(k^*)=B(k^*,\tau_\mu(k^*))=\frac{\tau_p}{\tau_s}-\frac{\tau_s+\tau_p}{\tau_s}\frac{(k^*+1)\tau_p\Lambda}{\left[1+\tau_p\Lambda\right]k^*+\tau_p\Lambda} \tag{A.8}$$

[33] Note that this holds true only when $1-\delta(\tau_\mu)\frac{1}{1+k}>0$. As such, this condition may not hold for some $k\in[-1,0]$. Consider $k\in[-1,0]$. If $1-\delta(\tau_\mu)\frac{1}{1+k}<0$, then speculator j will choose $d_j=1$ if and only if $s_j+b(k)s_p\leq s_0'(k)$, whereby the best-response function $b(k)$ remains the same. As will soon be shown, we cannot find any $k^*\in[-1,0]$ such that $b(k^*)=k^*$ so that any $k\in[-1,0]$ such that $1-\delta(\tau_\mu)\frac{1}{1+k}<0$ cannot constitute an equilibrium. Another possibility is that $1-\delta(\tau_\mu)\frac{1}{1+k}=0$. In this case, the regime change condition in (A.3) is independent of θ but is still dependent on s_p, and therefore the best response of speculator i is to trade only on the public signal s_p. This cannot support a symmetric equilibrium because that best response cannot be consistent with the other speculators' choice of $k\in[-1,0]$. For these reasons, taking into account the possibilities that $1-\delta(\tau_\mu)\frac{1}{1+k}\leq 0$ will not extend the set of equilibria.

must be a solution to the following quadratic equation:

$$(\Lambda+\frac{1}{\tau_p})k^{*2}+(2\Lambda-\frac{1}{\tau_s})k^*+\Lambda=0. \tag{A.9}$$

It is obvious that we can find the solutions k_1^* and k_2^*, as shown in (16), under parameter condition (15). One way to prove $k_1^*, k_2^* \in (0, k_B)$ is to focus on the quadratic equation (A.9). Under condition (15), these two solutions exist, and they must satisfy that $k_1^* \cdot k_2^* = \frac{\Lambda}{\Lambda+\frac{1}{\tau_p}} > 0$, and

$$k_1^*+k_2^*=\frac{\frac{1}{\tau_s}-2\Lambda}{\Lambda+\frac{1}{\tau_p}} \geq \frac{2\Lambda+4\Lambda\frac{\tau_s}{\tau_p}}{\Lambda+\frac{1}{\tau_p}}>0,$$

in which the first inequality comes from condition (15) and the second one is based on the fact that Λ, τ_s, and τ_p are all positive. Consequently, these two solutions must be strictly positive. By Property 2 in Lemma 4, we have $k^*=b(k^*)<k_B$ for $k^*=k_1^*, k_2^*$. □

In the following lemma, we present another perhaps more intuitive way of proving this result.

Lemma A.1. *Any solution to $k^*=b(k^*)$ must satisfy $k^* \in (0, k_B)$.*

Proof. Recall that $k_0=-\frac{\tau_p\Lambda}{1+\tau_p\Lambda} \in (-1,0)$, and $b(k)$ continuously increases with k in the range of $k \in (-\infty, k_0) \cup (k_0, +\infty)$. First, consider any $k \in (-\infty, k_0)$. From (A.7), we know that $\lim_{k\to-\infty} b(k)=(1-k_0)\frac{\tau_p}{\tau_s}-k_0>-1$, $b(-1)=k_B>0$, and $\lim_{k\uparrow k_0} b(k)=+\infty$. Provided that $b(k)$ is increasing in this range, we know that $b(k)>-1\geq k$ for any $k\leq -1$. Moreover, for $k \in (-1, k_0)$, $b(k)>0>k$. Therefore, $k^* \notin (-\infty, k_0)$. Next, consider any $k \in (k_0, 0]$. Since $b(0)=-1$, $\lim_{k\downarrow k_0} b(k)=-\infty$, and $b(k)$ is increasing in this range, we know $b(k)<-1<k$. Therefore, $k^* \notin (k_0, 0]$.

As such, $k^* \notin (-\infty, 0]$, or equivalently, if k^* exists, $k^* \in (0, +\infty)$. By Property 2 in Lemma 4, if k^* exists, it must be that $k^* \in (0, k_B)$. □

Proof of Proposition 3. Consider any equilibrium with $k=k_1^*, k_2^*$. Then, based on (4) and (9), we have

$$u^*(\mu_{k^*}(A))=\frac{\tau_\mu\Lambda}{1+\tau_\mu\Lambda}\left(\left[\frac{1}{1+k^*}\left(\frac{1}{\sqrt{\tau_s}}\Phi^{-1}(A)+s_0(k^*)\right)\right]-\frac{\sigma_u}{2}(z_h+z_l)\right)$$

since $k^*>0$, $u^*(\mu_{k^*}(A))$ is strictly increasing in A. Thus, by Lemma 2, strategic substitutability holds. □

Proof of Proposition 4. Note that $\rho_k=(1+\frac{1}{k})^2\tau_p$, which is strictly decreasing in k for $k \in (0,+\infty)$. Then, it is obvious that $\rho_{k^*}>\rho_{k_B}$ since $k^* \in (0, k_B)$ (see Lemma 5). □

Proof of Proposition 5. The ex post regime change condition in (18) can be established simply by plugging $\mu=\mu_k^*(A(\theta,\varepsilon_p))=\theta+\frac{k^*}{1+k^*}\frac{1}{\sqrt{\tau_p}}\varepsilon_p$ (see (5)) and $\delta(\tau_\mu)=\delta(\rho_k^*)=\frac{\rho_k^*\Lambda}{1+\rho_k^*\Lambda}$ into condition (11). □

Proof of Corollary 1. In equilibrium with $k^*=k_1^*, k_2^*$, based on (3) and (10), we know that $A(\theta,\varepsilon_p)=\Phi\left(\sqrt{\tau_s}[(1+k^*)\theta + {}^*\sigma_p\varepsilon_p-s_0]\right)$, and

$$m(\theta,\varepsilon_p)=\Phi\left(\frac{\sqrt{\tau_u}}{1+\tau_\mu\Lambda}\left(\theta-\tau_\mu\Lambda\left[\frac{k^*}{1+k^*}\sigma_p\varepsilon_p-\frac{\sigma_u}{2}(z_h+z_l)\right]\right)\right).$$

As $k^*>0$, it is easy to see that $\frac{\partial A}{\partial\varepsilon_p}>0$ and $\frac{\partial m}{\partial\varepsilon_p}<0$. □

Proof of Proposition 6. Note that the condition for the existence of k^* (15), or $\frac{1}{\tau_s^2}-4\Lambda\left(\frac{1}{\tau_s}+\frac{1}{\tau_p}\right)\geq 0$, essentially means, for any $\tau_s<\frac{1}{4\Lambda}$, $\tau_p\geq\underline{\tau_p}\equiv\frac{4\Lambda\tau_s^2}{1-4\Lambda\tau_s}$. In what follows, we prove that there exists some $\tilde{\tau}_p>\underline{\tau_p}$ such that $\rho_{k_2^*}$ strictly decreases with τ_p for any $\tau_p\in[\underline{\tau_p},\tilde{\tau}_p)$.

First, observe that

$$\frac{\mathrm{d}\rho_{k_2^*}}{\mathrm{d}\tau_p}=(1+\frac{1}{k_2^*})^2-2(1+\frac{1}{k_2^*})\frac{1}{k_2^{*2}}\tau_p\frac{\partial k_2^*}{\partial\tau_p}=(1+\frac{1}{k_2^*})(1+\frac{1}{k_2^*}-\frac{2}{\tau_p\sqrt{\frac{1}{\tau_s^2}-4\Lambda\left(\frac{1}{\tau_s}+\frac{1}{\tau_p}\right)}}).$$

Next, let

$$f(\tau_p) \equiv 1 + \frac{1}{k_2^*(\tau_p)} - \frac{2}{\tau_p\sqrt{\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_p}\right)}} = \frac{\frac{1}{\tau_s} - \sqrt{\frac{1}{\tau_s^2} - 4\Lambda(\frac{1}{\tau_s} + \frac{1}{\tau_p})}}{2\Lambda} - \frac{2}{\tau_p\sqrt{\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_p}\right)}}.$$

Given $k_2^* > 0$, $\text{sgn}(\frac{\mathrm{d}\rho_{k_2^*}}{\mathrm{d}\tau_p}) = \text{sgn}((1+\frac{1}{k_2^*})f(\tau_p)) = \text{sgn}(f(\tau_p))$. By definition of $f(\cdot)$, we have $f(\underline{\tau_p}) = -\infty$ and $\lim_{\tau_p \to +\infty} f(\tau_p) = \frac{\frac{1}{\tau_s} - \sqrt{\frac{1}{\tau_s^2} - 4\Lambda\frac{1}{\tau_s}}}{2\Lambda} > 0$. Furthermore, $f(\cdot)$ is a strictly increasing function because

$$\frac{\mathrm{d}f(\tau_p)}{\mathrm{d}\tau_p} = \frac{1}{\tau_p^2\sqrt{\frac{1}{\tau_s^2} - 4\Lambda(\frac{1}{\tau_s} + \frac{1}{\tau_p})}} + \frac{4\Lambda}{\tau_p^3\left(\frac{1}{\tau_s^2} - 4\Lambda(\frac{1}{\tau_s} + \frac{1}{\tau_p})\right)^{\frac{3}{2}}} > 0.$$

Therefore, there exists a unique $\tilde{\tau}_p \in (\underline{\tau_p}, +\infty)$ such that $f(\tilde{\tau}_p) = 0$. Based on monotonicity, for any $\tau_p < \tilde{\tau}_p$, $f(\tau_p) < 0$ and, accordingly, $\frac{\mathrm{d}\rho_{k_2^*}}{\mathrm{d}\tau_p} < 0$. This completes the proof. □

Proof of Proposition 7. First, under any strategy $d_j = \mathbb{1}\{s_j + k_x x_j \geq x_0(k)\}$, the size of the speculation is

$$A(\theta, \varepsilon_q) = \Phi\left(\sqrt{\frac{\tau_s\tau_\eta}{\tau_s + k_x^2\tau_\eta}}\left[(1+k_x)\theta + k_x\sigma_q\varepsilon_q - x_0(k)\right]\right).$$

As such, we can construct the public signal μ (see (19)):

$$\mu = \mu_{k_x}(A) = \theta + \frac{k_x}{1+k_x}\sigma_q\varepsilon_q = \frac{1}{1+k_x}\theta + \frac{k_x}{1+k_x}(x_j - \sigma_\eta\eta_j), \tag{A.10}$$

and its precision is $\tau_\mu = (\frac{k_x}{1+k_x})^2\tau_q$. Based on the new definition of μ and τ_μ, the posterior belief of the voters is the same as $G(\theta|u_i, \mu)$ given in (7) and, thus, the voters' equilibrium strategy features the same $\hat{u}(\cdot)$ as the one present in Lemma 3. Accordingly, the condition for $e = \mathcal{E}$ is identical to the one given in (11).

Any speculator j, after observing s_j and x_j, forms a posterior belief of θ and $\sigma_\eta\eta_j$ as follows[34]:

$$\theta|s_j, x_j \sim \mathcal{N}(\frac{\tau_s s_j + \tau_x x_j}{\tau_s + \tau_x}, \frac{1}{\tau_s + \tau_x}), \text{ and } \sigma_\eta\eta_j|s_j, x_j \sim \mathcal{N}(\frac{\tau_q\tau_s(x_j - s_j)}{\tau_q\tau_s + \tau_q\tau_\eta + \tau_s\tau_\eta}, \frac{1}{\tau_\eta + \frac{\tau_s\tau_q}{\tau_s + \tau_q}}). \tag{A.11}$$

Plugging μ into the regime change condition, $e = \mathcal{E}$ occurs if and only if

$$\theta + \frac{\delta(\tau_\mu)\frac{k_x}{1+k_x}}{1 - \delta(\tau_\mu)\frac{1}{1+k_x}}\sigma_\eta\eta_j \geq \frac{\delta(\tau_\mu)\left[\frac{k_x}{1+k_x}x_j - \frac{\sigma_u}{2}(z_h + z_l)\right] + \sigma_u z_h}{(1 - \delta(\tau_\mu)\frac{1}{1+k_x})}.$$

Based on the posterior belief, speculator j would choose $d_j = 1$ if and only if

$$\mathbb{P}\left(\frac{\tau_s s_j + \tau_x x_j}{\tau_s + \tau_x} + \frac{\delta(\tau_\mu)\frac{k_x}{1+k_x}}{1 - \delta(\tau_\mu)\frac{1}{1+k_x}}\frac{\tau_q\tau_s(x_j - s_j)}{\tau_q\tau_\eta + \tau_s\tau_\eta + \tau_q\tau_s} \geq \frac{1}{\Omega}\frac{\delta(\tau_\mu)\left[\frac{k_x}{1+k_x}x_j - \frac{\sigma_u}{2}(z_h + z_l)\right] + \sigma_u z_h}{1 - \delta(\tau_\mu)\frac{1}{1+k_x}}\middle| s_j, x_j\right) \geq c,$$

in which Ω presents the standard deviation of the posterior distribution of $\theta + \frac{\delta(\tau_\mu)\frac{k_x}{1+k_x}}{1-\delta(\tau_\mu)\frac{1}{1+k_x}}\sigma_\eta\eta_j$ conditional on s_j and x_j, which can be solved following (A.11). Abusing the notation of $b(\cdot)$ and $B(\cdot,\cdot)$, the above condition can be rewritten as $s_j + b(k_x)x_j \geq x_0(k_x)$, in which

$$b(k_x) = B(k_x, \tau_\mu) \equiv \frac{\tau_x}{\tau_s} - \frac{\tau_q + \tau_s + \frac{\tau_x}{\tau_\eta}}{\frac{1+k_x}{k_x\tau_\mu\Lambda}(1 + \frac{\tau_q}{\tau_\eta})\tau_s + \tau_s}, \tag{A.12}$$

and

[34] For the posterior belief of $\sigma_\eta\eta_j$, one can think in the following way: speculator j has a prior belief $\sigma_\eta\eta_j \sim \mathcal{N}(0, \sigma_\eta^2)$ and also a noisy signal $(x_j - s_j)$, whereby $\sigma_\eta\eta_j = (x_j - s_j) - \sigma_q\varepsilon_q + \sigma_s\varepsilon_j \sim \mathcal{N}(x_j - s_j, \sigma_q^2 + \sigma_s^2)$.

$$x_0(k_x) \equiv \sqrt{\frac{1}{\tau_s+\tau_x} + \left(\frac{\delta(\tau_\mu)\frac{k_x}{1+k_x}}{1-\delta(\tau_\mu)\frac{1}{1+k_x}}\right)^2 \frac{1}{\tau_\eta + \frac{1}{\sigma_q^2+\sigma_s^2}}}\frac{\tau_x+\tau_s}{\tau_s}\Phi^{-1}(c) + \frac{\tau_s+\tau_x}{\tau_s}\frac{\sigma_u z_h - \frac{\sigma_u}{2}\delta(\tau_\mu(k_x))(z_h+z_l)}{1-\delta(\tau_\mu(k_x))\frac{1}{1+k_x}}. \quad (A.13)$$

Following the same procedures as we did for Lemma 4 and letting $k'_B \equiv \frac{\tau_x}{\tau_s}$, one can easily check that all properties hold true for the new function $B(k_x, \tau_\mu)$. Plugging $\frac{1}{\tau_\mu} = (\frac{k_x}{1+k_x})^2\frac{1}{\tau_q}$ into (A.12), the solution k_x^* must satisfy that

$$k_x^* = b(k_x^*) = \frac{\frac{k_x^*}{\Lambda(1+k_x^*)} - \tau_s}{\frac{k_x^*}{\tau_q\Lambda(1+k_x^*)}(\tau_q+\tau_\eta)+\tau_\eta}\frac{\tau_\eta}{\tau_s}.$$

This condition can be simplified to

$$(\Lambda + \frac{1}{\tau_q} + \frac{1}{\tau_\eta})k_x^{*2} + (2\Lambda - \frac{1}{\tau_s})k_x^* + \Lambda = 0,$$

which is the same as (A.9) if we replace $\frac{1}{\tau_p}$ in (A.9) with $\frac{1}{\tau_q} + \frac{1}{\tau_\eta} = \frac{1}{\tau_x}$. Therefore, by replacing τ_p with τ_s in (15) and (16), we have the condition for the existence of k_x^* and the explicit solution of k_x^*, respectively. Following the same arguments as in the proof of Lemma 5, we can show that $k_x^* \in (0, k'_B)$. □

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