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Term structure of interest rates with short-run and long-run risks[☆]

Olesya V. Grishchenko [a,*], Zhaogang Song [b], Hao Zhou [c,d]

[a] Division of Monetary Affairs, Federal Reserve Board, Washington, DC 20551, USA
[b] Carey School of Business, John Hopkins University, 100 International Drive, Baltimore, MD 21202, USA
[c] SUSTech Business School, Southern University of Science and Technology, PR China
[d] PBC School of Finance, Tsinghua University, 43 Chengfu Road, Haidian District, Beijing, 100083, PR China

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Abstract

We find that interest rate variance risk premium (IRVRP) — the difference between implied and realized variances of interest rates — is a strong predictor of U.S. Treasury bond returns of maturities ranging between one and ten years for return horizons up to six months. IRVRP is not subsumed by other predictors such as forward rate spread or equity variance risk premium. These results are robust in a number of dimensions. We rationalize our findings within a consumption-based model with long-run risk, economic uncertainty, and inflation non-neutrality. In the model IRVRP is related to short-run risk only, while standard forward-rate-based factors are associated with both short-run and long-run risks in the economy. Our model qualitatively replicates the predictability pattern of IRVRP for bond returns.

JEL classification: G12; G13; G14

Abbreviations: IRVRP, Interest Rate Variance Risk Premium; EVRP, Equity Variance Risk Premium; FS, Forward Spread; HPR, Holding Period Return; BY, Bansal and Yaron (2004); BTZ, Bollerslev, Tauchen, and Zhou (2009).

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* Corresponding author.
E-mail addresses: olesya.v.grishchenko@frb.gov (O.V. Grishchenko), zsong8@jhu.edu (Z. Song), zhouh7@sustech.edu.cn (H. Zhou).
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1. Introduction

The failure of the expectations hypothesis first documented by Fama and Bliss[1] and Campbell and Shiller[2] has attracted enormous attention in the asset pricing literature over the past decades. Various plausible risk factors that appear to capture bond return predictability—forward spread,[1] forward rates factor,[3] jump risk measure,[4] hidden term structure factor,[5] and macroeconomic factor(s),[6–8] among many others—have been proposed. Despite this impressive progress, the fundamental challenge for uncovering a particular economic mechanism behind bond return variation still remains. Understanding such an economic mechanism is equally important for market participants as well as for monetary policy makers. Our paper focuses on (a) finding a robust empirical factor that would capture bond return variation across maturities and for different horizons; (b) exploiting the informational content of interest rate variance risk premium (IRVRP) constructed from the U.S. interest rate swaptions data; and (c) rationalizing the this empirical factor within a consumption-based long-run risk model.

The interest rate derivatives markets represent the largest segment of the U.S.fixed-income market because these financial derivatives represent an important tool for corporate treasurers, asset managers, and public institutions for hedging interest rate risk exposure.[a] According to the 2021 Bank of International Settlements Semiannual Survey of the over-the-counter derivatives market, the total global net outstanding notional value of interest rate swaps and swaptions exceeded 397 and 39 trillion of US denominated dollars, respectively.[b] In addition, a 2009 survey by the International Swaps and Derivatives Association reports that 88.3% of the Fortune Global 500 companies use swaps and swaptions for hedging interest rate risk. Last but not least, Dai and Singleton[9] pointed to similarities between the U.S. Treasury yields, swap rates, and swaption prices. Thus, it appears likely that the interest-rate derivatives markets can be informative for explaining variation in U.S. Treasury yields.

Our first and most important empirical finding connects the variation in the relatively short-horizon (one-to six-month) bond risk premiums to variation in IRVRP — the difference between the risk-neutral and objective expectations of variation in interest rates. IRVRP always loads positively on Treasury excess returns in the data, and high (low) values of IRVRP are associated with subsequent high (low) Treasury excess returns. Depending on the return horizon and maturity of Treasury securities, IRVRP alone explains a nontrivial share of the variation in bond returns.

Predictive power of IRVRP is the strongest for near-term (2–5 years) bonds and relatively short-horizon holding period returns (HPR). For one-month HPR, IRVRP explains between 56 and 75 percent of variation; for three-month HPR — between 28 and 62 percent of variation; and for six-month HPR — 10 to 38 percent of variation.[c]

Predictive power of IRVRP diminishes somewhat for longer-term bonds. That said, IRVRP still explains a considerable, albeit smaller, amount of variation, than for intermediate maturities. For one-, three-, and six-month HPR for longer maturities, it explains 32 to 50, 13 to 23, and 5 to 8 percent of variation, respectively. IRVRP is also significant for longest maturities (15- to 20-year) for one- and three-month HPR, but not for six-month HPR. IRVRP is not significant for any maturities at one-year horizons.

To sum up, IRVRP appears to capture relatively *short-run* variation in U.S. Treasury excess returns across the entire term structure. This informational and predictive value is most important for relatively short holding periods of U.S. Treasury securities.

Our second empirical result is that equity variance risk premium (EVRP) — a robust predictor for equity excess returns[10]—does not strongly predict variation in Treasury excess returns. That said, its predictive value for bond excess returns increases slightly with maturity of U.S. Treasury securities.

Our third empirical finding is that forward spread (FS), a classical predictor of U.S. Treasury excess returns, also explains a nontrivial share of bond return variation. However, corresponding R^2 statistics are much lower than those implied by IRVRP regressions. In addition, FS-implied R^2's increase with maturity with most explanatory power concentrated at maturities of three to five years.

[a] Interest rate swaps play a central role in the whole financial system as swap rates reflect term financing rates of major financial institutions. In fact, the floating leg of a plain vanilla swap is usually tied to the 3-month LIBOR rate, which serves as a benchmark rate for corporate treasurers, mortgage lenders, and credit card agencies.

[b] See https://www.bis.org/statistics/derstats.htm in general, and https://stats.bis.org/statx/srs/table/d7?f=pdf, in particular.

[c] Higher levels of predictability correspond to shorter term bonds and vice versa.

Fig. 3 is the main figure in our paper that illustrates these empirical findings. To summarize, IRVRP tends to capture variation in bond returns at shorter horizons, thus we associate IRVRP with the *short-run risks*, while EVRP and FS appear to capture variation in bond returns at longer horizons, thus, we associate these factors with *long-run risks*. We also find that neither EVRP, nor FS subsume significance and predictive power of IRVRP. So, the informational content of IRVRP appears to go beyond standard predictors for equity and bond returns.

We also run a number of robustness checks. First, we perform a subsample analysis to verify that our results are not driven by short turbulent periods in the financial markets, such as financial crisis of 2008–2009. Second, we control for additional bond return predictors, such as Cochrane and Piazzesi[3] factor as well as two macro factors including economic growth, expected inflation, as well as Ludvigson and Ng[6] macro principal component factors. Lastly, we also consider an alternative data set of Treasury bond portfolios. None of these modifications of our empirical analysis materially change our results.

To rationalize our empirical findings, we propose a stylized general equilibrium model as an extension of long-run risk models in Bansal and Yaron[11] and short-run risk models in BTZ Bollerslev et al[10] Bansal and Yaron[11] emphasize importance of the long-run risk in consumption growth for explaining the equity premium, while Bollerslev et al[10] show that richer volatility dynamics in consumption growth can be successful in capturing short-horizon stock return predictability. Our model includes both long-run risk and certain nontrivial volatility dynamics in consumption growth. It generates a two-factor volatility structure for the endogenously determined bond risk premium, in which the factors are explicitly related to the underlying volatility dynamics of consumption growth where different volatility concepts load differently on the fundamental risk factors and capture separately short-run and long-run risks of Treasury excess returns. In particular, IRVRP effectively isolates the *short-run* risk factor associated with the volatility-of-volatility of consumption growth. The *long-run* risk factor associated with volatility of consumption growth appears to be captured by Fama and Bliss[1] forward spread.

Finally, our calibration exercise suggests that the model fits remarkably well the upward-sloping nominal yield curve. The key two ingredients in fitting the nominal yield curve are the presence of the long-run risk in the model and inflation non-neutrality. As such, the long-run risk state variable from the real side of the model affects nominal prices via inflation channel. We reasonably calibrate inflation process,[12] while leaving the real side model parameters similar to the ones in BY and BTZ. The most important feature of the inflation process is the negative correlation with consumption volatility shock, consistent with recent empirical findings.[13–15] Without this feature, the nominal yield curve is downward-sloping.

The idea of economic uncertainty as a potential risk factor has gained attention recently, both for explaining variation in stock returns[10,16,17] and in bond returns.[15,18,19] The last two papers are especially relevant to our study. Bansal and Shaliastovich[15] link bond excess return variation to variation in volatility of real activity and inflation—variables they interpret as uncertainty, although they do not explicitly model the uncertainty process. Giacoletti et al[19] find that dispersion of beliefs about future interest rates — interpreted as investors' uncertainty about interest rates—is distinct from information about the macroeconomy and can be useful in explaining variation in bond returns.

While bond pricing empirical literature (see, Fama and Bliss[1]; Campbell and Shiller[2] among numerous studies) has documented predictability of long-horizon bond returns, bond predictability in the short run did not receive much attention until very recently.[20–22] A growing literature argues for the existence of the short-run and long-run risk components of the aggregate volatility to study the variation of stock returns (see, Adrian and Rosenberg[23]; Christoffersen et al[24]; Branger et al[25]; Zhou and Zhu[26,27] among others). A recent paper by Ghysels et al[28] emphasizes a short-run volatility component of bond yields as a useful predictor for future excess returns, as opposed to a long-run volatility component that has no predictive power.

To the best of our knowledge, our paper is the first one that explores short-horizon bond return predictability and explains the empirical findings within a consumption-based structural framework of two-factor volatility dynamics. While the volatility-of-volatility of consumption growth (short-run risk factor) seems to drive the variation in the short-horizon Treasury excess returns, the variation in long-horizon returns appears to be mainly driven by a different kind, possibly a longer-run risk factor of consumption growth. The long-run risk factor with money-non-neutrality is also important for matching the term structure of nominal interest rates.

The rest of the paper is organized as follows. Section 2 describes all relevant data to our empirical exercise and methodology of constructing IRVRP. Section 3 presents our empirical results and robustness checks. Section 4 presents our long-run risk model with two different volatility factors and inflation non-neutrality, and derives asset pricing implications of the model. Section 5 discusses calibration of the U.S. Treasury yield curve implied by our model, Section 6 provides concluding remarks about the study and future directions for relevant research.

2. Data and our measures

2.1. Measure of the interest rate variance risk premium

2.1.1. Methodology

We measure IRVRP as the difference between the market's expectation of the interest rate variation under the risk-neutral measure and that under the physical measure. To capture the risk-neutral expectation, we employ a model-free approach and construct an implied variance measure of swap rates using interest rate swaptions, similar to Bollerslev et al[10] and Carr and Wu[29] in measuring equity variance risk premium using equity options. Specifically, let $D(t, T_m)$ be the time-t price of a zero-coupon bond maturing at time T_m, and $S_{m,n}(t)$ be the time-t forward swap rate, i.e., the rate for a fixed versus floating interest rate swap contract with a start date T_m and maturity date T_n. The forward swap rate becomes the spot swap rate $S_{m,n}(T_m)$ at time $t = T_m$.

A swaption gives its holder the right but not the obligation to enter into an interest rate swap either as a fixed leg (payer swaption) or as a floating leg (receiver swaption) with a pre-specified fixed coupon rate. In particular, let T_m be the expiration date of the swaption, K be the coupon rate on the swap, and T_n be the final maturity date of the swap. The payer swaption allows the holder to enter into a swap at time T_m with a remaining term of $T_n - T_m$ and to pay the fixed annuity of K. At time t, this swaption is usually called a $(T_m - t)$ into $(T_n - T_m)$ payer swaption, also known as a $(T_m - t)$ by $(T_n - T_m)$ payer swaption, where $(T_m - t)$ is the option maturity and $(T_n - T_m)$ is the tenor of the underlying swap. Equivalently, the payer (receiver) swaption allows the holder to receive (pay) periodic coupon payments according to the floating influential interest rates. Hence, the payer (receive) swaption is effectively a call (put) option on the interest rate. By analogy to equity options, payer swaptions contain valuable information on the upside movements of interest rates, whereas receiver swaptions are about downside. The difference from equity options is that the underlying security of a swaption is a forward interest rate swap contract that has a maturity $(T_n - T_m)$, but the underlying of equity index option – the S&P 500 index – has an infinite maturity.

Let $\mathcal{P}_{m,n}(t;K)$ and $\mathcal{R}_{m,n}(t;K)$ denote the time-t value of a European payer and receiver swaption, respectively, expiring at T_m with strike K on a forward start swap for the time period between T_m and T_n. As shown by Li and Song[30] and Mele and Obayashi[31] extending the algorithm used by CBOE in constructing VIX to interest rate swaps, the market's risk-neutral expectation of the interest rate variation over $[t, T_m]$ can be computed as the following "model-free" portfolio of swaptions:

$$\mathbb{IV}_{m,n}(t) \equiv \frac{2}{A_{m,n}(t)}\left\{ \int\limits_{K > S_{m,n}(t)} \frac{1}{K^2}\mathcal{P}_{m,n}(t;K)dK + \int\limits_{K < S_{m,n}(t)} \frac{1}{K^2}\mathcal{R}_{m,n}(t;K)dK \right\}, \tag{1}$$

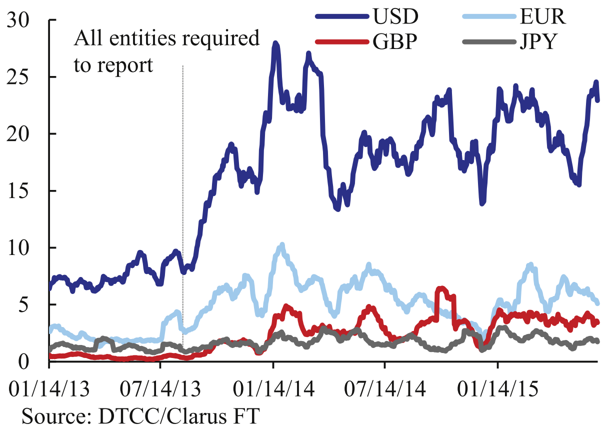
where $A_{m,n}(t) \equiv \sum_{j=m+1}^{n} D(t, T_j)$ is the present value of an annuity associated with the fixed leg of the forward swap contract.[d]

In order to quantify the variation of interest rates under the physical measure, we follow Bollerslev et al[10] to use the realized variance measure. Specifically, let $s_{m,n}(t)$ be the logarithm of the forward swap rate $S_{m,n}(t)$. The realized variation over the comparable to the $\mathbb{IV}_{m,n}(t)$ discrete-time interval $[t - (T_m - t), t]$ can then be measured in a "model-free" way as follows:

$$\mathbb{RV}_{m,n}(t) = \frac{1}{T_m - t}\sum_{i=1}^{M}\left[s_{m,n}\left(t - (T_m - t) + \frac{i}{M}(T_m - t)\right) - s_{m,n}\left(t - (T_m - t) + \frac{i-1}{M}(T_m - t)\right)\right]^2, \tag{2}$$

which will converge to the quadratic variation of $s_{m,n}$ over the interval $[t - (T_m - t), t]$ as $M \to \infty$, that is, on an increasing number of within-the-period observations (see Carr and Madan,[32] Barndorff-Nielsen and Shephard,[33] Hansen and Lunde[34] and so on).

[d] To be precise, this calculates the market's expectation of the swap rate variation over [t, T_m] under the so-called annuity measure $\mathbb{A}^{m,n}$, which is an equivalent probability measure to the risk-neutral measure $\mathbb{Q}$ such that $\frac{d\mathbb{A}^{m,n}}{d\mathbb{Q}} = e^{-\int_t^{T_m} r(s)ds}\frac{A_{m,n}(T_m)}{A_{m,n}(t)}$.



Mn. USD/day		Tenor								Sum
		1Y	2Y	3Y	5Y	Tenor bucket <10Y*	10Y	Tenor bucket 11Y-29Y	30Y	
Expiry	<1 M	0	32	152	146	64	804	0	58	1 257
	1M	0	23	85	293	111	1000	28	183	1 723
	2M	405	76	11	136	28	165	10	40	870
	3M	441	355	71	357	25	730	35	75	2 089
	4M-5M	900	200	26	125	27	107	0	4	1 390
	6M	399	360	6	303	22	508	5	68	1 671
	7M-11M	1534	60	22	37	15	149	22	56	1 894
	1Y	1130	392	31	192	51	400	12	71	2 280
	18M	95	92	55	57	0	82	0	2	383
	2Y	252	559	22	216	5	189	62	98	1 402
	30M-4Y	41	214	25	74	2	203	6	51	616
	5Y	72	46	0	83	3	148	16	60	428
	6Y-9Y	0	56	0	14	4	59	131	28	291
	10Y	0	30	0	16	0	9	25	12	91
	10Y<	11	18	0	0	0	38	0	0	68
	Sum	5 281	2 511	506	2 050	355	4 590	351	808	16 452

* Includes trades with non-standard tenor/expiry combinations.

Source: DTCC/Clarus FT

Fig. 1. Trading characteristics of interest rate swaptions market. This figure presents the trading characteristics of the interest rate swaptions market. The top panel presents the 4-week moving average of the daily traded swaptions volumes in the U.S., in billions of USD. The bottom panel presents the average traded amount of various combinations of swaptions expiry and underlying forward swap tenors in the period spanning July 1, 2015 to July 31, 2015, in millions of USD. The columns represent different tenors while the rows represent different expiries. Source: Depository Trust Clearing Corporation.

The interest rate variance risk premium measure is then computed as the difference between the market's risk-neutral and physical expectations of the swap rate variation over [t, T_m], proxied by $\mathbb{IV}_{m,n}(t)$ and $\mathbb{RV}_{m,n}(t)$, respectively:[e]

$$\mathrm{IRVRP}_{m,n}(t) \equiv \mathbb{IV}_{m,n}(t) - \mathbb{RV}_{m,n}(t). \tag{3}$$

2.1.2. Swaptions data and estimates

At year-end 2021, the BIS reported that the global net notional outstanding value of interest rate swaps and swaptions totaled approximately 397 trillion and 39 trillion of US-denominated dollars, respectively. Fig. 1 illustrates a more detailed look at the data. Top panel of Fig. 1 shows that the daily traded volume in the U.S. for all USD swaption types has fluctuated between \$15 and \$25 billion since the July 2013.[f] In comparison, U.S.-based trading activity in

[e] As we show in the next section, the estimates for $\mathbb{RV}_{m,n}(t)$ use the lagged time-t realized variance to proxy for the physical expectation of the future realized variance over [t, T_m].

[f] One major limitation of the data on the top panel of Fig. 1 is that the reporting rules of the Dodd–Frank Act implemented in 2013 allow firms to report a fixed cap amount on trades exceeding certain size thresholds to secure swap counterparty anonymity. On average 45 percent of swaption trades reported are capped.

currencies other than USD is limited. Bottom panel of Fig. 1 shows a heat map of the most traded tenor/expiry combinations for July 2015. The most traded swaption tenors are 1-, 2-, 5-, and 10-year swap rates. This might be expected since these tenors represent the most popular links between swap rates and benchmark Treasury securities. The volume in 1- and 10-year tenors stands out, indicating that active participants in the swaptions market are using swaptions for different purposes. For example, market commentaries suggest that short-expiry, short-tenor swaptions are used to express speculative trades, while longer-tenor swaptions are used to by quantitative investors, investors with exposures to the USD mortgage market and asset managers looking to hedge long-term liabilities against rate declines. Swaptions expiry volumes are more spread out, but shorter-tenor swaptions tend to have longer expiries than longer-tenor swaptions.

To construct the implied variance measure $\mathbb{IV}_{m,n}(t)$, we combine monthly (end-of-month) observations of (European) swaption prices from J.P. Morgan and Barclays Capital, two of the largest inter-dealer brokers in interest rate derivatives markets.[g] The swaption prices from J.P. Morgan are available from March 1993 with five strikes, namely, at-the-money-forward (ATMF), ATMF ± 100, and ATMF ± 50 basis points. The swaption prices from Barclays are available from January 2005 with thirteen strikes, namely, ATMF, ATMF ± 200, ATMF ± 150, ATMF ± 100, ATMF ± 75, ATMF ± 50, and ATMF ± 25 basis points. In our empirical analysis, we use swaption prices from J.P. Morgan from March 1993 through December 2004 and those from Barclays from January 2005 to February 2013, to obtain the maximum sample coverage.[h] Motivated by the fact illustrated on the top panel of Fig. 1 that both short- and long-tenors are actively traded, we use 12-month swaptions on multiple tenors, namely, 1, 2, 5, 10, and 20 years. The use of multiple tenors in the construction of our measure is likely to eliminate idiosyncratic movements associated with one single tenor and help capture the common dynamics of the market volatility risk.

To compute $A_{m,n}(t)$ that is needed to compute the implied variance measure (see equation (1)), we obtain LIBOR rates with maturities of 3, 6, 9, and 12 months, as well as 2-, 3-, 4-, 5-, 7-, 10-, 15-, 20-, 25-, and 30-year spot swap rates over our sample period, from J.P. Morgan and Barclays. We use a standard bootstrap procedure to obtain zero-coupon curves from the swap rates, and then compute $A_{m,n}(t)$.[i] We also compute the forward swap rates $S_{m,n}(t)$ based on these bootstrapped zero-coupon curves.

We approximate the integral involved in equation (1) by a discrete sum, a standard practice in the literature.[10,29,35] To obtain prices of payer swaption $\mathcal{P}_{m,n}(t;K)$ and receiver swaption $\mathcal{R}_{m,n}(t;K)$ on a dense set of strikes for the accuracy of the approximation, we follow the literature to interpolate implied volatilities across the range of observed strikes and use implied volatility of the lowest (highest) available strike to replace those of the strikes below (above). Specifically, we generate 200 implied volatility points that are equally spaced over a strike range with moneyness between $0.9 \times S_{m,n}(t)$ and $1.1 \times S_{m,n}(t)$, where $S_{m,n}(t)$ is the current forward swap rate on each day. This implied volatility/strike grid together with $A_{m,n}(t)$ and forward swap rates allows us to compute the empirical counterpart of the implied variance $\mathbb{IV}_{m,n}(t)$ in equation (1).

We then construct the realized variance $\mathbb{RV}_{m,n}(t)$ in equation (2) using daily series of 12-month forward swap rates on 1-, 2-, 5-, 10-, and 20-year tenors, with $M = 22$ (We use high-frequency intraday series of swap rates for robustness checks in Subsection 3.4). We take the difference between the implied and realized variance measures to compute the interest rate variance risk premium measure $\text{IRVRP}_{m,n}(t)$, according to equation (3), for each of the five tenors $n = 1$, 2, 5, 10, and 20 years. Our estimates for $\mathbb{RV}_{m,n}(t)$ use the lagged time-t realized variance to proxy for the physical expectation of the future realized variance over [t, T_m].[j] To obtain the market-level measure of interest rate variance risk premium, we then use the simple average of the five individual measures:

[g] Market participants quote the swaption prices using both the log-normal implied volatility of Black[36] and the normalized (absolute or basis point) implied volatility of a pricing formula based on normal distribution.

[h] All our empirical results remain little changed if we use the J.P. Morgan swaption data exclusively.

[i] We first use a standard cubic spline algorithm to interpolate the swap rates at semiannual intervals from one year to 30 years. We then solve for the zero curve by bootstrapping the interpolated par curve with swap rates as par bond yields. The day count convention is 30/360 for the fixed leg, and Actual/360 for the floating leg.

[j] Alternatively, we follow the literature and use the *heterogeneous autoregressive volatility model of realized volatility* (HAR-RV) of Andersen et al[37] and Corsi[38] in computing the physical expectation of the future realized variance (see Andersen et al[10] and Bollerslev et al[39] for more discussions). Results using these alternative estimates are similar.

Table 1
Summary statistics.

	Mean	SD	Min	Max	AR(1)	Mean	SD	Min	Max	AR(1)
A: asset returns										
	1-month holding period					3-month holding period				
2y	0.77	5.99	−16.01	19.05	0.18	0.68	3.66	−9.90	11.34	0.76
3y	1.40	9.80	−27.83	28.31	0.15	1.30	5.97	−16.37	17.67	0.75
4y	2.01	13.43	−38.06	39.46	0.13	1.91	8.08	−22.22	24.92	0.74
5y	2.58	16.88	−48.80	56.49	0.12	2.49	10.02	−27.54	31.07	0.73
6y	3.10	20.21	−62.85	74.66	0.10	3.02	11.83	−32.42	36.20	0.72
7y	3.57	23.46	−76.91	93.23	0.09	3.49	13.53	−36.92	40.47	0.71
8y	3.97	26.65	−90.85	111.54	0.08	3.89	15.16	−41.09	44.46	0.70
9y	4.31	29.79	−104.55	129.12	0.07	4.24	16.73	−44.97	50.33	0.69
10y	4.59	32.86	−117.88	145.61	0.06	4.54	18.25	−48.59	58.33	0.68
15y	5.49	46.81	−175.37	207.12	0.03	5.46	25.31	−63.17	94.25	0.67
20y	5.99	58.32	−219.92	236.57	0.03	5.99	32.14	−74.55	127.20	0.67
Equity	6.02	52.50	−204.90	130.50	0.09	6.08	31.79	−133.26	93.31	0.73
	6-month holding period					12-month holding period				
2y	0.53	2.38	−4.36	7.64	0.87	0.23	1.18	−2.70	3.12	0.94
3y	1.15	4.06	−7.44	13.98	0.86	0.86	2.33	−5.15	6.69	0.93
4y	1.74	5.59	−10.57	19.56	0.85	1.46	3.34	−7.10	9.67	0.92
5y	2.29	7.00	−13.51	24.31	0.84	2.01	4.24	−8.67	12.12	0.91
6y	2.79	8.31	−16.20	28.25	0.83	2.50	5.07	−10.36	14.13	0.91
7y	3.23	9.53	−18.71	31.50	0.82	2.93	5.85	−11.94	15.90	0.90
8y	3.60	10.69	−21.04	34.15	0.81	3.30	6.58	−13.56	17.64	0.89
9y	3.93	11.79	−23.24	36.29	0.81	3.62	7.28	−15.09	19.14	0.89
10y	4.20	12.85	−25.30	37.99	0.80	3.88	7.95	−16.55	20.80	0.88
15y	5.07	17.65	−33.96	53.83	0.79	4.72	10.86	−23.18	30.68	0.87
20y	5.54	22.32	−52.03	71.49	0.79	5.16	13.55	−30.21	39.01	0.87
Equity	6.09	24.04	−102.99	68.33	0.86	6.16	18.11	−54.24	42.65	0.94
B: variance risk premium measures and forward spreads										
	Mean	SD	Min	Max	AR(1)					
IRVRP	1.54	0.91	0.58	3.86	0.97					
EVRP	0.17	0.23	−2.19	1.16	0.23					
FS 2y	0.46	0.53	−0.64	1.96	0.93					
FS 5y	1.62	1.37	−0.56	4.09	0.97					
FS 10y	2.67	1.80	−0.43	5.72	0.98					
FS 20y	2.40	1.72	−0.80	5.35	0.97					

Note: this table presents summary statistics, including the mean, standard deviation (SD), minimum (Min), maximum (Max), and AR(1) coefficient, of our interest rate variance risk premium measure (IRVRP), the equity variance risk premium measure of Bollerslev et al[10] using S&P500 index options, and forward spreads equal to the difference between one-year forward rate τ years ahead and the one-year zero coupon yield (in Panel A), and of the Treasury and equity market excess returns (in Panel B). Both the forward spreads are excess returns are in percentage points. We consider forward spreads and Treasury excess returns with tenors of $\tau = 2, 3, 4, 5, 6, 7, 8, 9, 10, 15$, and 20 years. Four holding horizons of returns are included, 1-, 3-, 6-, and 12-months. Data is monthly and runs from March 1993 through February 2013.

$$\mathrm{IRVRP}_t = \sum_{n=1,2,5,10,20} \mathrm{IRVRP}_{m,n}(t) \Big/ 5, \tag{4}$$

where $m = 12$ months.

Panel A of Table 1 reports summary statistics of IRVRP. We observe that IRVRP seems to be quite persistent with an AR(1) coefficient of 0.97. The top panel of Fig. 2 plots monthly series of the interest-rate variance risk premium measure IRVRP_t. We observe that IRVRP increased dramatically around 2002 and during the recent financial crisis. In addition, IRVRP also increased notably amid the European financial crisis in the second half of 2011.

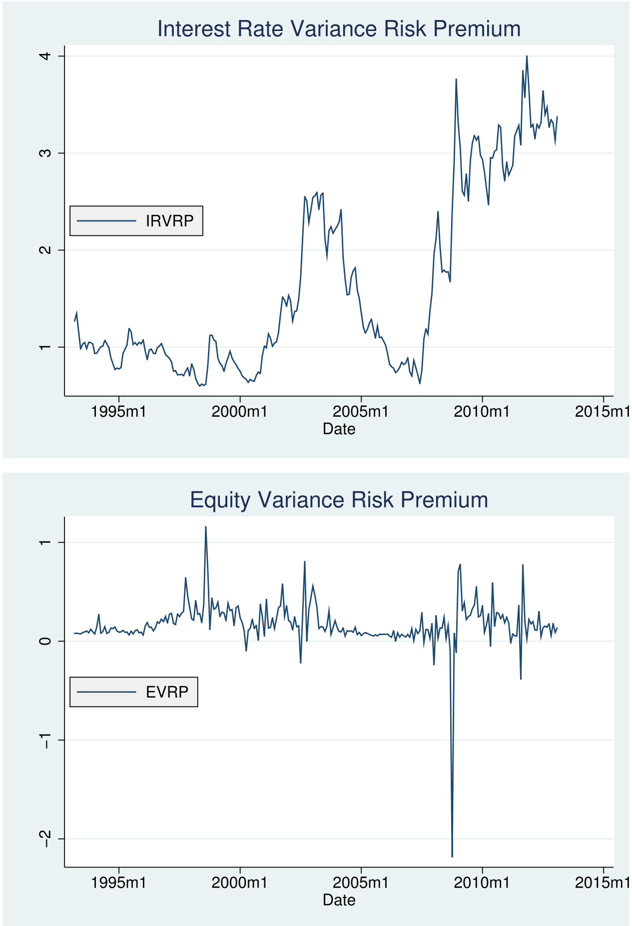


Fig. 2. Time series of the interest rate and equity variance risk premium. This figure plots monthly series of the interest rate variance risk premium (top panel) and equity variance risk premium (bottom panel). The former is computed as the simple average of the five individual interest rate variance risk premium measures on 1-, 2-, 5-, 10-, and 20-year tenors based on 12-month swaptions on corresponding tenors, while the latter is based on S&P 500 index options, as in Bollerslev et al[10] The sample period is March 1993–February 2013.

2.2. Asset returns

To compute Treasury bond returns, we use the zero−coupon Treasury yield data of Gürkaynak et al[40] with 2-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-, 15-, and 20-year maturities from March 1993 to February 2013. Specifically, let $p_t^{(\tau)}$ be the log price of a $\tau -$ year zero-coupon Treasury security at time t. Its $h -$ period log return is

$$r_{t+h}^{(\tau)} = p_{t+h}^{(\tau-h)} - p_t^{(\tau)}, \tag{5}$$

where $h = 1$, 3, 6, and 12 months. The corresponding excess returns are

$$rx_{t+h}^{(\tau)} = r_{t+h}^{(\tau)} - y_t^{(h)}, \tag{6}$$

where $y_t^{(h)}$ is the $h -$ period zero-coupon rate at time t. Furthermore, we use the continuously compounded returns on the S&P 500 index at the monthly frequency, including dividends, from Center for Research in Security Press (CRSP), as equity market returns.

Summary statistics of annualized excess returns (in percentage points) of the Treasury securities and equity market are presented in Panel B of Table 1. We observe that (time-series) average excess returns increase with the tenor of the underlying security for all four different holding horizons. Holding the tenor fixed, the average excess returns also increase monotonically with the holding horizon. In particular, average excess returns of Treasuries are mostly negative up to the 6-month holding horizon, and turn positive for the 12-month holding horizon. Moreover, the autocorrelation of the excess returns series is higher for shorter tenors and longer holding horizons, though being low for long-tenor assets including the Treasuries of longer than 10 years and equity market at the 1-month holding horizon.

2.3. Additional return predictors

In our empirical analysis, we compare our interest rate variance risk premium mainly with two established return predictors in the literature, the equity variance risk premium of Bollerslev et al[10] and the forward spread of Fama and Bliss.[1] The equity variance risk premium is also constructed as the difference between option-implied variance and realized variance, and hence may capture a similar fashion of volatility risk as our interest rate variance risk premium. It is thus important to investigate if the IRVRP is associated with a distinctive channel of volatility risk and possess different return predictive power. The forward spread is considered because our interest rate variance risk premium is constructed using derivative prices on interest rates, and hence it is important to study whether the IRVRP only captures information that is already in the yield curve.

The bottom panel of Fig. 2 plots monthly series of the equity variance risk premium measure, and panel A of Table 1 reports its summary statistics. We observe that the equity variance risk premium spikes around 1998, and reaches a deeply negative value in 2008. Moreover, it is much less persistent, with the AR(1) coefficient about 0.23. In sum, the interest rate and equity variance risk premium seems to exhibit distinctive dynamics.

Panel A of Table 1 also reports the summary statistics of the forward spreads, defined as the difference between one-year forward rate τ (2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years ahead and the one-year zero coupon yield, using the Treasury yield data of Gürkaynak et al.[40] We observe that the forward spread increases with the tenor τ, and is mostly as persistent as the interest rate variance risk premium, with AR(1) coefficients between 0.92 and 0.97.

In addition to the interest rate and equity variance risk premium as well as the forward spreads, we also consider two sets of bond and equity return predictors, respectively, as additional controls. For bond returns, we include the economic growth measured by the three-month moving average of the Chicago Fed National Activity Index and the expected inflation measured by the forecast consensus of future inflation from Blue Chip Financial Forecasts. These two macro variables have been shown to drive the term structure dynamics significantly.[6,41] For the equity market return, we include traditional predictors including the log dividend price ratio, the log earnings price ratio, the net equity expansion factor of Goyal and Welch[42] (obtained from Amit Goyal's webpage), and the default spread equal to the difference between Moody's BAA and AAA corporate bond spreads.

3. Empirical results

In this section, we report empirical results of (both bond and equity) return predictive regressions for 1-, 3-, 6-, and 12-month holding horizons. We start with simple univariate regressions to document the role of our main explanatory variable, i.e., interest rate variance risk premium. We then include the equity variance risk premium and forward spread in the predictive regressions to document the differential predictive power of these three factors for different tenors of the underlying assets. Finally, we control for other well-known predictors of bond and equity risk premia for robustness.

The start date for all regressions is March 1993. All regressions are standardized, in the sense that each variable is first demeaned and then divided by its standard deviation. Such standardized regressions make slope coefficients comparable across different regressors, allowing a comparison of both statistical and economic significance. We report *t*-statistics adjusted for Newey and West[43] standard errors, with the optimal lag length determined according to Newey and West[44] for each estimated coefficient.

3.1. Return predictability with interest rate variance risk premium

We first assess the predictive power of the interest rate variance risk premium for Treasury and equity excess returns by the following univariate regressions:

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)\cdot \mathrm{IRVRP}_t + \varepsilon_{t+h}^{(\tau)}, \tag{7}$$

Table 2
Univariate predictive regressions: interest rate variance risk premium.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
1-month holding period												
IRVRP	0.871**	0.836**	0.794**	0.752**	0.711**	0.672**	0.635**	0.600**	0.568**	0.437**	0.346**	0.068
	(12.570)	(12.374)	(12.407)	(12.630)	(13.101)	(13.590)	(12.868)	(11.269)	(9.648)	(9.319)	(6.355)	(1.273)
R^2	0.759	0.699	0.631	0.565	0.505	0.451	0.403	0.360	0.322	0.191	0.120	0.005
3-month holding period												
IRVRP	0.792**	0.682**	0.595**	0.531**	0.484**	0.447**	0.416**	0.388**	0.364**	0.259**	0.172*	0.125
	(10.532)	(9.436)	(8.611)	(7.941)	(7.435)	(6.990)	(6.549)	(6.125)	(5.710)	(3.707)	(2.067)	(1.532)
R^2	0.627	0.465	0.354	0.282	0.234	0.199	0.173	0.151	0.132	0.067	0.030	0.016
6-month holding period												
IRVRP	0.621**	0.455**	0.369**	0.320**	0.289**	0.267**	0.249**	0.234*	0.221*	0.156	0.089	0.179
	(7.093)	(5.196)	(4.235)	(3.639)	(3.230)	(2.930)	(2.698)	(2.508)	(2.343)	(1.602)	(0.844)	(1.554)
R^2	0.386	0.207	0.136	0.102	0.083	0.071	0.062	0.055	0.049	0.024	0.008	0.032
12-month holding period												
IRVRP	−0.037	0.019	0.060	0.089	0.109	0.122	0.130	0.135	0.136	0.112	0.057	0.254^{+}
	(−0.307)	(0.151)	(0.473)	(0.666)	(0.779)	(0.842)	(0.877)	(0.892)	(0.894)	(0.741)	(0.374)	(1.725)
R^2	0.001	0.000	0.004	0.008	0.012	0.015	0.017	0.018	0.019	0.012	0.003	0.065

Note: this table reports the univariate regression $rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)\cdot\text{IRVRP}_t + \varepsilon^{(\tau)}_{t+h}$,
where $rx^{(\tau)}_{t+h}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$ tenor, and IRVRP_t is the interest rate variance risk premium measure. The t-statistics presented in parentheses are calculated using Newey and West [43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^{+}$ for $p < 0.1$, where p is the p-value.

where $rx^{(\tau)}_{t+h}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$ tenor, and IRVRP_t is the interest rate variance risk premium measure.

The regression results are reported in Table 2. We observe that IRVRP significantly predicts short-horizon bond excess returns positively with solid t-statistics based on Newey-West robust standard errors, up to 6-month holding horizons. For example, one standard deviation increase in IRVRP leads to a 201 (=0.568 × 3.53%), 181 (=0.364 × 4.96%), and 146 (=0.221 × 6.60%) basis point increase (slope coefficient times the standard deviation of the 10-year bond excess return) increase in expected 10-year bond excess returns, at the 1-, 3-, and 6-month holding horizons, respectively. Adjusted R^2s range from 5% for the 6-month holding horizon to 32% for the 1-month holding horizon.

Furthermore, both the economic and statistical significance of IRVRP monotonically decrease with the tenor of the asset. For example, the regression coefficient, t-statistics, and adjusted R^2s all decreasing functions of the asset tenor, and IRVRP loses the significance for equity market returns that has a tenor of ∞. Top panels of Fig. 3 plot the univariate regression results for the 3-month holding horizon, with the estimated coefficient $\beta^{\tau}_1(3)$ of IRVRP in the top left panel, and the associated adjusted R^2 in the top right panel, against the tenor τ. The regression coefficients monotonically decrease from about 0.8 to 0.2 for Treasuries when the maturity increases from 2 years to 20 years and to about 0.1 for the equity market return. Correspondingly, the adjusted R^2 decreases from about 60% to 3% and to less than 2% for Treasuries and equities, respectively.

To summarize, IRVRP is a strong predictor of short-horizon asset excess returns, and its predictive power has a sharp monotonically decreasing pattern with the tenor of the asset.

3.2. Interest rate vs equity variance risk premium

Our interest rate variance risk premium is constructed in a similar way to the equity variance risk premium – both are the difference between option-implied variance and realized variance. Moreover, as shown in the last subsection, the predictive power of IRVRP remains the strongest at short-holding horizons, the same pattern as the EVRP's predictive power for the equity market return as first established by Bollerslev et al.[10] Hence, it is natural to expect that IRVRP and EVRP both capture a *similar* fashion of "short-horizon" volatility risk. But do they capture the *same* type of volatility risk? If not, what is the channel in distinguishing them?

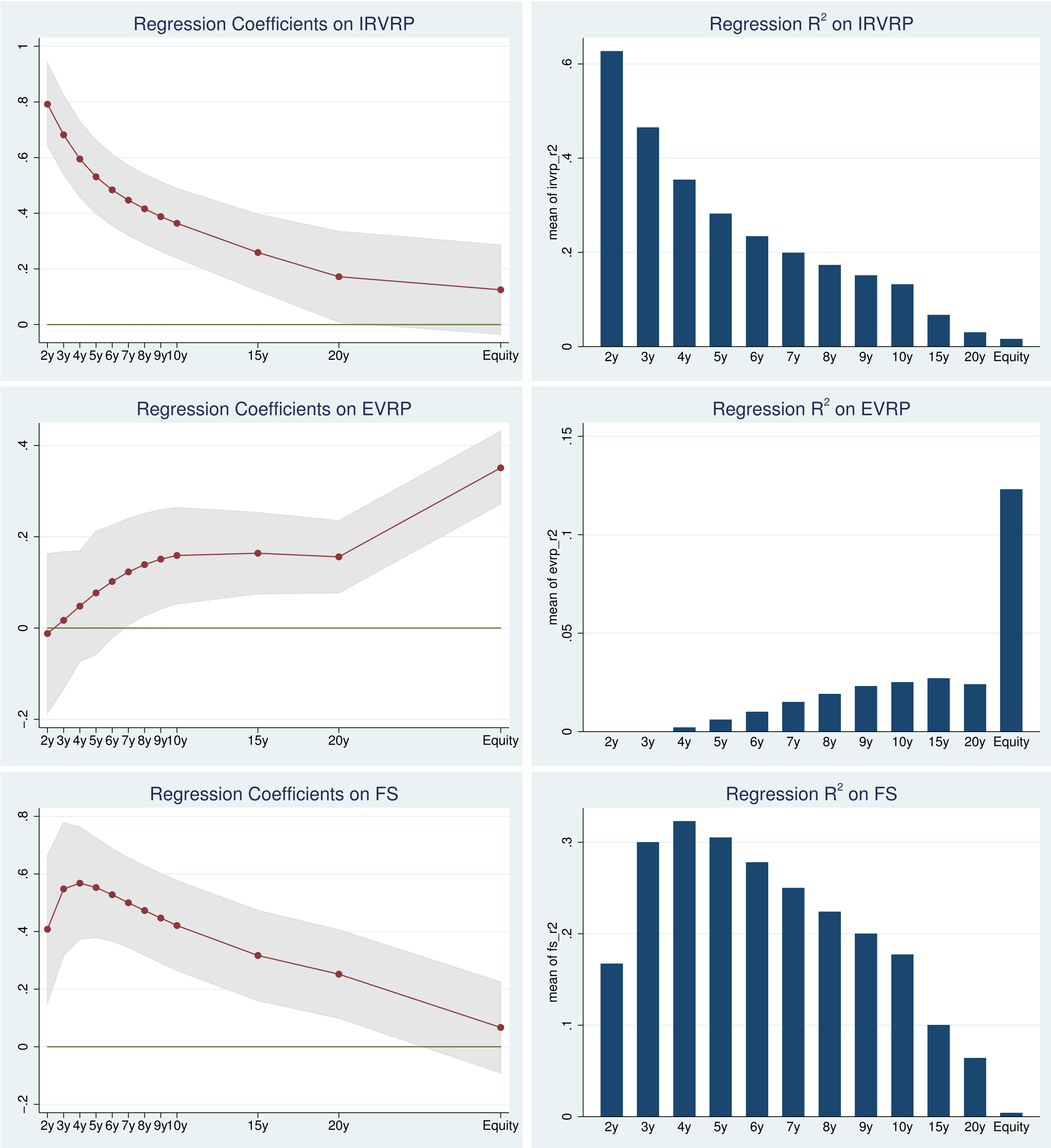


Fig. 3. Univariate regression coefficients. This figure plots the estimated coefficients (left panels) and adjusted R^2s (right panels) of univariate regressions of the Treasury and equity market excess returns on the interest rate variance risk premium, the equity variance risk premium, and the forward spread, in top, middle, and bottom panels, respectively. The shaded areas in the left panels represent confidence levels of the regression coefficients at the 95% significance level. All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. The holding horizon of the excess returns is three months, and both the estimated coefficients and adjusted R^2s are plotted against the asset tenor τ, equal to 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20 years for Treasuries, and ∞ for the equity market portfolio.

Table 3
Univariate predictive regressions: equity variance risk premium.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
1-month holding period												
EVRP	0.023	0.009	−0.010	−0.031	−0.052	−0.073	−0.092	−0.109	−0.124+	−0.173**	−0.199**	0.265**
	(0.255)	(0.101)	(−0.116)	(−0.388)	(−0.688)	(−0.995)	(−1.294)	(−1.579)	(−1.845)	(−2.946)	(−3.762)	(5.727)
R^2	0.001	0.000	0.000	0.001	0.003	0.005	0.008	0.012	0.015	0.030	0.039	0.070
3-month holding period												
EVRP	0.012	−0.017	−0.048	−0.077	−0.102	−0.123*	−0.139*	−0.151**	−0.159**	−0.164**	−0.156**	0.351**
	(0.130)	(−0.221)	(−0.774)	(−1.115)	(−1.631)	(−2.075)	(−2.439)	(−2.725)	(−2.948)	(−3.597)	(−3.876)	(8.673)
R^2	0.000	0.000	0.002	0.006	0.010	0.015	0.019	0.023	0.025	0.027	0.024	0.123
6-month holding period												
EVRP	0.014	−0.020	−0.052	−0.079	−0.103	−0.122+	−0.137*	−0.147**	−0.154**	−0.150*	−0.128+	0.274**
	(0.139)	(−0.224)	(−0.640)	(−1.088)	(−1.525)	(−1.928)	(−2.297)	(−2.800)	(−2.940)	(−2.453)	(−1.849)	(5.286)
R^2	0.000	0.000	0.003	0.006	0.011	0.015	0.019	0.022	0.024	0.023	0.016	0.075
12-month holding period												
EVRP	0.032	0.015	0.004	−0.009	−0.021	−0.032	−0.042	−0.050	−0.056	−0.066	−0.058	0.156**
	(0.342)	(0.166)	(0.045)	(−0.101)	(−0.215)	(−0.336)	(−0.459)	(−0.563)	(−0.650)	(−0.872)	(−0.859)	(2.576)
R^2	0.001	0.000	0.000	0.000	0.000	0.001	0.002	0.003	0.003	0.004	0.003	0.024

Note: this table reports the univariate regression $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h) \cdot \mathrm{EVRP}_t + \varepsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$ tenor, and EVRP_t is the equity variance risk premium measure constructed by Bollerslev et al[10] using S&P 500 index options. The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^+$ for $p < 0.1$, where p is the p-value.

In this subsection, we study the predictive power of the equity variance risk premium for both the bond and equity market excess returns. Most importantly, we document distinguishing patterns of return predictive power of IRVRP and EVRP, which shed light on different potential channels of volatility risk in driving asset prices. In particular, we first consider the univariate regressions:

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h) \cdot \mathrm{EVRP}_t + \varepsilon_{t+h}^{(\tau)}, \tag{8}$$

where $rx_{t+h}^{(\tau)}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$ tenor, and EVRP_t is the equity variance risk premium measure constructed by Bollerslev et al[10] using S&P 500 index options.

The regression results are reported in Table 3. We observe that EVRP significantly predicts short-horizon excess returns of the long-term Treasury securities and equity market return. The regression coefficient for the equity market return is positive, and remains the most significant at the 3-month holding horizon with t-statistics about 8.67 and adjusted R^2 about 12%, consistent with Bollerslev et al.[10] For bond returns, the regression coefficient is negative, consistent with the negative correlation between the equity and Treasury in the recent decades, as documented in Campbell et al.[45]

To compare with the return predictive power of IRVRP, we report the univariate regression results for 3-month holding horizon in middle panels of Fig. 3, with the estimated coefficient $\beta_1^{\tau}(3)$ of EVRP in the middle left panel, and the associated adjusted R^2 in the middle right panel, against the tenor τ. For the convenience of comparing the economic significance, we report the absolute values of the regression coefficients on EVRP for bonds. We find that the regression coefficients and adjusted R^2s of EVRP all monotonically *increase* with the tenor of the asset, in sharp contrast to the *decreasing* predictive power of IRVRP. This sharp contrast suggests distinguishing economic effects of interest rate and equity variance risk premium: IRVRP seems to capture the volatility risk of short tenor, while EVRP seems capture the volatility risk of long tenor.

Given the distinctive pattern of return predictive power of the interest rate and equity variance risk premium, we expect the return predictability of IRVRP is robust to the inclusion of EVRP. To verify this conjecture, we run the following multivariate return predictive regressions:

Table 4
Multivariate predictive regressions: interest rate and equity variance risk premium.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
1-month holding period												
IRVRP	0.871**	0.836**	0.795**	0.753**	0.713**	0.674**	0.638**	0.604**	0.571**	0.442**	0.352**	0.061
	(12.384)	(12.238)	(12.481)	(12.731)	(12.932)	(13.068)	(13.100)	(13.010)	(12.796)	(10.353)	(5.346)	(1.221)
EVRP	−0.001	−0.014	−0.032	−0.051	−0.072+	−0.092*	−0.110*	−0.126*	−0.140**	−0.185**	−0.208**	0.263**
	(−0.043)	(−0.513)	(−1.015)	(−1.398)	(−1.697)	(−1.960)	(−2.200)	(−2.423)	(−2.632)	(−3.454)	(−3.210)	(5.999)
R^2	0.759	0.699	0.632	0.568	0.510	0.459	0.415	0.376	0.342	0.225	0.163	0.074
3-month holding period												
IRVRP	0.792**	0.683**	0.596**	0.534**	0.487**	0.450**	0.420**	0.393**	0.369**	0.264**	0.176*	0.116
	(10.606)	(9.458)	(8.613)	(7.842)	(7.106)	(6.597)	(6.135)	(5.903)	(5.596)	(3.689)	(2.105)	(1.645)
EVRP	−0.010	−0.036	−0.064*	−0.092**	−0.116**	−0.136**	−0.151**	−0.162**	−0.169**	−0.171**	−0.161**	0.348**
	(−0.361)	(−1.341)	(−2.366)	(−3.055)	(−3.382)	(−3.606)	(−3.722)	(−3.896)	(−4.011)	(−4.065)	(−3.936)	(7.839)
R^2	0.627	0.466	0.358	0.290	0.247	0.218	0.195	0.177	0.161	0.096	0.055	0.137
6-month holding period												
IRVRP	0.621**	0.456**	0.371**	0.322**	0.292**	0.270**	0.253**	0.238*	0.225*	0.160	0.093	0.172+
	(7.061)	(4.995)	(3.937)	(3.305)	(2.914)	(2.732)	(2.626)	(2.469)	(2.308)	(1.595)	(0.860)	(1.763)
EVRP	−0.003	−0.033	−0.062	−0.088+	−0.111*	−0.130**	−0.144**	−0.154**	−0.160**	−0.155**	−0.130+	0.269**
	(−0.055)	(−0.626)	(−1.216)	(−1.743)	(−2.204)	(−2.621)	(−2.997)	(−3.234)	(−3.354)	(−2.779)	(−1.880)	(4.616)
R^2	0.386	0.208	0.140	0.110	0.096	0.088	0.083	0.078	0.074	0.048	0.025	0.105
12-month holding period												
IRVRP	−0.038	0.018	0.060	0.089	0.109	0.123	0.132	0.136	0.138	0.113	0.058	0.250+
	(−0.310)	(0.145)	(0.459)	(0.648)	(0.767)	(0.843)	(0.878)	(0.894)	(0.896)	(0.745)	(0.381)	(1.843)
EVRP	0.033	0.015	0.002	−0.011	−0.024	−0.036	−0.046	−0.054	−0.060	−0.069	−0.060	0.149*
	(0.321)	(0.151)	(0.021)	(−0.120)	(−0.269)	(−0.418)	(−0.555)	(−0.675)	(−0.776)	(−1.003)	(−0.910)	(2.014)
R^2	0.002	0.001	0.004	0.008	0.012	0.016	0.019	0.021	0.022	0.017	0.007	0.087

Note: this table reports the multivariate regression.
$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)\cdot\text{IRVRP}_t + \beta_2^{(\tau)}(h)\cdot\text{EVRP}_t + \varepsilon_{t+h}^{(\tau)}$,
where $rx_{t+h}^{(\tau)}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$ tenor, IRVRP$_t$ is the interest rate variance risk premium measure, and EVRP$_t$ is the equity variance risk premium measure constructed by Bollerslev et al[10] using S&P 500 index options. The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^+$ for $p < 0.1$, where p is the p-value.

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)\cdot\text{IRVRP}_t + \beta_2^{(\tau)}(h)\cdot\text{EVRP}_t + \varepsilon_{t+h}^{(\tau)}. \tag{9}$$

Results are reported in Table 4. Controlling for the equity variance risk premium, the predictability of the interest rate variance risk premium is almost unchanged. Its monotonically decreasing significance as a function of the asset tenor remains the same.

3.3. Interest rate variance risk premium vs forward spread

It has been well established in the literature that yield-based factors are strong predictors of bond returns.[1,3,46] As our interest rate variance risk premium measure is constructed using prices of swaptions that are derivatives on yields in principal, it is natural to ask whether IRVRP is a mere reflection of information already contained in the yield curve.

Most term structure models use the first three principal components of the yield curve as factors because they capture most of the variation in yields. Among the three principal components, the slope factor or the spread between long-term and short-term yields has been shown to possess significant predictive power for bond risk premia.[1,46] In this subsection, we study the predictive power of the forward spread, as proposed in Fama and Bliss,[1] for both the bond and equity market excess returns, and compare its return predictive power with that of the interest rate variance risk premium. Importantly, we document that the pattern of return predictive power of IRVRP is distinct from and robust to that

Table 5
Univariate predictive regressions: forward spread.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
1-month holding period												
FS	0.409**	0.612**	0.688**	0.706**	0.698**	0.678**	0.652**	0.624**	0.594**	0.459**	0.366**	0.039
	(3.246)	(7.997)	(9.152)	(10.639)	(11.851)	(12.142)	(11.975)	(11.330)	(10.675)	(8.233)	(6.398)	(0.777)
R^2	0.167	0.374	0.474	0.499	0.488	0.460	0.425	0.389	0.353	0.210	0.134	0.002
3-month holding period												
FS	0.408**	0.548**	0.568**	0.553**	0.528**	0.500**	0.473**	0.447**	0.421**	0.317**	0.252**	0.067
	(3.122)	(4.657)	(5.712)	(6.287)	(6.439)	(6.291)	(5.991)	(5.640)	(5.291)	(3.968)	(3.219)	(0.830)
R^2	0.167	0.300	0.323	0.305	0.278	0.250	0.224	0.200	0.177	0.100	0.064	0.004
6-month holding period												
FS	0.360*	0.440**	0.438**	0.420**	0.399**	0.378**	0.357**	0.338**	0.319**	0.248*	0.208*	0.084
	(2.217)	(3.020)	(3.371)	(3.548)	(3.569)	(3.483)	(3.340)	(3.175)	(3.008)	(2.361)	(1.975)	(0.845)
R^2	0.129	0.193	0.192	0.176	0.159	0.143	0.128	0.114	0.102	0.061	0.043	0.007
12-month holding period												
FS	0.049	0.100	0.155	0.198	0.225	0.240^{+}	0.246^{+}	0.246^{+}	0.241^{+}	0.204	0.181	0.199
	(0.248)	(0.521)	(0.890)	(1.244)	(1.519)	(1.697)	(1.786)	(1.810)	(1.793)	(1.503)	(1.265)	(1.496)
R^2	0.002	0.010	0.024	0.039	0.051	0.058	0.061	0.060	0.058	0.042	0.033	0.039

Note: this table reports the univariate regression.
$rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)\cdot \mathrm{FS}^{(\tau)}_t + \varepsilon^{(\tau)}_{t+h}$,
where $rx^{(\tau)}_{t+h}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau=\infty$ tenor, and $\mathrm{FS}^{(\tau)}_t$ is the forward spread between the one-year forward rate τ years ahead and the one-year zero coupon yield. The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^{+}$ for $p < 0.1$, where p is the p-value.

of the FS (We also differentiate the return predictive power of interest rate variance risk premium from that of the ten-shape yield factor of Cochrane and Piazzesi[3] as a robustness check in Subsection 3.4).

In particular, we first consider the following univariate regressions:

$$rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)\cdot \mathrm{FS}^{(\tau)}_t + \varepsilon^{(\tau)}_{t+h}, \tag{10}$$

where $rx^{(\tau)}_{t+h}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau=\infty$ tenor, and $\mathrm{FS}^{(\tau)}_t$ is the forward spread between the one-year forward rate τ-year ahead and the on-year zero coupon yield.

The regression results are reported in Table 5. We observe that the forward spread significantly predicts short-horizon Treasury excess returns, similar to both the interest rate and equity variance risk premium. The regression coefficient is positive with solid t-statistics based on Newey-West robust standard errors, consistent with Fama and Bliss.[1] Different from the interest rate and equity variance risk premium, however, neither the economic nor statistical significance of the forward spread is a monotonic function of the asset tenor; instead, its predictive power seems to peak at the medium tenor, around five years.

To have a sharp comparison, we report the univariate regression results of the forward spread for the 3-month holding horizon in bottom panels of Fig. 3, with the estimated coefficient $\beta^{\tau}_1(3)$ of FS in the bottom left panel, and the associated adjusted R^2 in the bottom right panel, against the tenor τ. We observe that both the regression coefficients and adjusted R^2s of the forward spread increase from short tenor to medium tenor, and hence decrease moving to long tenor.

This distinctive pattern of return predictive power suggests that our interest rate variance risk premium captures a distinctive economic channel of risk premia than the forward spread, a well-established yield-based factor in the literature. To formally substantiate this conclusion, we run the following multivariate return predictive regressions:

$$rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)\cdot \mathrm{IRVRP}_t + \beta^{(\tau)}_2(h)\cdot \mathrm{FS}^{(\tau)}_t + \varepsilon^{(\tau)}_{t+h}. \tag{11}$$

Results are reported in Table 6. We find that controlling for the forward spread, the predictability of the interest rate variance risk premium weakens somewhat, especially at the 6-month holding horizon. However, the strong return predictive power remains strongly at the very short holding horizons, especially at the 1-month horizon. Moreover, the

Table 6
Multivariate predictive regressions: interest rate variance risk premium and forward spread.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
1-month holding period												
IRVRP	0.830**	0.704**	0.597**	0.509**	0.439**	0.382**	0.336**	0.300**	0.272**	0.209**	0.172*	0.083
	(14.542)	(11.418)	(8.106)	(5.998)	(4.762)	(4.005)	(3.471)	(3.127)	(2.890)	(2.619)	(2.131)	(0.816)
FS	0.290**	0.339**	0.369**	0.389**	0.401**	0.405**	0.403**	0.396**	0.385**	0.300**	0.243**	−0.021
	(4.592)	(5.512)	(5.545)	(5.392)	(5.282)	(5.205)	(5.111)	(5.041)	(4.903)	(4.072)	(3.238)	(−0.217)
R^2	0.842	0.796	0.728	0.658	0.592	0.531	0.476	0.427	0.383	0.228	0.148	0.005
3-month holding period												
IRVRP	0.749**	0.552**	0.407**	0.305*	0.233+	0.181	0.144	0.117	0.098	0.042	−0.019	0.159
	(11.286)	(6.424)	(3.777)	(2.443)	(1.704)	(1.270)	(0.977)	(0.784)	(0.655)	(0.302)	(−0.136)	(1.225)
FS	0.301**	0.333**	0.350**	0.363**	0.370**	0.371**	0.367**	0.358*	0.346*	0.285*	0.266*	−0.047
	(3.319)	(3.252)	(3.077)	(2.961)	(2.880)	(2.756)	(2.668)	(2.567)	(2.456)	(2.125)	(2.007)	(−0.347)
R^2	0.715	0.559	0.441	0.362	0.308	0.266	0.233	0.205	0.181	0.101	0.064	0.017
6-month holding period												
IRVRP	0.582**	0.334**	0.189	0.095	0.033	−0.008	−0.035	−0.051	−0.060	−0.079	−0.124	0.246+
	(6.707)	(2.916)	(1.339)	(0.592)	(0.188)	(−0.046)	(−0.182)	(−0.259)	(−0.302)	(−0.415)	(−0.664)	(1.942)
FS	0.276+	0.310+	0.337*	0.361*	0.377*	0.384*	0.383*	0.376+	0.365+	0.308	0.296	−0.093
	(1.959)	(1.940)	(1.969)	(2.006)	(2.023)	(2.008)	(1.961)	(1.892)	(1.815)	(1.555)	(1.509)	(−0.633)
R^2	0.461	0.288	0.217	0.182	0.160	0.143	0.128	0.115	0.103	0.064	0.050	0.036
12-month holding period												
IRVRP	−0.045	−0.024	−0.032	−0.056	−0.082	−0.103	−0.116	−0.121	−0.120	−0.106	−0.152	0.230
	(−0.372)	(−0.162)	(−0.177)	(−0.271)	(−0.361)	(−0.421)	(−0.450)	(−0.453)	(−0.442)	(−0.404)	(−0.618)	(1.384)
FS	0.056	0.109	0.172	0.232	0.281	0.314	0.332	0.337	0.334	0.285	0.290	0.033
	(0.281)	(0.499)	(0.742)	(0.960)	(1.121)	(1.219)	(1.260)	(1.262)	(1.241)	(1.108)	(1.121)	(0.188)
R^2	0.004	0.011	0.025	0.041	0.054	0.063	0.067	0.067	0.064	0.046	0.044	0.065

Note: this table reports the univariate regression.
$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)\cdot\text{IRVRP}_t + \beta_2^{(\tau)}(h)\cdot\text{FS}_t^{(\tau)} + \varepsilon_{t+h}^{(\tau)}$,
where $rx_{t+h}^{(\tau)}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$ tenor, IRVRP_t is the measure of interest rate variance risk premium, and $\text{FS}_t^{(\tau)}$ is the forward spread between the one-year forward rate τ years ahead and the one-year zero coupon yield. The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and + for $p < 0.1$, where p is the p-value.

pattern of monotonically decreasing significance as a function of the asset tenor remains the same.[11] This empirical result is rationalized within our long-run-risk type model presented in Section 4, where we show that the IRVRP is driven by only one state variable, namely, vol-of-vol factor q_t (see equation (A.45) in Appendix A.7), that reflects the nature of short-run risks, whereas the forward spread is the function of all state variables (see equation (A.34) in Appendix A.6), and thus captures both short- and long-run risks.

3.4. Robustness checks

3.4.1. Subsample analysis

One may be concerned that the recent financial crisis could be the single important driver of our empirical results. To address this concern, we run the return predictive regressions for two subsamples, from March 1993 to December 2003 and from January 2004 to February 2013. Table 7 reports the subsample regression results. We find that the interest rate variance risk premium has similar significant return predictive power in the two subsample periods, and is in fact even stronger, if anything, in the first subsample excluding the recent financial crisis. The monotonically decreasing significance of the interest rate variance risk premium as a function of the asset tenor remains little changed.

[11] We also run multivariate regressions of asset excess returns on the interest rate variance risk premium, controlling for both the forward spreads and the equity variance risk premium, as reported in Table B1. Results further confirm the differential return predictive power of our interest rate variance risk premium factor from the other two.

Table 7
Subsample analysis.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20yy	Equity
A: sample period: 1993–2003 (N = 129)												
1-month holding period												
IRVRP	1.290**	1.228**	1.150**	1.069**	0.991**	0.919**	0.854**	0.794**	0.741**	0.550**	0.432**	0.125
	(10.013)	(10.074)	(9.656)	(9.213)	(6.931)	(8.193)	(8.774)	(9.772)	(9.387)	(7.666)	(6.209)	(1.100)
R^2	0.662	0.576	0.488	0.411	0.349	0.299	0.258	0.225	0.198	0.111	0.067	0.004
3-month holding period												
IRVRP	1.226**	1.052**	0.892**	0.767**	0.670**	0.594**	0.533**	0.481**	0.438**	0.291*	0.195^{+}	0.166
	(9.278)	(11.816)	(9.789)	(7.986)	(6.662)	(5.697)	(4.970)	(4.403)	(3.946)	(2.525)	(1.705)	(0.891)
R^2	0.517	0.349	0.241	0.175	0.133	0.105	0.084	0.069	0.058	0.026	0.013	0.009
6-month holding period												
IRVRP	1.004**	0.713**	0.533**	0.420**	0.341*	0.284^{+}	0.240	0.205	0.176	0.078	0.005	0.147
	(6.438)	(4.770)	(3.373)	(2.604)	(2.108)	(1.757)	(1.492)	(1.282)	(1.109)	(0.508)	(0.035)	(0.574)
R^2	0.287	0.135	0.075	0.047	0.031	0.022	0.016	0.012	0.009	0.002	0.000	0.008
12-month holding period												
IRVRP	0.061	0.039	0.033	0.028	0.025	0.021	0.017	0.012	0.007	−0.032	−0.085	0.247
	(0.248)	(0.162)	(0.144)	(0.133)	(0.123)	(0.110)	(0.092)	(0.069)	(0.039)	(−0.195)	(−0.527)	(0.808)
R^2	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.019
B: sample period: 2004–2013 (N = 110)												
1-month holding period												
IRVRP	0.752**	0.721**	0.687**	0.653**	0.622**	0.592**	0.564**	0.536**	0.509**	0.389**	0.294**	0.119^{+}
	(9.136)	(8.848)	(8.597)	(8.445)	(8.359)	(8.282)	(8.165)	(7.978)	(7.717)	(5.723)	(3.679)	(1.911)
R^2	0.798	0.733	0.656	0.580	0.509	0.445	0.387	0.337	0.294	0.156	0.087	0.017
3-month holding period												
IRVRP	0.677**	0.577**	0.501**	0.450**	0.414**	0.387**	0.365**	0.345**	0.326**	0.224*	0.122	0.230*
	(7.099)	(5.425)	(4.543)	(4.102)	(3.875)	(3.738)	(3.621)	(3.484)	(3.308)	(2.029)	(0.893)	(2.164)
R^2	0.666	0.480	0.353	0.276	0.228	0.194	0.167	0.146	0.127	0.056	0.015	0.054
6-month holding period												
IRVRP	0.547**	0.386**	0.305*	0.263^{+}	0.239^{+}	0.224	0.214	0.205	0.196	0.136	0.056	0.344*
	(5.300)	(2.771)	(2.133)	(1.911)	(1.955)	(1.590)	(1.487)	(1.454)	(1.420)	(0.986)	(0.357)	(2.023)
R^2	0.440	0.220	0.134	0.097	0.078	0.067	0.059	0.053	0.048	0.021	0.003	0.110
12-month holding period												
IRVRP	0.023	0.033	0.058	0.084	0.107	0.126	0.141	0.152	0.159	0.141	0.068	0.452**
	(0.116)	(0.239)	(0.341)	(0.562)	(0.780)	(0.684)	(0.709)	(0.754)	(0.804)	(0.751)	(0.350)	(3.517)
R^2	0.001	0.002	0.006	0.012	0.018	0.025	0.030	0.034	0.036	0.025	0.005	0.216

Note: this table presents results of the univariate regression (7) for two subsample periods: March 1993 to December 2003 and January 2004 to February 2013. The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^{+}$ for $p < 0.1$, where p is the p-value.

3.4.2. *Realized variance based on high-frequency swap rates*

Our baseline measure of interest rate variance risk premium has the realized variance measure constructed using daily series of swap rates. In this section, we check the robustness of our results using high-frequency data of swap rates to construct the measure of realized variance. In particular, we obtain the intraday 10-year swap rates from February 11, 2002 to January 31, 2013, provided by Barclays. We follow the literature to use the 5-min series to strike a balance between the accuracy that increases with frequency in econometric theory and the microstructure issues such as price discreteness, bid-ask spreads, and non-synchronous trading effects that increase with frequency in practice.[10,34,47] In consequence, we use about 80 5-min observations each day from 8:20 a.m. to 3:00 p.m. US EST to estimate the realized variance according to equation (2). We then take the difference between the implied variance using swaptions on 10-year swap rate and this realized variance to get the alternative measure of interest rate variance risk premium, denoted as IRVRP_t^{5-min}.

Table 8 repeats our baseline regression (7) using the alternative measure IRVRP_t^{5-min} of interest rate variance risk premium. The return predictive power becomes weaker than the baseline results in Table 2, probably because IRVRP_t^{5-min} only captures information in the 10-year tenor while our baseline measure of interest rate variance risk

Table 8
Realized variance based on high-frequency swap rates.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
1-month holding period												
$IRVRP^{5-min}$	0.600**	0.566**	0.531**	0.499**	0.469**	0.442**	0.416**	0.392**	0.368**	0.263**	0.181^{+}	0.115
	(4.992)	(4.751)	(4.568)	(4.432)	(4.188)	(3.992)	(3.898)	(3.698)	(3.566)	(2.667)	(1.695)	(1.373)
R^2	0.505	0.434	0.363	0.303	0.254	0.214	0.181	0.154	0.131	0.061	0.028	0.013
3-month holding period												
$IRVRP^{5-min}$	0.505**	0.401**	0.326*	0.277^{+}	0.245^{+}	0.223^{+}	0.205	0.189	0.174	0.087	−0.007	0.255*
	(3.503)	(2.690)	(2.196)	(1.928)	(1.789)	(1.698)	(1.622)	(1.542)	(1.461)	(0.763)	(−0.060)	(2.537)
R^2	0.348	0.204	0.127	0.088	0.067	0.054	0.044	0.037	0.031	0.007	0.000	0.055
6-month holding period												
$IRVRP^{5-min}$	0.373*	0.213	0.130	0.085	0.058	0.042	0.031	0.023	0.015	−0.040	−0.118	0.377**
	(2.156)	(1.237)	(0.774)	(0.522)	(0.376)	(0.282)	(0.217)	(0.161)	(0.105)	(−0.321)	(−0.807)	(2.816)
R^2	0.177	0.054	0.020	0.008	0.004	0.002	0.001	0.001	0.000	0.002	0.014	0.116
12-month holding period												
$IRVRP^{5-min}$	−0.069	−0.066	−0.061	−0.057	−0.052	−0.047	−0.043	−0.041	−0.040	−0.074	−0.153	0.453**
	(−0.365)	(−0.364)	(−0.350)	(−0.336)	(−0.320)	(−0.321)	(−0.595)	(−0.266)	(−0.237)	(−0.417)	(−0.831)	(3.870)
R^2	0.006	0.006	0.005	0.004	0.004	0.003	0.002	0.002	0.002	0.006	0.025	0.198

Note: this table presents results of the univariate regression (7) using the alternative measure of interest rate variance risk premium, $IRVRP^{5-min}$. The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^{+}$ for $p < 0.1$, where p is the p-value.

premium combines information from multiple tenors. Yet, we still observe pretty significant predictive power of $IRVRP_t^{5-min}$ at the 1-month horizon. Most importantly, the sharp monotonically decreasing significance with the asset tenor remains the same as the baseline regressions.

3.4.3. Alternative treasury datasets

We check the robustness of the return predictive power of the interest rate variance risk premium using two alternative Treasury datasets in this section. First, we use the Fama–Bliss discount bond database from CRSP, which contains Treasury notes with maturities of 1, 2, 3, 4, and 5 years, to compute the Treasury excess returns. The Fama–Bliss database is also used in Fama and Bliss,[1] Cochrane and Piazzesi,[3] and so on. Panel A of Table 9 repeats our baseline regression (7) using the Treasury excess returns based on the Fama–Bliss database. We find that both the significant predictive power and the monotonically decreasing significance with the asset tenor, of the interest rate variance risk premium, remain the same.

Second, we use actual traded bonds from the Fama dataset in CRSP, different from the interpolated bond yields in both the Gürkaynak et al[40] and Fama–Bliss datasets, to compute the Treasury excess returns. This dataset combines actually traded Treasury bonds into portfolios of different maturity buckets, and computes an equal weighted average of 1-month holding period returns of all bonds in the portfolio. We obtain monthly return series and compute returns in excess of the 3-month T-bill rate. Panel B of Table 9 repeats our baseline regression (7) using the Treasury excess returns based on this Fama bond portfolios. We observe that the interest rate variance risk premium still possess strong and significant return predictive power, with t-statistics above 11 and adjusted R^2s above 37%. The monotonically decreasing significance with the asset tenor remains the same.

3.4.4. Control for additional return predictors

We now control for additional predictors of Treasury and equity returns. For Treasuries, we control for the tent-shape yield factor of Cochrane and Piazzesi[3] (CP) as well as two macro factors including the economic growth (GRO) and the expected inflation (INF) (Table B2 in the Appendix reports the results controlling for the macro principal component factors of Ludvigson and Ng.[6] For the equity market return, we control for traditional predictors including the log dividend price ratio (d/p), the log earnings price ratio (e/p), the net equity expansion (NTIS) factor of Goyal and Welch,[42] and the default spread (DS).

Table 10 reports the regression of the Treasury and equity excess returns on the interest rate variance risk premium, controlling for these additional return predictors. We find that the interest rate variance risk premium still possess strong

Table 9
Bond returns using the Fama–Bliss and Fama maturity portfolio data sets.

A: Fama–Bliss

	2y	3y	4y	5y
1-month holding period				
IRVRP	0.888**	0.850**	0.802**	0.768**
	(11.792)	(11.686)	(11.693)	(13.364)
R^2	0.747	0.685	0.610	0.559
3-month holding period				
IRVRP	0.816**	0.696**	0.602**	0.552**
	(10.652)	(9.624)	(8.315)	(8.119)
R^2	0.612	0.445	0.333	0.280
6-month holding period				
IRVRP	0.648**	0.476**	0.397**	0.369**
	(7.607)	(5.759)	(4.697)	(4.458)
R^2	0.372	0.201	0.140	0.121
12-month holding period				
IRVRP	0.028	0.073	0.139	0.216^{+}
	(0.225)	(0.586)	(1.105)	(1.709)
R^2	0.001	0.004	0.015	0.037

B: Fama maturity portfolios

	$\tau < 2$y	$\tau < 5$y	5y$\leq \tau <$ 10y	$\tau \geq$ 10y
IRVRP	0.886**	0.821**	0.787**	0.614**
	(12.325)	(13.319)	(13.719)	(11.546)
R^2	0.784	0.674	0.620	0.377

Note: this table presents results of the univariate regression (7) using two alternative Treasury datasets to calculate the Treasury excess returns. The first is the Fama–Bliss discount bond database from CRSP, which contains Treasury notes with maturities of 1, 2, 3, 4, and 5 years, while the second is the Fama dataset from CRSP, which contains actually traded Treasury securities combined into portfolios of different maturity buckets. The *t*-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^{+}$ for $p < 0.1$, where p is the p-value.

and significant return predictive power for Treasuries, controlling for the CP, GRO, and INF factors. Its return predictive power for the equity market excess returns becomes more significant, with *t*-statistics around two at the 1- and 3-month holding horizons, controlling for the traditional equity return predictors. The monotonically decreasing significance with the asset tenor remains the same as the baseline results.

4. Model and asset pricing

We have shown in Section 3 that the interest rate variance risk premium robustly predicts nominal Treasury bond returns especially at short maturities. We have also shown that its predictive power declines with the maturity of the Treasury security and the holding period horizon. In addition, we have shown that other predictors, such as forward spread, drive longer-maturity Treasury bond returns and at longer holding-period horizons. To reconcile these findings with the theory, we propose a consumption-based asset pricing model, which explains these predictability patterns with short-run and long-run risk factors. Specifically, we propose a long-run-risk-type consumption-based asset pricing model with consumption (short-run growth) risk, expected consumption (long-run growth) risk, consumption volatility (long-run volatility) risk, and consumption volatility-of-volatility (short-run volatility) risk. Such a framework delivers a two-factor structure for the bond risk premium, which is perfectly spanned by the interest rate variance risk premium (loaded only on short-run volatility) and forward spread (loaded on both short-run and long-run volatilities).

4.1. Preferences

We consider a discrete-time endowment economy with recursive preferences introduced by Kreps and Porteus,[48] Epstein and Zin,[49] and Weil:[50]

Table 10
Additional controls.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity	
A: 1-month holding period													
IRVRP	0.872**	0.845**	0.816**	0.788**	0.761**	0.735**	0.709**	0.683**	0.657**	0.539**	0.449**	IRVRP	0.174*
	(12.066)	(12.726)	(13.441)	(13.967)	(14.148)	(13.946)	(13.456)	(12.693)	(11.749)	(8.656)	(6.187)		(2.075)
CP	4.519*	5.425*	6.302**	7.083**	7.718**	8.189**	8.500**	8.673**	8.734**	8.187**	7.438*	d/p	0.599^{+}
	(2.082)	(2.533)	(2.981)	(3.343)	(3.562)	(3.637)	(3.608)	(3.511)	(3.391)	(2.820)	(2.367)		(1.754)
GRO	0.084	0.084	0.093	0.106	0.119	0.131	0.141	0.149^{+}	0.155^{+}	0.165*	0.163^{+}	e/p	−0.199
	(0.965)	(0.945)	(1.031)	(1.166)	(1.321)	(1.476)	(1.621)	(1.756)	(1.883)	(2.283)	(1.779)		(−0.708)
INF	−0.241	−0.233	−0.215	−0.193	−0.168	−0.143	−0.120	−0.100	−0.082	−0.025	0.003	NTIS	2.925
	(−1.078)	(−1.113)	(−1.104)	(−1.056)	(−0.978)	(−0.881)	(−0.776)	(−0.669)	(−0.568)	(−0.177)	(0.019)		(0.565)
												DS	−51.132
													(−1.631)
R^2	0.778	0.722	0.660	0.600	0.546	0.496	0.451	0.410	0.372	0.236	0.158		0.046
B: 3-month holding period													
IRVRP	0.789**	0.698**	0.637**	0.598**	0.574**	0.556**	0.540**	0.525**	0.509**	0.415**	0.320**	IRVRP	0.253^{+}
	(9.541)	(7.929)	(6.729)	(6.554)	(6.129)	(5.429)	(5.470)	(5.248)	(5.034)	(3.895)	(2.745)		(1.956)
CP	6.147*	8.000**	9.417**	10.486**	11.254**	11.746**	11.993**	12.036**	11.914**	10.018**	7.884**	d/p	0.926^{+}
	(2.191)	(2.644)	(3.071)	(3.529)	(3.888)	(4.224)	(4.527)	(4.619)	(4.627)	(4.055)	(3.219)		(1.754)
GRO	0.043	0.059	0.091	0.124	0.154	0.177^{+}	0.195*	0.209*	0.218*	0.230*	0.225*	e/p	−0.265
	(0.411)	(0.536)	(0.834)	(1.018)	(1.354)	(1.720)	(2.053)	(2.265)	(2.421)	(2.465)	(2.131)		(−0.738)
INF	−0.248	−0.241	−0.217	−0.185	−0.149	−0.113	−0.079	−0.048	−0.020	0.067	0.101	NTIS	9.554
	(−1.141)	(−1.149)	(−1.030)	(−0.742)	(−0.579)	(−0.518)	(−0.396)	(−0.243)	(−0.105)	(0.363)	(0.564)		(1.049)
												DS	−52.260
													(−1.282)
R^2	0.659	0.514	0.417	0.357	0.318	0.290	0.267	0.247	0.227	0.140	0.081		0.129

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity	
C: 6-month holding period													
IRVRP	0.599**	0.461**	0.408**	0.390**	0.386**	0.387**	0.389**	0.389**	0.386**	0.341*	0.269^{+}	IRVRP	0.204^{+}
	(5.071)	(3.888)	(3.078)	(2.891)	(2.821)	(2.803)	(2.799)	(2.793)	(2.776)	(2.447)	(1.856)		(1.686)
CP	9.119*	11.384**	12.868**	13.948**	14.709**	15.180**	15.395**	15.392**	15.212**	12.831**	10.021**	d/p	0.985
	(2.257)	(2.773)	(2.917)	(3.336)	(3.759)	(4.148)	(4.466)	(4.681)	(4.776)	(3.903)	(2.636)		(1.447)
GRO	−0.073	−0.027	0.036	0.091	0.137	0.173^{+}	0.200*	0.221*	0.236*	0.264**	0.272*	e/p	0.046
	(−0.475)	(−0.170)	(0.284)	(0.784)	(1.261)	(1.700)	(2.078)	(2.372)	(2.570)	(2.576)	(2.346)		(0.132)
INF	−0.264	−0.268	−0.255	−0.229	−0.195	−0.158	−0.121	−0.085	−0.052	0.062	0.111	NTIS	20.553
	(−1.064)	(−0.823)	(−0.911)	(−0.824)	(−0.707)	(−0.578)	(−0.447)	(−0.319)	(−0.198)	(0.247)	(0.464)		(1.591)
												DS	15.334
													(0.353)
R^2	0.474	0.321	0.264	0.242	0.231	0.225	0.218	0.211	0.201	0.139	0.088		0.236
D: 12-month holding period													
IRVRP	−0.021	0.041	0.113	0.176	0.230	0.273	0.307^{+}	0.333*	0.351*	0.367*	0.323*	IRVRP	0.181
	(−0.109)	(0.219)	(0.617)	(0.990)	(1.328)	(1.623)	(1.873)	(2.078)	(2.238)	(2.477)	(2.143)		(1.494)
CP	14.480**	16.796**	18.837**	20.508**	21.750**	22.575**	23.033**	23.196**	23.135**	21.181**	18.785**	d/p	1.199^{+}
	(2.770)	(3.155)	(3.849)	(4.564)	(5.204)	(5.688)	(5.983)	(6.104)	(6.092)	(5.232)	(4.277)		(1.673)
GRO	−0.109	−0.018	0.084	0.174*	0.248**	0.308**	0.356**	0.393**	0.422**	0.512**	0.578**	e/p	0.432
	(−0.610)	(−0.140)	(0.813)	(2.006)	(3.263)	(4.279)	(4.820)	(4.941)	(4.837)	(4.146)	(4.274)		(1.345)
INF	−0.203	−0.368	−0.437	−0.458	−0.450	−0.426	−0.393	−0.359	−0.325	−0.214	−0.187	NTIS	27.882*
	(−0.425)	(−0.799)	(−0.950)	(−1.011)	(−1.014)	(−0.982)	(−0.928)	(−0.862)	(−0.794)	(−0.545)	(−0.491)		(2.179)
												DS	67.754
													(1.555)
R^2	0.203	0.240	0.276	0.310	0.338	0.359	0.370	0.375	0.373	0.323	0.276		0.408

Note: this table presents results of the multivariate regression.
$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)\cdot\mathrm{IRVRP}_t + \beta_2^{(\tau)}(h)\cdot\mathrm{Factor}_t + \varepsilon_{t+h}^{(\tau)}$,
where $rx_{t+h}^{(\tau)}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$ tenor, and IRVRP_t is the measure of interest rate variance risk premium. For Treasuries, the Factor_t contains the CP variable as well as two macro factors GRO and INF. For the equity, Factor_t contains the log dividend price ratio (d/p), the log earnings price ratio (e/p), the net equity expansion (NTIS) factor of Goyal and Welch[42] (obtain from Amit Goyal's webpage), and the default spread (DS). The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^{+}$ for $p < 0.1$, where p is the p-value.

$$U_t = \left[\left(1 - \delta \right) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(\mathrm{E}_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{12}$$

where δ is the time discount factor, $\gamma \geq 0$ is the risk aversion parameter, $\psi \geq 0$ is the intertemporal elasticity of substitution (IES), and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. Preference for early resolution of uncertainty implies $\gamma > \frac{1}{\psi}$, which, in general, implies $\theta < 1$. We will assume throughout the paper that $\gamma > 1$ and $\psi > 1$, which implies $\theta < 0$ and refer to preference for early resolution of uncertainty as consistent with $\theta < 0$.[12] A special case of recursive preferences—expected utility—corresponds to the case of $\gamma = \frac{1}{\psi}$ $(\theta = 1)$.

Epstein and Zin[49] show that the log-linearized form of the associated real stochastic discount factor m_t is given by:

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{c,t+1}, \tag{13}$$

where $g_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right)$ is the log growth of the aggregate consumption, $r_{c,t+1}$ is the log return on an aggregate wealth portfolio that delivers aggregate consumption as its dividend each time period. Note that the return on wealth is different from the observed return on the market portfolio because aggregate consumption is not equal to aggregate dividends. Consequently, the return on wealth is not observable in the data. The nominal discount factor $m_{t+1}^{\$}$ is equal to the real discount factor minus expected inflation π_{t+1}:

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}. \tag{14}$$

4.2. Economy dynamics

To solve for the equilibrium asset prices we specify consumption and inflation dynamics. Consumption dynamics features time-varying consumption growth rate g_t and expected consumption growth rate x_t, time-varying volatility of consumption growth $\sigma_{g,t}^2$ and time-varying volatility-of-volatility of consumption growth q_t:

$$\begin{aligned} x_{t+1} &= \rho_x x_t + \varphi_e \sigma_{g,t} z_{x,t+1}, \\ g_{t+1} &= \mu_g + x_t + \sigma_{g,t} z_{g,t+1}, \\ \sigma_{g,t+1}^2 &= a_\sigma + \rho_\sigma \sigma_{g,t}^2 + \sqrt{q_t} z_{\sigma,t+1}, \\ q_{t+1} &= a_q + \rho_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1}, \end{aligned} \tag{15}$$

where the parameters satisfy $a_\sigma > 0$, $a_q > 0$, $|\rho_\sigma| < 1$, $|\rho_q| < 1$ and $\varphi_q > 0$. The vector of shocks $(z_{x,t+1}, z_{g,t+1}, z_{\sigma,t+1}, z_{q,t+1})$ follows *i.i.d.* normal distribution with zero mean and unit variance and shocks are assumed to be uncorrelated among themselves. The second pair of equations in equation (15) is new compared to Bansal and Yaron[11] and Bansal and Shaliastovich.[15] Stochastic volatility $\sigma_{g,t+1}^2$ represents time-varying economic uncertainty in consumption growth with time-varying volatility-of-volatility (vol-of-vol) measured by q_t.[13] Since $\sigma_{g,t}^2$ directly affects variation in x_t, the predictable component in consumption growth, we will refer to $\sigma_{g,t}^2$ as the state variable that captures *the long-run risk*. The volatility-of-volatility process q_t can be thought of as the volatility risk or, *the short-run risk*. As we saw earlier, this terminology was supported by our empirical findings.

In order for the real economy model (15) to have realistic implications for nominal bond risk premiums, we conjecture a fairly rich inflation process motivated by previous literature. Indeed, Bansal and Shaliastovich[15] allow for

[p] BKY (Bansal, Kiku, and Yaron[58]) discuss the wide range of regression-based estimates of the IES in the literature and their sensitivity to the presence of measurement errors. They argue that a better approach is undertaken in Bansal et al[51] and Hansen et al[52] who use a large set of instruments to estimate conditional Euler equations for the real bond and find that the IES is larger than one. Beeler and Campbell[53] disagree in a sense that aggregate consumption growth does not appear to respond to the real risk-free rate fluctuations in a manner consistent with IES being greater than one. They report, however, that their instrumental variables estimation approach of the BKY model yields the median estimates above 1.3.

[q] Recent studies provided empirical support in favor of time-varying consumption growth volatility, e.g., Bekaert and Liu,[54] Bansal et al,[57] Lettau et al,[55] Bekaert et al,[56] among others.

expected inflation shocks to be correlated (negatively) with expected consumption growth, and Zhou[12] allows for a vol-of-vol shock to affect inflation. We incorporate both of these features into expected inflation dynamics π_{t+1}:

$$\pi_{t+1} = a_\pi + \rho_\pi \pi_t + \varphi_\pi z_{\pi,t+1} + \varphi_{\pi g}\sigma_{g,t} z_{g,t+1} + \varphi_{\pi\sigma}\sqrt{q_t} z_{\sigma,t+1}, \tag{16}$$

where ρ_π is a persistence and $\frac{a_\pi}{1-\rho_\pi}$ is the long-run mean of the inflation process. There are three shocks that drive inflation process: (1) a constant volatility part φ_π with an autonomous shock $z_{\pi,t+1}$; (2) a stochastic volatility part $\varphi_{\pi\sigma}\sigma_{g,t}$ that works through consumption growth channel $z_{g,t+1}$; and (3) another stochastic volatility part $\varphi_{\pi\sigma}\sqrt{q_t}$ that works through the volatility channel $z_{\sigma,t+1}$. Exogenous inflation shock $z_{\pi,t+1}$ does not generate inflation risk premium even in the presence of the time-varying volatility of this shock.[14] In contrast, the second and the third shocks generate inflation risk premium because real side shocks (stochastic volatility of consumption growth and uncertainty) affect inflation. In addition, since $\varphi_{\pi g}$ and $\varphi_{\pi\sigma}$ control inflation exposures to the growth and uncertainty risks, this process implicitly violates inflation neutrality in the short run, but not in the long run.[15]

4.3. Pricing kernel

In equilibrium, the log wealth-consumption ratio z_t is affine in expected consumption growth x_t, stochastic volatility of consumption growth σ_t^2, and the vol-of-vol factor q_t:

$$z_t = A_0 + A_x x_t + A_\sigma \sigma_{g,t}^2 + A_q q_t. \tag{17}$$

Campbell and Shiller[59] show that the return on this asset can be approximated as follows:

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \tag{18}$$

where $\kappa_0 = \ln(1+\exp(\bar{z})) - \kappa_1 \bar{z}$, $\kappa_1 = \frac{\exp(\bar{z}))}{1+\exp(\bar{z})}$, and $\bar{z}$ is the average wealth-consumption ratio:

$$\bar{z} = A_0(\bar{z}) + A_\sigma(\bar{z})\bar{\sigma}^2 + A_q(\bar{z})\bar{q}. \tag{19}$$

The equilibrium loadings for equation (17) are derived in Appendix A.1:

$$\begin{aligned} A_x &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho_x}, \\ A_\sigma &= \frac{1}{2\theta(1-\kappa_1\rho_\sigma)}\left[\left(\theta-\frac{\theta}{\psi}\right)^2 + (\theta\kappa_1 A_x \varphi_e)^2\right], \\ A_q &= \frac{1-\kappa_1\rho_q - \sqrt{(1-\kappa_1\rho_q)^2 - \theta^2\kappa_1^4\varphi_q^2 A_\sigma^2}}{\theta(\kappa_1\varphi_q)^2}. \end{aligned} \tag{20}$$

As in Bansal and Yaron,[11] recursive preferences along with the early resolution of uncertainty are crucial in determining the sign of the equilibrium loadings of the state variables in our model. When the intertemporal elasticity of substitution $\psi > 1$, the intertemporal substitution effect dominates the wealth effect. In response to higher expected consumption growth, agents invest more and, consequently, wealth-consumption ratio increases. Therefore, the wealth-consumption ratio loading on the expected consumption growth is positive ($A_x > 0$) whereas loadings on the volatility and volatility-of-volatility of consumption growth are both negative ($A_\sigma < 0$ and $A_q < 0$) as in times of high volatility and/or uncertainty agents sell off risky assets driving the wealth-consumption ratio down.[16]

[r] The inability of the expected inflation process with only one (autonomous) shock even with stochastic volatility to generate inflation risk premium is examined in Zhou[61].

[s] There is no violation of inflation neutrality in the long run because unconditional expectation of inflation process (16) is $E\pi_t = \frac{a_\pi}{1-\rho_\pi}$.

[t] The solution for A_q represents one of a pair of roots of a quadratic equation, but we pick the one presented in equation (20) as the more meaningful one. We elaborate on this choice in Section A.1.

The persistence of expected growth shock ρ_x and time-varying volatility ρ_σ magnify the effect of the changes in these state variables on the valuation ratio since investors perceive such macroeconomic changes as long-lasting. Contrary to that, persistence of the volatility-of-volatility, ρ_q, roughly cancels out in the A_q loading. This provides further support for interpretation of q_t as a state variable that captures relatively short-run economic risks.[17]

Using the solution for the wealth-consumption ratio above, we show in Appendix A.3 that the conditional mean of the stochastic discount factor m_{t+1} is linear in the fundamental state variables and the innovation in m_{t+1} pins down the fundamental sources of (and compensations for) risks in the economy:

$$m_{t+1} - \mathrm{E}_t[m_{t+1}] = -\lambda_g \sigma_{g,t} z_{g,t+1} - \lambda_x \sigma_{g,t} z_{x,t+1} - \lambda_\sigma \sqrt{q_t} z_{\sigma,t+1} - \lambda_q \sqrt{q_t} z_{q,t+1}, \tag{21}$$

where the quantities of risks are time-varying volatility and volatility-of-volatility of consumption growth, $\sigma_{g,t}$ and $\sqrt{q_t}$, respectively; and λ_g, λ_x, λ_σ, λ_q represent the market prices of risk of consumption growth, expected consumption growth, volatility, and volatility-of-volatility:

$$\begin{array}{ll} \lambda_g = \gamma, & \lambda_\sigma = (1-\theta)\kappa_1 A_\sigma, \\ \lambda_x = (1-\theta)\kappa_1 A_x \varphi_e, & \lambda_q = (1-\theta)\kappa_1 A_q \varphi_q. \end{array} \tag{22}$$

The market price of consumption risk λ_g is equal to the coefficient of relative risk aversion γ. Other risk prices crucially depend on our preference assumptions.

When agents have preference for early resolution of uncertainty ($\theta < 0$), the market price of expected consumption risk is positive: $\lambda_x > 0$. In this case, positive shocks to consumption and expected consumption cause risk premium to decrease as agents buy risky assets and drive wealth-consumption ratio up. On the contrary, market prices of risk of volatility and volatility-of-volatility are negative ($\lambda_\sigma < 0$ and $\lambda_q < 0$): Consistent with the so-called leverage effect, in response to either type of volatility positive shock, agents sell risky assets and drive wealth-consumption ratio down and volatility risk premiums up. It is worth noting that these effects are not based on the statistical linkages between return and volatility, as the endowment and volatility shocks are uncorrelated; but arise endogenously in the equilibrium. In the absence of recursive preference for an early resolution of uncertainty ($\gamma = \frac{1}{\psi}$ and $\theta = 1$), there would be no compensations for investors for baring risks in expected consumption, volatility, or volatility-of-volatility.

4.4. Asset prices

We focus in this paper on the nominal yield curve and nominal bond return predictability. Hence in this section we provide the model solutions for the nominal quantities in our economy.[18]

Nominal risk-free rate. The nominal risk-free rate is the negative of the (log) price of the nominal one-period bond. Thus, it is equal to the real risk-free rate plus inflation compensation. The closed form expression for the nominal risk-free rate is derived in Appendix A.3:

$$\begin{aligned} r^{\$}_{f,t} &= -\theta \ln \delta + \gamma\mu_g + a_\pi - (\theta-1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] - \frac{1}{2}\varphi_\pi^2 \\ &\quad + [\gamma - (\theta-1)A_x(\kappa_1\rho_x - 1)]x_t \\ &\quad + \left[-(\theta-1)A_\sigma(\kappa_1\rho_\sigma - 1) - \frac{1}{2}\gamma^2 - \frac{1}{2}(\theta-1)^2(\kappa_1 A_x \varphi_e)^2 - \frac{1}{2}\varphi_{\pi g}^2 - \gamma\varphi_{\pi g}\right]\sigma_{g,t}^2 \\ &\quad + \left[-(\theta-1)A_q(\kappa_1\rho_q - 1) - \frac{1}{2}(\theta-1)^2\kappa_1^2(A_\sigma^2 + A_q^2\varphi_q^2) - \frac{1}{2}\varphi_{\pi\sigma}^2 + (\theta-1)\kappa_1 A_\sigma \varphi_{\pi\sigma}\right] q_t \\ &\quad + \rho_\pi \pi_t. \end{aligned} \tag{23}$$

Since inflation is not an autonomous process, it affects loadings on σ_t^2 and q_t in equation (23) via additional terms, related to $\varphi_{\pi g}$ and $\varphi_{\pi\sigma}$ coefficients, respectively, besides having a direct effect on the nominal rates, $\rho_\pi \pi_t$. This results in inflation short-run non-neutrality, which means that inflation is affected by future real growth in the economy.

[u] Bansal, Kiku, and Yaron[48] check that their approximate solutions are very accurate when compared against numerical solutions, used, e.g., in Binsbergen et al.[60]

[v] The corresponding real quantities are can be computed similarly and available upon request.

Nominal n − period bond price. A general recursion for solving for the $n-$period nominal bond price is as follows:

$$P_t^{\$,n} = \mathrm{E}_t\left[M_{t+1}^{\$} P_{t+1}^{\$,n-1}\right]. \tag{24}$$

We assume that the (log) price of the $n-$period nominal bond $p_t^{\$,n}$ follows an affine representation of the real state variables x_t, σ_t^2, q_t and inflation π_t:

$$p_t^{\$,n} = B_0^{\$,n} + B_1^{\$,n} x_t + B_2^{\$,n}\sigma_t^2 + B_3^{\$,n} q_t + B_4^{\$,n}\pi_t. \tag{25}$$

We solve for the nominal bond state loadings $B_i^{\$,n},\ i = 0, \ldots, 4$ using initial conditions $B_i^{\$,0} = 0,\ i = 0, \ldots, 4$ (since $p_t^{\$,0} = 0$) and the above recursion, see Appendix A.4.

Nominal bond risk premium. Nominal bond risk premium $\mathrm{brp}_t^{\$,n}$ is given by the negative of covariance between the nominal pricing kernel $m_{t+1}^{\$,n-1}$ and the nominal bond price $p_{t+1}^{\$,n-1}$ (see Appendix A.5 for details):

$$\begin{aligned}
\mathrm{brp}_t^{\$,n} &= -\mathrm{Cov}_t\left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}\right] \\
&= \left[(\gamma + \varphi_{\pi g})B_4^{\$,n-1}\varphi_{\pi g} - (\theta - 1)\kappa_1 A_x B_1^{\$,n-1}\varphi_e^2\right]\sigma_{g,t}^2 \\
&-\left[\left((\theta-1)\kappa_1 A_\sigma - \varphi_{\pi\sigma}\right)\left(B_2^{\$,n-1} + B_4^{\$,n-1}\varphi_{\pi\sigma}\right) + (\theta-1)\kappa_1 A_q B_3^{\$,n-1}\varphi_q^2\right] q_t \\
&+B_4^{\$,n-1}\varphi_\pi^2 \\
&\equiv \beta_1^{\$,n-1}\sigma_{g,t}^2 + \beta_2^{\$,n-1} q_t + B_4^{\$,n-1}\varphi_\pi^2.
\end{aligned} \tag{26}$$

Bond risk premium (26) is driven by two volatility factors: consumption volatility factor $\sigma_{g,t}^2$ and volatility-of-volatility factor q_{19}.[w]

The effect of expected growth risk captured by A_x equilibrium loading on the wealth-consumption ratio amplifies the overall contribution of the consumption risk, $\sigma_{g,t}$. This effect is absent in Zhou[12] and Mueller et al,[61] and thus, makes it more difficult to explain the upward sloping term structure of the nominal yield curve. The two volatility factors $\sigma_{g,t}^2$ and q_t are inherently latent factors in bond risk premium. While consumption volatility risk $\sigma_{g,t}^2$ represents the classic risk-return tradeoff and is the standard factor in consumption-based models, q_t factor did not receive a lot of attention with the exception of Bollerslev et al[10] paper. The next section demonstrates how q_t factor can be empirically isolated from $\sigma_{g,t}^2$ factor.

Interest rate variance risk premium. Bollerslev et al[10] show that the equity variance risk premium—the difference in expectations of the equity variance under risk-neutral and physical measures—is driven entirely by the vol-of-vol factor q_t and is a useful predictor of time variation in aggregate stock returns. Motivated by this result, we derive IRVRP that also loads entirely on the q_t factor in our model.

By definition, IRVRP is the covariance of the nominal bond return variance $\sigma_{r\$,t+1}^2$ with the nominal stochastic discount factor $m_{t+1}^{\$}$, which in terms of the parameters is given by the following formula (see Appendix A.7 for derivation):

$$\begin{aligned}
\mathrm{IRVRP}_t^{\$,n}\left[\sigma_{r\$,t+1}^2\right] &= \mathrm{E}_t^{\mathbb{Q}}\left[\sigma_{r\$,t+1}^2\right] - \mathrm{E}_t\left[\sigma_{r\$,t+1}^2\right] = \mathrm{Cov}_t\left[\sigma_{r\$,t+1}^2, m_{t+1}^{\$}\right] \\
&= (\theta-1)\kappa_1\left\{\left(A_\sigma - \varphi_{\pi\sigma}\right)\left[\left(B_1^{\$,n-2}\varphi_e\right)^2 + \left(B_4^{\$,n-2}\varphi_{\pi g}\right)^2\right]\right. \\
&\left.+A_q\varphi_q^2\left[\left(B_2^{\$,n-2} + B_4^{\$,n-2}\varphi_{\pi\sigma}\right)^2 + \left(B_3^{\$,n-2}\varphi_q\right)^2\right]\right\} q_t.
\end{aligned} \tag{27}$$

The first and central observation here is that the time variation in IRVRP is solely due to the time variation in q_t state variable. If volatility-of-volatility is constant, $q_t = q$, equation (27) reduces to a constant $(\theta - 1)\kappa_1\left\{\left(A_\sigma - \varphi_{\pi\sigma}\right)\left[\left(B_1^{\$,n-2}\varphi_e\right)^2 + \left(B_4^{\$,n-2}\varphi_{\pi g}\right)^2\right]\right\}q$, contrary to empirical evidence presented earlier in the paper that the interest rate variance risk premium is time-varying. The second observation is that, although consumption growth risk $\sigma_{g,t}^2$ does not affect the nominal bond variance risk premium directly, it still has an indirect effect through the pricing solution. If consumption volatility $\sigma_{g,t}^2$ is not stochastic, then the wealth-consumption ratio equilibrium loadings $A_\sigma = 0$

[w] The third constant term provides a correction for inflation risk through φ_π, due to the autonomous inflation shock π_t.

Table 11
Model calibration.

Type	Parameters	BY	BTZ	GSZ
Panel A: real economy dynamics				
Preferences	δ	0.997	0.997	0.997
	γ	10	10	8
	ψ	1.5	1.5	1.5
Endowment	μ_g	0.0015	0.0015	0.0015
	ρ_x	0.979	0	0.979
	φ_e	0.001	0	0.001
	a_σ	1.20463e-05	1.20463e-05	1.20463e-05
	ρ_σ	0.978	0.978	0.978
Uncertainty	a_q		2e-07	2e-10
	ρ_q		0.8	0.8
	φ_q		0.001	0.0001
Panel B: inflation dynamics				
Constant	a_π			8.33333e-05
Persistence	ρ_π			0.95
Autonomous	φ_π			0.0013
Consumption	$\varphi_{\pi g}$			−0.0385
Uncertainty	$\varphi_{\pi\sigma}$			28.5044
Panel C: Campbell–Shiller constants				
	κ_0	0.3251	0.3251	0.3251
	κ_1	0.9	0.9	0.9

Note: this table reports the calibrated parameters used in previous studies and in our paper. Column "BY" refers to the choice of parameters in Bansal and Yaron,[11] column "BTZ" — to that in Bollerslev et al,[10] and column "GSZ" refers to our choice of parameters.

and $A_q = 0$ by construction, and bond variance risk premium is zero. The last observation is that if there is no recursive preference ($\theta = 1$), then IRVRP is zero by construction. Lastly, positivity of IRVRP is guaranteed by negative θ along with negative values of A_σ and A_q.

5. Calibration

In this section we discuss calibration of the nominal yield curve implied by our model (15) and inflation process (16). We consider two benchmark cases, Bansal and Yaron[11] and Bollerslev et al.[10] Compared to BY, BTZ incorporate the time-varying vol-of-vol factor, but in the absence of the long-run risk channel. We differ from BTZ in two aspects: (1) we have the long-run risk state variable in the real side model; and (2) we model inflation process in order to derive implications for the nominal bond prices. We present all three models' parameters (BY, BTZ, and ours) in Table 11.

5.1. Calibration parameters

Panel A provides calibration values for the real economy dynamics. We set preference parameters $\delta = 0.997$, $\gamma = 8$, and $\psi = 1.5$.[x] Consumption growth parameters $\mu_g = 0.0015$, $\rho_x = 0.979$, $\varphi_e = 0.001$ are consistent with BY (and BTZ except for $\rho_x = \varphi_e \equiv 0$). Volatility persistence $\rho_\sigma = 0.978$ is the same as in BY and BTZ, and $a_\sigma = (1-\rho_\sigma)\mathrm{E}\sigma$ is set so that the unconditional expectation $\mathrm{E}\sigma_t^2 = 0.0234^2$, which is slightly higher than in BY and BTZ because we find that this value matches better the nominal yield curve in the model. We set the expected volatility-of-volatility level $\mathrm{E}q = a_q(1-\rho_q)^{-1} = 10^{-9}$ so that $a_q = 2^{-10}$ given $\rho_q = 0.8$. In addition, $\varphi_q = 10^{-4}$. Our choice of ρ_σ and ρ_q is broadly consistent with the estimates of Bollerslev et al,[62] who find that the long-run risk (proxied by $\sigma_{g,t}^2$) is more persistent than the short-run risk (proxied by q_t). Thus, the calibrated model is connected with the earlier empirical section where we show that these two types of risks in the nominal bond premium are disentangled.

Panel B provides calibration parameters for the inflation dynamics. We set the average annualized inflation rate $\mathrm{E}\pi = 2\%$ and persistence parameter $\rho_\pi = 0.95$ in accordance with the current Fed's inflation target and Great

[x] BY and BTZ use $\gamma = 10$, but in our model slightly lower value of γ works reasonably well.

Moderation period overall.[y] Implied a_π on a monthly basis is equal to $\mathrm{E}\pi(1-\rho_\pi) = 0.02/12 \times 0.05 = 8 \times 10^{-5}$. The total unconditional variance of the inflation process (16) is given by:

$$\mathrm{Var}(\pi) = \frac{1}{1-\rho_\pi^2}\left(\varphi_\pi^2 + \varphi_{\pi g}^2 \mathrm{E}\sigma^2 + \varphi_{\pi\sigma}^2 \mathrm{E}q\right). \tag{28}$$

We calibrate variance-related parameters of equation (28) so that the total annualized unconditional inflation volatility is 2%. Since $\rho_\pi = 0.95$, $\mathrm{E}\sigma^2 = 0.0234^2$, $\mathrm{E}q = 10^{-9}$, the term in parentheses in equation (28) on a monthly basis is: $\varphi_\pi^2 + \varphi_{\pi g}^2 \times 0.0234^2 + \varphi_{\pi\sigma}^2 \times 10^{-9} = 0.02^2/12 \times (1-0.95^2) = 3.25 \times 10^{-6}$. Further, we assume that the first (autonomous) shock contributes one half to the total variance while the other two shocks contribute equally to the remaining half of the total variance of the inflation process.[22] Thus, the contribution of the first shock to the total inflation variance is $0.5 \times 3.25 \times 10^{-6} = 1.625 \times 10^{-6}$, implying $\varphi_\pi = 0.0013$. The contribution of the second and third shocks are equal to each other and to $0.25 \times 3.25 \times 10^{-6} = 8.125 \times 10^{-7}$. Therefore, the implied $\varphi_{\pi g} = \left(8.125 \times 10^{-7}/0.0234^2\right)^{1/2} = -0.0385$. The negative sign of $\varphi_{\pi g}$ is motivated by previous empirical findings.[13–15] In particular, Bansal and Shaliastovich[15] use SPF survey data for one-year ahead consensus inflation forecast over 1969–2010 sample and a latent factor for the expected consumption growth to estimate relationship between the two. They find that expected inflation negatively affects future consumption growth thus suggesting non-neutrality of inflation. Last, the implied $\varphi_{\pi\sigma} = \left(8.125 \times 10^{-7}/10^{-9}\right)^{1/2} = 28.5$.

5.2. Calibration results

Fig. 4 reports our calibration results. Both panels show the average nominal yield curve out to 10 years (blue solid line (in the web version)) in sample period from January 1991 to December 2010 and the calibrated nominal yield curve (red dashed line (in the web version)) implied by our model (Panel A) and by our modified model in the absence of the long-run risk channel x_t (Panel B). Panel A shows that our model closely matches the average level of nominal yields and captures the slope of the yield curve too. The 1-, 5-, and 10-year model-implied yields are 3.71%, 5.14% and 5.58% relative to observed yields of 4%, 4.95%, and 5.65% at corresponding maturities. Panel B of Fig. 4 shows that, absent long-run risk, the model is not successful in fitting the nominal upward-sloping yield curve as it generates downward-sloping yield curve, even with the presence of time-varying economic uncertainty.[23]

To understand the effect of the long-run risk factor better, it is useful to write down the nominal yields as an affine combination of state variables:

$$y_t^{\$,n} = -\frac{1}{n}\left[B_0^{\$,n} + B_1^{\$,n} x_t + B_2^{\$,n} \sigma_t^2 + B_3^{\$,n} q_t + B_4^{\$,n} \pi_t\right], \tag{29}$$

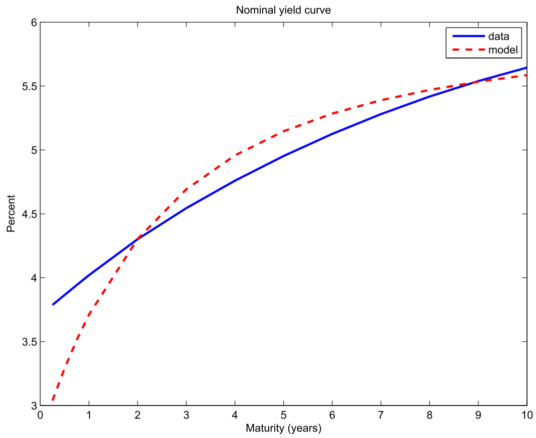
where $B_1^{\$,n}$, $B_2^{\$,n}$, $B_3^{\$,n}$, $B_4^{\$,n}$ are model-implied nominal bond price loadings provided in Appendix A.4. The equilibrium nominal yield loadings are plotted in Fig. 5.[zb] In our model, nominal yields hedge expected consumption and inflation risks. As the top left panel of Fig. 5 shows, nominal yields increase when expected consumption is high because $-\frac{1}{n}B_1^{\$,n} > 0$, but the effect declines with maturity. Intuitively, a negative shock to expected consumption drives bond prices up and yields down and a positive shock to expected consumption drives bond prices down and yields up. The same effect is obvious for expected inflation as $-\frac{1}{n}B_4^{\$,n} > 0$ with the effect declining with maturity (bottom right panel). The top right panel shows the effect of consumption volatility shock on nominal yields. The corresponding loading $-\frac{1}{n}B_2^{\$,n}$ manifests negative correlation of expected inflation and consumption growth that we discussed above. Positive consumption volatility shock has a negative effect on the yields, which decreases with maturity. Bottom left panel plots the effect of the consumption vol-of-vol shock, manifested by $-\frac{1}{n}B_3^{\$,n}$, on the yields. Positive consumption vol-of-vol

[y] These numbers may be justified by the data after 1980s and especially after 2008, when Fed launched unprecedented measures of accommodative monetary policy, namely, quantitative easing. Our expected inflation rate is lower than the one in Bansal and Shaliastovich,[15] who set it at 3.61% (see their Table 5).

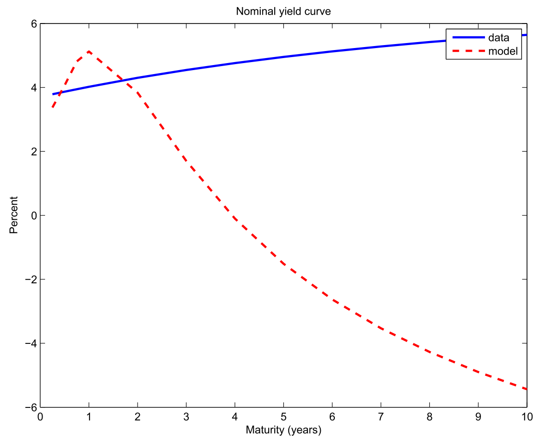
[z] Equal distribution of variance among the shocks results in slight overshooting of the model-implied interest rates levels relative to those in the sample.

[za] Bansal and Shaliastovich[15] fit the term structure of interest rates for the short- and intermediate-term yields (up to five years only), whereas our model quantitatively matches the level and slope of the nominal term structure from one- to ten-year interest rates.

[zb] Nominal yield loadings are nominal bond price loadings with a negative sign.



(a) With LRR component



(b) No LRR component

Fig. 4. The model-implied nominal yield curve. The figure plots the average zero-coupon nominal Treasury yield curve as observed in the data using the sample of January 1991–December 2010 monthly data as the solid blue line in both Panels (a) and (b). The figure also plots the model-implied yield curve with the long-run risk component (Panel (a)) and without the long-run risk component (Panel (b)) as the dashed red line.

shock has a positive and large effect on the short-term yields, and the effect declines with maturity, consistent with our empirical findings.

Fig. 6 shows the model-implied average real and nominal yi eld curves. The real yield curve is downward-sloping (as it has to be expected in the long-run risk models) while the nominal yield curve is upward sloping. The reason for such a difference is the negative correlation between the real and nominal side of the economy, $\varphi_{\pi g} < 0$. This parameterization is motivated by the fact that (at least) prior to the financial crisis period, the economy was driven by the supply shocks, when high inflation was associated with a bad economic state (low growth).[25] Because of this negative relationship, bonds used to serve as a hedge against low growth outcome, generating upward-sloping nominal yield curve.

[zc] Chen et al[63] document this empirically but note that the sign of this relationship switched from negative to positive after the financial crisis of 2008.

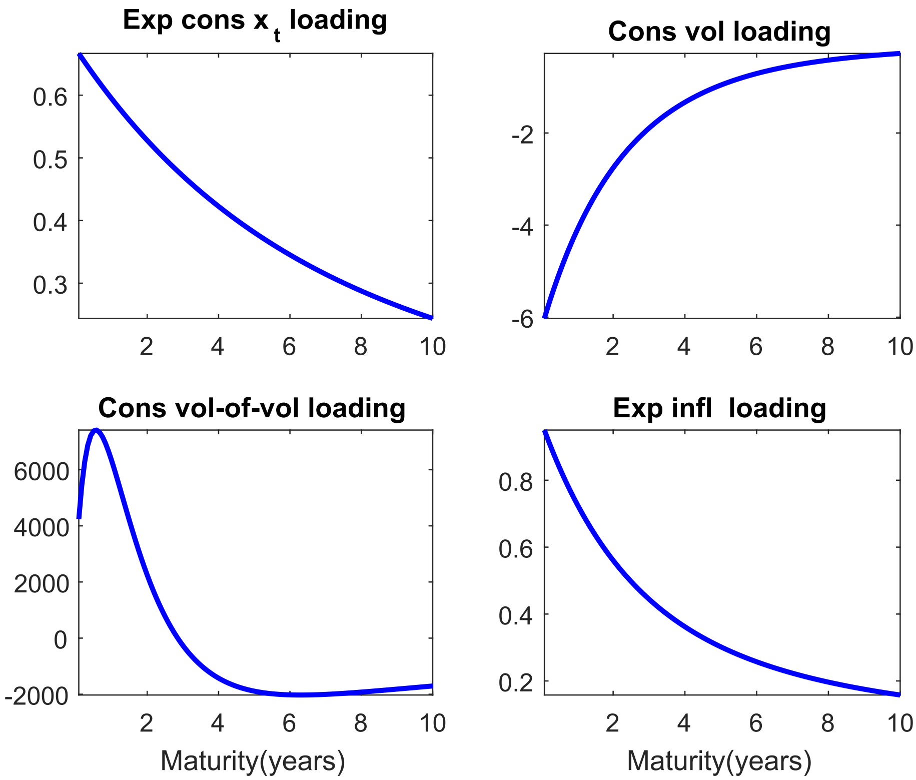


Fig. 5. Equilibrium nominal bond yield loadings. The figure plots the model-implied nominal bond yield loadings on expected consumption growth (top left panel), consumption volatility (top right panel), consumption volatility-of-volatility (bottom left panel), and expected inflation (bottom right panel).

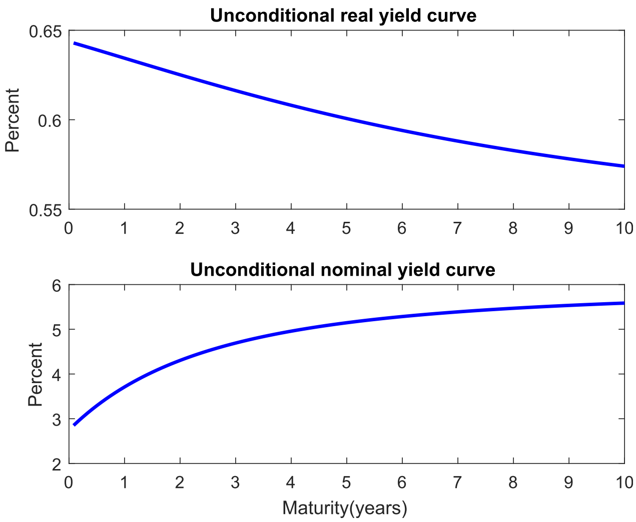


Fig. 6. Unconditional nominal and real yield curves. The figure plots the model-implied unconditional real (top panel) and nominal (bottom panel) yield curves.

6. Conclusion

We study the bond pricing implications in the context of the long-run risks asset-pricing model with two types of volatility risks—long-run consumption volatility and short-run consumption volatility-of-volatility risks—and inflation non-neutrality. The model is promising in explaining important stylized facts of the Treasury market returns.

First, our reasonably calibrated version of the model with long-run and short-run volatility risks matches well the upward-sloping yield curve out to ten years, and the long-run risk plus inflation non-neutrality appear to be the main driving forces behind this result. Second, the interest-rate variance risk premium (IRVRP) constructed from interest rate derivatives markets drives short-horizon (one- and three-month) Treasury excess returns, while other popular predictive variables, such as Fama–Bliss forward spread or Cochrane–Piazzesi forward-rate factor drive variation in longer-horizon (one-year) Treasury excess returns.

Inside our model, time-varying bond risk premium is driven by two volatility factors—volatility of consumption and volatility-of-volatility of consumption; whereas interest rate variance risk premium loads entirely on the vol-of-vol factor, forward rate loads on both consumption volatility and vol-of-vol factors plus growth and inflation factors. Since variance risk premium explains a significant part in variation in short-horizon Treasury excess returns, we interpret vol-of-vol factor as a short-run volatility risk factor. Since the forward-rate-related factors appear to explain time-variation in long-horizon Treasury excess returns, we interpret these factors as related to the long-run volatility risk factor. Thus, our model and empirical findings provide useful insights on different volatility risks in driving bond risk premium dynamics. These insights should be useful for market participants and monetary policy makers in practice.

Declaration of competing interest:

None.

A. Appendix

A.1. Solution for the consumption-wealth ratio coefficients

Euler equation imposes equilibrium restrictions on the asset prices:

$$\mathrm{E}[\exp(m_{t+1}+r_{t+1})]=1. \tag{A.1}$$

Since this Euler equation should hold for any asset, it should also hold for the wealth-consumption ratio z_t. Thus, the return on this asset $r_{c,t+1}$ should satisfy equation (A.1). Using it and the wealth return equation (18), obtain:

$$\mathrm{E}_t[\exp(m_{t+1}+r_{c,t+1})]=\mathrm{E}_t\left[\exp\left(\theta\ln\delta-\frac{\theta}{\psi}g_{t+1}+\theta r_{c,t+1}\right)\right]=1, \tag{A.2}$$

or, in log-linearized dynamics:

$$\mathrm{E}_t\left[\theta\ln\delta-\frac{\theta}{\psi}g_{t+1}+\theta r_{c,t+1}\right]+\frac{1}{2}\mathrm{Var}_t\left[\theta\ln\delta-\frac{\theta}{\psi}g_{t+1}+\theta r_{c,t+1}\right]=0. \tag{A.3}$$

Substituting out $r_{c,t+1}$ in terms of z_t dynamics (17) and consumption growth g_{t+1}, we can solve for the equilibrium wealth-consumption ratio loadings A_0, A_x, A_{σ^2}, A_q:

$$\begin{aligned}
&\mathrm{E}_t\Big[\theta \ln \delta - \frac{\theta}{\psi}(\mu_g + x_t + \sigma_{g,t} z_{g,t+1}) + \theta\Big(\kappa_0 + \kappa_1(A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{g,t+1} + A_q q_{t+1}) - \\
&\qquad A_0 - A_x x_t - A_\sigma \sigma^2_{g,t} - A_q q_t + \mu_g + x_t + \sigma_{g,t} z_{g,t+1}\Big)\Big] + \\
&\frac{1}{2}\mathrm{Var}_t\Big[\theta \ln \delta - \frac{\theta}{\psi}(\mu_g + x_t + \sigma_{g,t} z_{g,t+1}) + \theta\Big(\kappa_0 + \kappa_1(A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{g,t+1} + A_q q_{t+1}) - \\
&\qquad A_0 - A_x x_t - A_\sigma \sigma^2_{g,t} - A_q q_t + \mu_g + x_t + \sigma_{g,t} z_{g,t+1}\Big)\Big] = 0.
\end{aligned} \tag{A.4}$$

To solve for A_0, set constant terms under the expectation in equation (A.4) equal to zero:

$$\begin{aligned}
&\theta \ln \delta + \theta(\kappa_0 + \kappa_1(A_0 + A_\sigma a_\sigma + A_q a_q)) - A_0 + \left(\theta - \frac{\theta}{\psi}\right)\mu_g = 0 \quad \Rightarrow \\
&A_0 = \frac{1}{1-\kappa_1}\left[\ln \delta + \kappa_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q) + \left(1 - \frac{1}{\psi}\right)\mu_g\right].
\end{aligned} \tag{A.5}$$

To solve for A_x, match the terms in front of x_t:

$$-\frac{\theta}{\psi} + \theta(\kappa_1 A_x \rho_x - A_x + 1) = 0 \quad \Rightarrow \quad A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho_x}. \tag{A.6}$$

To solve for A_σ, match the terms in front of $\sigma^2_{g,t}$:

$$\begin{aligned}
&(\theta \kappa_1 A_\sigma \rho_\sigma - \theta A_\sigma)\sigma^2_{g,t} + \frac{1}{2}\mathrm{Var}_t\left[-\frac{\theta}{\psi}\sigma^2_{g,t} z_{g,t+1} + \theta \kappa_1 A_x \varphi_e \sigma_{g,t} z_{x,t+1} + \theta \sigma_{g,t} z_{g,t+1}\right] = \\
&\theta A_\sigma(\kappa_1 \rho_\sigma - 1)\sigma^2_{g,t} + \frac{1}{2}\mathrm{Var}_t\left[\left(\theta - \frac{\theta}{\psi}\right)\sigma_{g,t} z_{g,t+1} + \theta \kappa_1 A_x \varphi_e \sigma_{g,t} z_{x,t+1}\right] = 0 \quad \Rightarrow \\
&\theta A_\sigma(\kappa_1 \rho_\sigma - 1) + \frac{1}{2}\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta \kappa_1 A_x \varphi_e)^2\right] = 0 \quad \Rightarrow \\
&A_\sigma = \frac{1}{2\theta(1 - \kappa_1 \rho_\sigma)}\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta \kappa_1 A_x \varphi_e)^2\right].
\end{aligned} \tag{A.7}$$

To solve for A_q, match the terms in front of q_t and set equal to zero:

$$\begin{aligned}
&(\theta \kappa_1 A_q \rho_q - \theta A_q)q_t + \frac{1}{2}\mathrm{Var}_t[\theta \kappa_1 A_\sigma \sqrt{q_t}\, z_{\sigma_{t+1}} + \theta \kappa_1 A_q(\rho_q q_t + \phi_q \sqrt{q_t}\, z_{q_{t+1}}) - \theta A_q q_t] = \\
&\theta A_q(\kappa_1 \rho_q - 1)q_t + \frac{1}{2}\mathrm{Var}(\theta \kappa_1 A_\sigma \sqrt{q_t}\, z_{\sigma_{t+1}} + \theta \kappa_1 A_q \phi_q \sqrt{q_t}\, z_{q_{t+1}}) = 0 \quad [xrArr] \\
&\frac{1}{2}(\theta \kappa_1 \phi_q)^2 A_q^2 + \theta(\kappa_1 \rho_q - 1)A_q + \frac{1}{2}(\theta \kappa_1 A_\sigma)^2 = 0 \quad \text{or, equivalently,} \\
&(\theta \kappa_1 \phi_q)^2 A_q^2 + 2\theta(\kappa_1 \rho_q - 1)A_q + (\theta \kappa_1 A_\sigma)^2 = 0.
\end{aligned} \tag{A.8}$$

The solution for A_q represents the solution to a quadratic equation and is given by:

$$A_q^{\pm} = \frac{1 - \kappa_1 \rho_q \pm \sqrt{(1 - \kappa_1 \rho_q)^2 - (\theta \kappa_1^2 \varphi_q A_\sigma)^2}}{\theta(\kappa_1 \varphi_q)^2}. \tag{A.9}$$

As Tauchen[64] notes, a "positive" root A_q^+ has an unfortunate property $\lim_{\varphi_q \to 0} \varphi_q^2 A_q^+ \neq 0$, which is, essentially, a violation of the transversality condition in this setting: though uncertainty q_t vanishes with $\varphi_q \to 0$, the effect of it on prices is not. Therefore, we choose A_q^- root as a viable solution for A_q:

$$A_q = \frac{1-\kappa_1\rho_q - \sqrt{(1-\kappa_1\rho_q)^2 - \theta^2\kappa_1^4\varphi_q^2 A_\sigma^2}}{\theta(\kappa_1\varphi_q)^2}. \tag{A.10}$$

To insure that the determinant in equation (A.10) is positive, we also impose a constraint on φ_q —the magnitude of the shock $z_{q,t+1}$:

$$\varphi_q^2 \leq \frac{(1-\kappa_1\rho_q)^2}{\theta^2\kappa_1^4 A_\sigma^2}. \tag{A.11}$$

A.2. Solution for the real pricing kernel and the real risk-free rate

Using the solutions for A's obtained in equation (A.1), we solve for the expected value $\mathrm{E}_t[m_{t+1}]$ and the variance $\mathrm{Var}_t[m_{t+1}]$ of the real pricing kernel m_{t+1}:

$$\begin{aligned}
\mathrm{E}_t[m_{t+1}] &= \theta \ln \delta - \frac{\theta}{\psi}\mathrm{E}_t[g_{t+1}] + (\theta-1)\mathrm{E}_t[r_{c,t+1}] = \\
&= \theta \ln \delta - \frac{\theta}{\psi}(\mu_g + x_t) + (\theta-1)\mathrm{E}_t(\kappa_0 + \kappa_1 z_{t+1} + g_t - z_t) \\
&= \theta \ln \delta - \frac{\theta}{\psi}(\mu_g + x_t) + (\theta-1)\Big[\kappa_0 + \kappa_1(A_0 + A_x\rho_x x_t + A_\sigma(a_\sigma + \rho_\sigma\sigma_{g,t}^2) + A_q(a_q + \rho_q q_t)) \\
&\qquad + \mu_g + x_t - A_0 - A_x x_t - A_\sigma\sigma_{g,t}^2 - A_q q_t\Big] \\
&= \theta \ln \delta + \underbrace{\left(\left(\theta-1\right) - \frac{\theta}{\psi}\right)}_{-\gamma}\mu_g + (\theta-1)[\kappa_0 + (\kappa_1-1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] \\
&\quad - \frac{\theta}{\psi}x_t + (\theta-1)[(A_x(\kappa_1\rho_x - 1) + 1)x_t + A_\sigma(\kappa_1\rho_\sigma - 1)\sigma_{g,t}^2 + A_q(\kappa_1\rho_q - 1)q_t] \\
&= \theta \ln \delta - \gamma(\mu_g + x_t) + (\theta-1)[\kappa_0 + (\kappa_1-1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] \\
&\quad + (\theta-1)[A_x(\kappa_1\rho_x - 1)x_t + A_\sigma(\kappa_1\rho_\sigma - 1)\sigma_{g,t}^2 + A_q(\kappa_1\rho_q - 1)q_t].
\end{aligned} \tag{A.12}$$

and

$$\begin{aligned}
\mathrm{Var}_t[m_{t+1}] &= \mathrm{Var}_t\left[\theta \ln \delta - \frac{\theta}{\psi}g_{t+1} + (\theta-1)r_{c,t+1}\right] \\
&= \mathrm{Var}_t\left[-\frac{\theta}{\psi}g_{t+1} + (\theta-1)[\kappa_1(A_0 + A_x x_{t+1} + A_\sigma\sigma_{g,t+1}^2 + A_q q_{t+1}) + g_{t+1}]\right] \\
&= \mathrm{Var}_t\left[\left(\left(\theta-1\right) - \frac{\theta}{\psi}\right)\sigma_{g,t}z_{g,t+1} + (\theta-1)\kappa_1(A_x\varphi_e\sigma_{g,t}z_{x,t+1} + A_\sigma\sqrt{q_t}\, z_{\sigma,t+1} + A_q\varphi_q\sqrt{q_t}\, z_{q,t+1})\right] \\
&= \gamma^2\sigma_{g,t}^2 + (\theta-1)^2\kappa_1^2\Big[A_x^2\varphi_e^2\sigma_{g,t}^2 + (A_\sigma^2 + A_q^2\varphi_q^2)q_t\Big].
\end{aligned} \tag{A.13}$$

The real risk-free rate is the negative of the (log) real pricing kernel with the Jensen's correction. Using the solutions (A.12) and (A.13) for the real pricing kernel, the model-implied real risk-free rate is given by:

$$\begin{aligned} r_{f,t} &= -p_t^1 = -\mathrm{E}_t[m_{t+1}] - \frac{1}{2}\mathrm{Var}_t[m_{t+1}] \\ &= -\theta \ln \delta + \gamma\mu_g - (\theta-1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] \\ &\quad + [\gamma - (\theta-1)A_x(\kappa_1\rho_x - 1)]x_t \\ &\quad + \left[-(\theta-1)A_\sigma(\kappa_1\rho_\sigma - 1) - \frac{1}{2}(\theta-1)^2\kappa_1^2 A_x^2\varphi_e^2 - \frac{1}{2}\gamma^2\right]\sigma_{g,t}^2 \\ &\quad + \left[-(\theta-1)A_q(\kappa_1\rho_q - 1) - \frac{1}{2}(\theta-1)^2\kappa_1^2(A_\sigma^2 + A_q^2\varphi_q^2)\right]q_t. \end{aligned} \tag{A.14}$$

Note that the time variation of the risk-free rate (A.14) crucially depends on the assumption of the preference for early resolution of uncertainty ($\theta < 0$). Without it ($\theta = 1$) the time variation in the risk-free rate depends only on the variation of the predictable consumption growth component x_t (long-run risk) and equals to:

$r_{f,t} = -\ln\delta + \gamma(\mu_g + x_t) - \frac{1}{2}\gamma^2\sigma_{g,t}^2$. Moreover, in the absence of the long-run risk, it is nearly constant (ignoring the time-varying Jensen's inequality correction): $r_{f,t} = -\ln\delta + \gamma\mu_g - \frac{1}{2}\gamma^2\sigma_{g,t}^2$. In the steady state the real risk-free rate can be written as:

$$r_f = -[c_0\ c_1\ c_2\ c_3] \times [1\ \mathrm{E}_x\ \mathrm{E}_{\sigma^2}\ \mathrm{E}_q]'. \tag{A.15}$$

where steady-state loadings c_i, $i = 0, \ldots, 3$ are given by:

$$\begin{aligned} c_0 &= \theta\ln\delta - \gamma\mu_g + (\theta-1)[\kappa_0 + (\kappa_1-1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)], \\ c_1 &= -\gamma + (\theta-1)A_x(\kappa_1\rho_x - 1), \\ c_2 &= \frac{1}{2}\gamma^2 + \frac{1}{2}(\theta-1)^2\kappa_1^2A_x^2\varphi_e^2 + (\theta-1)A_\sigma(\kappa_1\rho_\sigma - 1), \\ c_3 &= \frac{1}{2}(\theta-1)^2\kappa_1^2(A_\sigma^2 + A_q^2\varphi_q^2) + (\theta-1)A_q(\kappa_1\rho_q - 1). \end{aligned} \tag{A.16}$$

A.3. Solution for the nominal one-period risk-free rate

Similarly to the real risk-free rate (A.14), the nominal one-period risk-free rate is the negative of the (log) nominal pricing kernel with the Jensen's correction:

$$\begin{aligned} r_{f,t}^{\$} &= -\mathrm{E}_t[m_{t+1}^{\$}] - \frac{1}{2}\mathrm{Var}_t[m_{t+1}^{\$}] \\ &= -\mathrm{E}_t[m_{t+1} - \pi_{t+1}] - \frac{1}{2}\mathrm{Var}_t[m_{t+1}] - \frac{1}{2}\mathrm{Var}_t[\pi_{t+1}] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}] \\ &= r_{f,t} + \mathrm{E}_t[\pi_{t+1}] - \frac{1}{2}\mathrm{Var}_t[\pi_{t+1}] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}] \\ &= r_{f,t} + a_\pi + \rho_\pi\pi_t - \frac{1}{2}[\varphi_\pi^2 + \varphi_{\pi g}^2\sigma_{g,t}^2 + \varphi_{\pi\sigma}^2 q_t] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}]. \end{aligned} \tag{A.17}$$

We need to compute the last term in equation (A.17) to complete the expression for the nominal risk-free rate in closed form:

$$\mathrm{Cov}_t[m_{t+1}, \pi_{t+1}] = \mathrm{E}_t[(m_{t+1} - \mathrm{E}_t m_{t+1}) \times (\pi_{t+1} - \mathrm{E}_t\pi_{t+1})]. \tag{A.18}$$

The unexpected components of the pricing kernel m_{t+1} and inflation π_{t+1} are given by:

$$\begin{aligned} m_{t+1}-\mathrm{E}_t[m_{t+1}] &= -\gamma\sigma_{g,t}z_{g,t+1}+(\theta-1)\kappa_1(A_x\varphi_e z_{x,t+1}+A_\sigma\sqrt{q_t}\,z_{\sigma,t+1}+A_q\varphi_q\sqrt{q_t}\,z_{q,t+1}),\\ \pi_{t+1}-\mathrm{E}_t[\pi_{t+1}] &= \varphi_\pi z_{\pi,t+1}+\varphi_{\pi g}\sigma_{g,t}z_{g,t+1}+\varphi_{\pi\sigma}\sqrt{q_t}z_{\sigma,t+1}, \end{aligned} \tag{A.19}$$

which implies for equation (A.18):

$$\mathrm{E}_t[(m_{t+1}-\mathrm{E}_t m_{t+1})\times(\pi_{t+1}-\mathrm{E}_t\pi_{t+1})] = -\gamma\varphi_{\pi g}\sigma^2_{g,t}+(\theta-1)\kappa_1 A_\sigma\varphi_{\pi\sigma}q_t. \tag{A.20}$$

Combining equation (A.14), (A.17), and (A.20) together , we obtain the closed-form expression for the nominal risk-free rate in terms of the model parameters and state variables:

$$\begin{aligned} r^{\$}_{f,t} &= -\theta\ln\delta+\gamma\mu_g+a_\pi-(\theta-1)[\kappa_0+(\kappa_1-1)A_0+\kappa_1(A_\sigma a_\sigma+A_q a_q)]-\frac{1}{2}\varphi^2_\pi\\ &+[\gamma-(\theta-1)A_x(\kappa_1\rho_x-1)]x_t\\ &+\left[-(\theta-1)A_\sigma(\kappa_1\rho_\sigma-1)-\frac{1}{2}\gamma^2-\frac{1}{2}(\theta-1)^2(\kappa_1A_x\varphi_e)^2-\frac{1}{2}\varphi^2_{\pi g}-\gamma\varphi_{\pi g}\right]\sigma^2_{g,t}\\ &+\left[-(\theta-1)A_q(\kappa_1\rho_q-1)-\frac{1}{2}(\theta-1)^2\kappa_1^2(A_\sigma^2+A_q^2\varphi_q^2)-\frac{1}{2}\varphi^2_{\pi\sigma}+(\theta-1)\kappa_1A_\sigma\varphi_{\pi\sigma}\right]q_t\\ &+\rho_\pi\pi_t. \end{aligned} \tag{A.21}$$

The nominal steady-state risk-free rate can be expressed similarly to that of the real risk-free rate:

$$r^{\$}_f = -[c^{\$}_0\ c^{\$}_1\ c^{\$}_2\ c^{\$}_3\ c^{\$}_4]\times[1\ \mathrm{E}_x\ \mathrm{E}_{\sigma^2}\ \mathrm{E}_q\ \mathrm{E}_\pi]', \tag{A.22}$$

where the nominal risk-free rate loadings $c^{\$}_i$, $i=0,\ldots,4$ are related to the real risk-free rate loadings c_i, $i=0,\ldots,3$ as:

$$\begin{aligned} c^{\$}_0 &= c_0-a_\pi+\frac{1}{2}\varphi^2_\pi,\\ c^{\$}_1 &= c_1,\\ c^{\$}_2 &= c_2+\frac{1}{2}\varphi^2_{\pi g}+\gamma\varphi_{\pi g},\\ c^{\$}_3 &= c_3+\frac{1}{2}\varphi^2_{\pi\sigma}-(\theta-1)\kappa_1A_\sigma\varphi_{\pi\sigma},\\ c^{\$}_4 &= -\rho_\pi. \end{aligned} \tag{A.23}$$

A.4. Solution for the nominal $n-$ period bond price

The nominal $n-$ period bond (log) price $p^{\$,n}_t$ is given by:

$$p^{\$,n}_t = \mathrm{E}_t\left[m^{\$}_{t+1}\right]+\frac{1}{2}\mathrm{Var}_t\left[m^{\$}_{t+1}\right]+\mathrm{E}_t\left[p^{\$,n-1}_{t+1}\right]+\frac{1}{2}\mathrm{Var}_t\left[p^{\$,n-1}_{t+1}\right]+\mathrm{Cov}_t\left[m^{\$}_{t+1},p^{\$,n-1}_{t+1}\right]. \tag{A.24}$$

The first and the second terms in equation (A.24) are known from the nominal risk-free rate calculations (A.17). The last three terms can be computed using an affine pricing conjecture:

$$p^{\$,n}_t = B^{\$,n}_0+B^{\$,n}_1x_t+B^{\$,n}_2\sigma^2_t+B^{\$,n}_3q_t+B^{\$,n}_4\pi_t. \tag{A.25}$$

Then the expected nominal bond price is:

$$
\begin{aligned}
\mathrm{E}_t\left[p_{t+1}^{\$,n-1}\right] &= B_0^{\$,n-1} + B_1^{\$,n-1}\rho_x x_t + B_2^{\$,n-1}(a_\sigma + \rho_\sigma \sigma_{g,t}^2) \\
&\quad + B_3^{\$,n-1}(a_q + \rho_q q_t) + B_4^{\$,n-1}(a_\pi + \rho_\pi \pi_t) \\
&= \left[B_0^{\$,n-1} + B_2^{\$,n-1}a_\sigma + B_3^{\$,n-1}a_q + B_4^{\$,n-1}a_\pi\right] \\
&\quad + B_1^{\$,n-1}\rho_x x_t + B_2^{\$,n-1}\rho_\sigma \sigma_{g,t}^2 + B_3^{\$,n-1}\rho_q q_t + B_4^{\$,n-1}\rho_\pi \pi_t.
\end{aligned} \tag{A.26}
$$

The shock to the nominal bond price is given by:

$$
\begin{aligned}
p_{t+1}^{\$,n-1} - \mathrm{E}_t\left[p_{t+1}^{\$,n-1}\right] &= B_1^{\$,n-1}\varphi_e \sigma_{g,t} z_{x,t+1} + B_2^{\$,n-1}\sqrt{q_t} z_{\sigma,t+1} + B_3^{\$,n-1}\varphi_q \sqrt{q_t} z_{q,t+1} \\
&\quad + B_4^{\$,n-1}\left[\varphi_\pi z_{\pi,t+1} + \varphi_{\pi g}\sigma_{g,t} z_{g,t+1} + \varphi_{\pi\sigma}\sqrt{q_t}\, z_{\sigma,t+1}\right].
\end{aligned} \tag{A.27}
$$

Thus, the variance of the nominal bond price is given by:

$$
\begin{aligned}
\mathrm{Var}_t\left[p_{t+1}^{\$,n-1}\right] &= \mathrm{E}_t\left[p_{t+1}^{\$,n-1} - \mathrm{E}_t\left[p_{t+1}^{\$,n-1}\right]\right]^2 = \left[\left(B_1^{\$,n-1}\varphi_e\right)^2 + \left(B_4^{\$,n-1}\varphi_{\pi g}\right)^2\right]\sigma_{g,t}^2 \\
&\quad + \left[\left(B_2^{\$,n-1} + B_4^{\$,n-1}\varphi_{\pi\sigma}\right)^2 + \left(B_3^{\$,n-1}\varphi_q\right)^2\right]q_t + \left(B_4^{\$,n-1}\varphi_\pi\right)^2.
\end{aligned} \tag{A.28}
$$

Lastly, covariance between between the nominal pricing kernel and the nominal bond price equals to:

$$
\mathrm{Cov}_t\left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}\right] = \mathrm{E}_t\left[\left(m_{t+1}^{\$} - \mathrm{E}_t m_{t+1}^{\$}\right) \times \left(p_{t+1}^{\$,n-1} - \mathrm{E}_t p_{t+1}^{\$,n-1}\right)\right], \tag{A.29}
$$

where the shock to the nominal pricing kernel in terms of state variables is:

$$
\begin{aligned}
& m_{t+1}^{\$} - \mathrm{E}_t m_{t+1}^{\$} = m_{t+1} - \mathrm{E}_t m_{t+1} - (\pi_{t+1} - \mathrm{E}_t \pi_{t+1}) \\
& = -\gamma\sigma_{g,t} z_{g,t+1} + (\theta-1)\kappa_1\left(A_x \varphi_e \sigma_{g,t} z_{x,t+1} + A_\sigma \sqrt{q_t}\, z_{\sigma,t+1} + A_q \varphi_q \sqrt{q_t}\, z_{q,t+1}\right) \\
& -\varphi_\pi z_{\pi,t+1} - \varphi_{\pi g}\sigma_{g,t} z_{g,t+1} - \varphi_{\pi\sigma}\sqrt{q_t} z_{\sigma,t+1},
\end{aligned} \tag{A.30}
$$

and the shock to the nominal bond price, $p_{t+1}^{\$,n-1} - \mathrm{E}_t p_{t+1}^{\$,n-1}$, is given in equation (A.27). Thus, a final expression for a covariance term in equation (A.24) is:

$$
\begin{aligned}
\mathrm{Cov}_t\left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}\right] &= \left[(\theta-1)\kappa_1 A_x B_1^{\$,n-1}\varphi_e^2 - (\gamma + \varphi_{\pi g})B_4^{\$,n-1}\varphi_{\pi g}\right]\sigma_{g,t} \\
+ \Big[\Big((\theta-1)\kappa_1 A_\sigma - \varphi_{\pi\sigma}\Big)&(B_2^{\$,n-1} + B_4^{\$,n-1}\varphi_{\pi\sigma}) + (\theta-1)\kappa_1 A_q B_3^{\$,n-1}\varphi_q^2 q_t\Big] q_t \\
&- B_4^{\$,n-1}\varphi_\pi^2.
\end{aligned} \tag{A.31}
$$

Combining together equations (A.17), (A.26), (A.28), and (A.31), the nominal $n-$ period bond price is:

$$
\begin{aligned}
B_0^{\$,n} &= c_0 - a_\pi + \left[B_0^{\$,n-1} + B_2^{\$,n-1}a_\sigma + B_3^{\$,n-1}a_q + B_4^{\$,n-1}a_\pi\right] + \frac{1}{2}\varphi_\pi^2\left(B_4^{\$,n-1} - 1\right)^2 \\
B_1^{\$,n} &= c_1 + B_1^{\$,n-1}\rho_x \\
B_2^{\$,n} &= B_2^{\$,n-1}\rho_\sigma + (\theta-1)A_\sigma(\kappa_1\rho_\sigma - 1) + \frac{1}{2}\varphi_e^2\left[(\theta-1)\kappa_1 A_x + B_1^{\$,n-1}\right]^2 + \frac{1}{2}\left[(\gamma + \varphi_{\pi g}) - B_4^{\$,n-1}\varphi_{\pi g}\right]^2 \\
B_3^{\$,n} &= B_3^{\$,n-1}\rho_q + (\theta-1)A_q(\kappa_1\rho_q - 1) + \frac{1}{2}\left[(\theta-1)\kappa_1 A_\sigma + B_2^{\$,n-1} + \varphi_{\pi\sigma}\left(B_4^{\$,n-1} - 1\right)\right]^2 \\
&\quad + \frac{1}{2}\varphi_q^2\left[(\theta-1)\kappa_1 A_q + B_3^{\$,n-1}\right]^2 \\
B_4^{\$,n} &= \rho_\pi\left(B_4^{\$,n-1} - 1\right).
\end{aligned} \tag{A.32}
$$

A.5. Solution for the nominal bond risk premium

Let $rx_{t+1}^{\$,n-1} = p_{t+1}^{\$,n-1} - p_t^{\$,n} - r_{f,t}^{\$}$ be the return on buying the $n-$ period nominal bond at time t for $p_t^{\$,n}$ and selling $(n-1)$-period bond at time $t+1$ for $p_{t+1}^{\$,n-1}$ in excess of the short nominal rate. Then the expected excess return $\mathrm{E}\left[rx_{t+1}^{\$,n-1}\right]$ is given by the negative of covariance between the nominal pricing kernel $m_{t+1}^{\$,n-1}$ and the nominal bond price $p_{t+1}^{\$,n-1}$ adjusted for the Jensen inequality correction $\frac{1}{2}\mathrm{Var}\left[rx_{t+1}^{\$,n-1}\right]$. Using equations (A.31) and (A.28), we obtain the closed-form expression for the nominal bond risk premium:

$$
\begin{aligned}
\mathrm{brp}_t^{\$,n} &= \mathrm{E}\left[rx_{t+1}^{\$,n-1}\right] + \frac{1}{2}\mathrm{Var}\left[rx_{t+1}^{\$,n-1}\right] = -\mathrm{Cov}_t\left[m_{t+1}^{\$}, rx_{t+1}^{\$,n-1}\right] \\
&= -\mathrm{Cov}_t\left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}\right] \\
&= \left[(\gamma+\varphi_{\pi g})B_4^{\$,n-1}\varphi_{\pi g} - (\theta-1)\kappa_1 A_x B_1^{\$,n-1}\varphi_e^2\right]\sigma_{g,t}^2 \\
&\quad -\left[\left((\theta-1)\kappa_1 A_\sigma - \varphi_{\pi\sigma}\right)\left(B_2^{\$,n-1} + B_4^{\$,n-1}\varphi_{\pi\sigma}\right) + (\theta-1)\kappa_1 A_q B_3^{\$,n-1}\varphi_q^2\right]q_t \\
&\quad + B_4^{\$,n-1}\varphi_\pi^2 \\
&\equiv \beta_1^{\$,n}\sigma_{g,t}^2 + \beta_2^{\$,n}q_t + B_4^{\$,n-1}\varphi_\pi^2.
\end{aligned}
\tag{A.33}
$$

A.6. Solution for the forward spread

Let $f_t^{\$,n}$ be the one-year forward rate starting $n-1$ periods ahead: $f_t^{\$,n} = ny_t^{\$,n} - (n-1)y_t^{\$,n-1}$, then the forward spread $FS_t^{\$,n} = f_t^{\$,n} - y_t^{\$,1}$ is the difference between the forward rate and the one-period nominal bond yield $y_t^{\$,1}$. Using the affine pricing conjecture (A.25), show that the forward spread is the function of all four model state variables:

$$
\begin{aligned}
&FS_t^{\$,n} = ny_t^{\$,n} - (n-1)y_t^{\$,n-1} - y_t^{\$,1} = p_t^{\$,n-1} - p_t^{\$,n} - y_t^{\$,1} = \\
&\left(B_0^{\$,n-1} - B_0^{\$,n}\right) + \left(B_1^{\$,n-1} - B_1^{\$,n}\right)x_t + \left(B_2^{\$,n-1} - B_2^{\$,n}\right)\sigma_{g,t}^2 + \left(B_3^{\$,n-1} - B_3^{\$,n}\right)q_t + \\
&\left(B_4^{\$,n-1} - B_4^{\$,n}\right)\pi_t - \\
&\left(-\left(c_0 - a_\pi + \frac{1}{2}\varphi_\pi^2\right) - c_1 x_t - \left(c_2 + \frac{1}{2}\varphi_{\pi g}^2 + \gamma\varphi_{\pi g}\right)\sigma_{g,t}^2 - \left(c_3 + \frac{1}{2}\varphi_{\pi\sigma}^2 - (\theta-1)\kappa_1 A_\sigma \varphi_{\pi\sigma}\right)q_t + \rho_\pi \pi_t\right) \\
&= F_0^{\$,n} + F_1^{\$,n}x_t + F_2^{\$,n}\sigma_{g,t}^2 + F_3^{\$,n}q_t + F_4^{\$,n}\pi_t.
\end{aligned}
\tag{A.34}
$$

Using the nominal bond loadings in equation (A.32), obtain the constant term and the factors for the state variables in the forward spread equation (A.34). The constant term $F_0^{\$,n}$ in equation (A.34) is:

$$
\begin{aligned}
&F_0^{\$,n} = \left(B_0^{\$,n-1} - B_0^{\$,n}\right) + \left(c_0 - a_\pi + \frac{1}{2}\varphi_\pi^2\right) = \\
&-\left[B_2^{\$,n-1}a_\sigma + B_3^{\$,n-1}a_q + B_4^{\$,n-1}a_\pi\right] + \frac{1}{2}\varphi_\pi^2\left(1 - \left(B_4^{\$,n-1} - 1\right)^2\right).
\end{aligned}
\tag{A.35}
$$

The $F_1^{\$,n}$ loading on x_t is:

$$
F_1^{\$,n} = \left(B_1^{\$,n-1} - B_1^{\$,n}\right) + c_1 = B_1^{\$,n-1}(1-\rho_x).
\tag{A.36}
$$

The $F_2^{\$,n}$ loading on $\sigma_{g,t}^2$ is:

$$F_2^{\$,n} = \left(B_2^{\$,n-1} - B_2^{\$,n}\right) + \left(c_2 + \frac{1}{2}\varphi_{\pi g}^2 + \gamma\varphi_{\pi g}\right) = B_2^{\$,n-1}(1-\rho_\sigma) + \left(c_2 + \frac{1}{2}\varphi_{\pi g}^2 + \gamma\varphi_{\pi g}\right)$$
$$-(\theta-1)A_\sigma(\kappa_1\rho_\sigma - 1) - \frac{1}{2}\varphi_e^2\left[(\theta-1)\kappa_1 A_x + B_1^{\$,n-1}\right]^2 - \frac{1}{2}\left[(\gamma+\varphi_{\pi g}) - B_4^{\$,n-1}\varphi_{\pi g}\right]^2. \tag{A.37}$$

The $F_3^{\$,n}$ loading on q_t is:

$$F_3^{\$,n} = \left(B_3^{\$,n-1} - B_3^{\$,n}\right) + \left(c_3 + \frac{1}{2}\varphi_{\pi\sigma}^2 - (\theta-1)\kappa_1 A_\sigma \varphi_{\pi\sigma}\right) =$$
$$B_3^{\$,n-1}(1-\rho_q) + \left(c_3 + \frac{1}{2}\varphi_{\pi\sigma}^2 - (\theta-1)\kappa_1 A_\sigma\varphi_{\pi\sigma}\right) - (\theta-1)A_q(\kappa_1\rho_q - 1) -$$
$$\frac{1}{2}\left[(\theta-1)\kappa_1 A_\sigma + B_2^{\$,n-1} + \varphi_{\pi\sigma}\left(B_4^{\$,n-1} - 1\right)\right]^2 - \frac{1}{2}\varphi_q^2\left[(\theta-1)\kappa_1 A_q + B_3^{\$,n-1}\right]^2. \tag{A.38}$$

and the $F_4^{\$,n}$ loading on π_t term is:

$$F_4^{\$,n} = \left(B_4^{\$,n-1} - B_4^{\$,n}\right) - \rho_\pi = B_4^{\$,n-1} - \rho_\pi\left(B_4^{\$,n-1} - 1\right) - \rho_\pi = B_4^{\$,n-1}(1-\rho_\pi). \tag{A.39}$$

A.7. Solution for the interest rate variance risk premium (IRVRP)

Define $\sigma_{r\$,t}^2 = \text{Var}_t\left[r_{t,t+1}^{\$,n}\right]$, where $r_{t,t+1}^{\$,n} = p_{t+1}^{\$,n-1} - p_t^{\$,n}$, so $\sigma_{r\$,t}^2 = \text{Var}_t\left[p_{t+1}^{\$,n-1}\right]$. Similarly, $\sigma_{r\$,t+1}^2 = \text{Var}_{t+1}\left[p_{t+2}^{\$,n-2}\right]$. We need the conditional variance at time $t+1$ because time-t conditional variance is known and therefore, variance risk premium is constant. To derive the interest rate variance risk premium recall the affine pricing conjecture (A.25):

$$p_t^{\$,n} = B_0^{\$,n} + B_1^{\$,n}x_t + B_2^{\$,n}\sigma_t^2 + B_3^{\$,n}q_t + B_4^{\$,n}\pi_t. \tag{A.40}$$

Therefore:

$$\sigma_{r\$,t+1}^2 = \text{Var}_{t+1}\left[p_{t+2}^{\$,n-2}\right] = \text{E}_{t+1}\left[p_{t+2}^{\$,n-2} - \text{E}_t\left[p_{t+2}^{\$,n-2}\right]\right]^2 =$$
$$\left[\left(B_1^{\$,n-2}\varphi_e\right)^2 + \left(B_4^{\$,n-2}\varphi_{\pi g}\right)^2\right]\sigma_{g,t+1}^2 + \left[\left(B_2^{\$,n-2} + B_4^{\$,n-2}\varphi_{\pi\sigma}\right)^2 + \left(B_3^{\$,n-2}\varphi_q\right)^2\right]q_{t+1}$$
$$+\left(B_4^{\$,n-2}\varphi_\pi\right)^2, \tag{A.41}$$

and its expectation equals to:

$$\text{E}_t\left[\sigma_{r\$,t+1}^2\right] = \left[\left(B_1^{\$,n-2}\varphi_e\right)^2 + \left(B_4^{\$,n-2}\varphi_{\pi g}\right)^2\right]\left(a_\sigma + \rho_\sigma\sigma_{g,t}^2\right)$$
$$+\left[\left(B_2^{\$,n-2} + B_4^{\$,n-2}\varphi_{\pi\sigma}\right)^2 + \left(B_3^{\$,n-2}\varphi_q\right)^2\right]\left(a_q + \rho_q q_t\right) + \left(B_4^{\$,n-2}\varphi_\pi\right)^2. \tag{A.42}$$

The IRVRP is defined as the difference in expectations of the variance under risk-neutral $\mathbb{Q}$ and actual measures, which is given by the covariance between the variance of the nominal bond price and the nominal stochastic discount factor:

$$\text{E}_t^{\mathbb{Q}}\left[\sigma_{r\$,t+1}^2\right] - \text{E}_t\left[\sigma_{r\$,t+1}^2\right] = \text{Cov}_t\left[\sigma_{r\$,t+1}^2, m_{t+1}^{\$}\right]$$
$$= \text{E}_t\left[\left(\sigma_{r\$,t+1}^2 - \text{E}_t\sigma_{r\$,t+1}^2\right) \times \left(m_{t+1}^{\$} - \text{E}_t m_{t+1}^{\$}\right)\right]. \tag{A.43}$$

The unexpected part of the variance of the nominal bond price is given by:

$$\begin{aligned}\sigma^2_{r\$,t+1}-\mathrm{E}_t\sigma^2_{r\$,t+1} &= \left[\left(B_1^{\$,n-2}\varphi_e\right)^2+\left(B_4^{\$,n-2}\varphi_{\pi g}\right)^2\right]\sqrt{q_t}z_{\sigma,t+1}\\ &+\left[\left(B_2^{\$,n-2}+B_4^{\$,n-2}\varphi_{\pi\sigma}\right)^2+\left(B_3^{\$,n-2}\varphi_q\right)^2\right]\varphi_q\sqrt{q_t}z_{q,t+1}\end{aligned} \tag{A.44}$$

and the unexpected part of the nominal pricing kernel is given by (A.30). Taking the expectation of the product of equations (A.30) and (A.44), we obtain the IRVRP (A.43) in the closed form as a function of model parameters:

$$\begin{aligned}&\mathrm{IRVRP}_t^{\$,n}\left[\sigma^2_{r\$,t+1}\right]=\mathrm{E}_t^{\mathbb{Q}}\left[\sigma^2_{r\$,t+1}\right]-\mathrm{E}_t\left[\sigma^2_{r\$,t+1}\right]=\\ &(\theta-1)\kappa_1\left\{\left(A_\sigma-\varphi_{\pi\sigma}\right)\left[\left(B_1^{\$,n-2}\varphi_e\right)^2+\left(B_4^{\$,n-2}\varphi_{\pi g}\right)^2\right]+\right.\\ &\left.A_q\varphi_q^2\left[\left(B_2^{\$,n-2}+B_4^{\$,n-2}\varphi_{\pi\sigma}\right)^2+\left(B_3^{\$,n-2}\varphi_q\right)^2\right]\right\}q_t=\\ &(\theta-1)\kappa_1\nu^{\$,n}q_t.\end{aligned} \tag{A.45}$$

Note that the IRVRP in (A.45) has a superscript n, so it is maturity-dependent. Although the premium is a function of $(n-2)-$ period bond price parameters, it is indexed by n superscript because it is assessed at time t when the nominal bond has n periods to maturity. The notational leap from $n-2$ to n occurs because we assess time-t expectation of the time-$(t+1)$ conditional variance $\sigma^2_{r\$,t+1}$ of the time-$(t+2)$ nominal bond price $p_{t+2}^{\$,n-2}$.

A.8. Model-implied predictive regression coefficients

In this subsection we provide model-implied coefficients of the predictive regression of the nominal bond risk premium (A.33) on the IRVRP, equity variance risk premium (EVRP), and the forward spread $FS_t^{\$,n}$.

Predictability by the IRVRP factor. Consider the regression:

$$\mathrm{brp}_t^{\$,n}=a^n+b^n\times\mathrm{IRVRP}_t^{\$,n}\left[\sigma^2_{r\$,t+1}\right]+\varepsilon. \tag{A.46}$$

Ignoring the error term, the slope and R^2 coefficients for the regression (A.46) are given by:

$$b^n=\frac{\mathrm{Cov}\left[\mathrm{brp}_t^{\$,n},\mathrm{IRVRP}_t^{\$,n}\left[\sigma^2_{r\$,t+1}\right]\right]}{\mathrm{Var}\left[\mathrm{IRVRP}_t^{\$,n}\left[\sigma^2_{r\$,t+1}\right]\right]}\qquad \left(R^2\right)^n=\frac{(b^n)^2\times\mathrm{Var}\left[\mathrm{IRVRP}_t^{\$,n}\left[\sigma^2_{r\$,t+1}\right]\right]}{\mathrm{Var}\left[\mathrm{brp}_t^{\$,n}\right]}. \tag{A.47}$$

Using IRVRP_t and $\mathrm{brp}_t^{\$,n}$ in equations (A.45) and (A.33), respectively, in terms of state variables, obtain:

$$\begin{aligned}&\mathrm{Cov}\left[\mathrm{brp}_t^{\$,n},\mathrm{IRVRP}_t^{\$,n}\left[\sigma^2_{r\$,t+1}\right]\right]=\\ &\mathrm{Cov}\left[\beta_1^{\$,n}\sigma^2_{g,t}+\beta_2^{\$,n}q_t+B_4^{\$,n-1}\varphi_\pi^2,(\theta-1)\kappa_1\nu^{\$,n}q_t\right]=(\theta-1)\kappa_1\nu^{\$,n}\beta_2^{\$,n}\mathrm{Var}(q_t),\end{aligned} \tag{A.48}$$

because $\mathrm{Cov}\left[\sigma^2_{g,t},q_t\right]=0$. The model-implied variance of IRVRP is:

$$\mathrm{Var}\left[\mathrm{IRVRP}_t^{\$,n}\left[\sigma^2_{r\$,t+1}\right]\right]=(\theta-1)^2\kappa_1^2(\nu^{\$,n})^2\mathrm{Var}(q_t), \tag{A.49}$$

and the model-implied variance of $\mathrm{brp}_t^{\$,n}$ is:

$$\mathrm{Var}\left[\mathrm{brp}_t^{\$,n}\right]=\left(\beta_1^{\$,n}\right)^2\mathrm{Var}(\sigma^2_{g,t})+\left(\beta_2^{\$,n}\right)^2\mathrm{Var}(q_t). \tag{A.50}$$

Then the model-implied slope coefficient b^n of the regression (A.46) is:

$$b^n=\frac{(\theta-1)\kappa_1\nu^{\$,n}\beta_2^{\$,n}\mathrm{Var}(q_t)}{(\theta-1)^2\kappa_1^2(\nu^{\$,n})^2\mathrm{Var}(q_t)}=\frac{\beta_2^{\$,n}}{(\theta-1)\kappa_1\nu^{\$,n}}, \tag{A.51}$$

and the regression fit coefficient R^2 of the regression (A.46) is:

$$\left(R^2\right)^n = \frac{\left(\frac{\beta_2^{\$,n}}{(\theta-1)\kappa_1\nu^{\$,n}}\right)^2 \times (\theta-1)^2\kappa_1^2(\nu^{\$,n})^2\mathrm{Var}(q_t)}{\left(\beta_1^{\$,n}\right)^2\mathrm{Var}(\sigma_{g,t}^2)+\left(\beta_2^{\$,n}\right)^2\mathrm{Var}(q_t)} = \frac{\left(\beta_2^{\$,n}\right)^2\mathrm{Var}(q_t)}{\left(\beta_1^{\$,n}\right)^2\mathrm{Var}(\sigma_{g,t}^2)+\left(\beta_2^{\$,n}\right)^2\mathrm{Var}(q_t)}. \tag{A.52}$$

Predictability by the EVRP factor. Consider the regression:

$$\mathrm{brp}_t^{\$,n} = a^n + b^n \times \mathrm{EVRP}_t + \varepsilon. \tag{A.53}$$

As IRVRP, EVRP in the model is also driven solely by the q_t factor:

$$\begin{aligned}\mathrm{EVRP}_t &= \mathrm{E}_t^{\mathbb{Q}}\left[\sigma_{r,t+1}^2\right]-\mathrm{E}_t^{\P}\left[\sigma_{r,t+1}^2\right]\\ &= (\theta-1)\kappa_1\left[A_\sigma(1+\kappa_1^2A_x^2\varphi_e^2)+A_q\kappa_1^2\varphi_q^2(A_\sigma^2+A_q^2\varphi_q^2)\right]q_t.\end{aligned} \tag{A.54}$$

Thus, b^n and $\left(R^2\right)^n$ for the regression (A.53) are provided by:

$$b^n = \frac{\mathrm{Cov}\left[\mathrm{brp}_t^{\$,n},\mathrm{EVRP}_t\right]}{\mathrm{Var}[\mathrm{EVRP}_t]} = \frac{\beta_2^{\$,n}}{(\theta-1)\kappa_1\left[A_\sigma(1+\kappa_1^2A_x^2\varphi_e^2)+A_q\kappa_1^2\varphi_q^2(A_\sigma^2+A_q^2\varphi_q^2)\right]}, \tag{A.55}$$

$$\left(R^2\right)^n = \frac{(b^n)^2\times\mathrm{Var}[\mathrm{EVRP}_t]}{\mathrm{Var}\left[\mathrm{brp}_t^{\$,n}\right]} = \frac{\left(\beta_2^{\$,n}\right)^2\mathrm{Var}(q_t)}{\left(\beta_1^{\$,n}\right)^2\mathrm{Var}(\sigma_{g,t}^2)+\left(\beta_2^{\$,n}\right)^2\mathrm{Var}(q_t)}. \tag{A.56}$$

Note that the regression fit coefficients (A.52) and (A.56) are identical because both IRVRP and EVRP are driven by the same state variable, q_t.

Predictability by the FS factor. Consider the regression:

$$\mathrm{brp}_t^{\$,n} = a^n + b^n \times FS_t^{\$,n} + \varepsilon. \tag{A.57}$$

Ignoring the error term, the slope and R^2 coefficients for the regression (A.57) are given by:

$$\begin{aligned} b^n &= \frac{\mathrm{Cov}\left[\mathrm{brp}_t^{\$,n},FS_t^{\$,n}\right]}{\mathrm{Var}\left[FS_t^{\$,n}\right]},\\ \left(R^2\right)^n &= \frac{(b^n)^2\times\mathrm{Var}\left[FS_t^{\$,n}\right]}{\mathrm{Var}\left[\mathrm{brp}_t^{\$,n}\right]}.\end{aligned} \tag{A.58}$$

Using $\mathrm{brp}_t^{\$,n}$ and $FS_t^{\$,n}$ in terms of model state variables in equations (A.33) and (A.34), respectively, obtain:

$$\mathrm{Cov}\left[\mathrm{brp}_t^{\$,n},FS_t^{\$,n}\right] = \beta_1^{\$,n-1}F_2^{\$,n}\mathrm{Var}(\sigma_{g,t}^2)+\beta_2^{\$,n-1}F_3^{\$,n}\mathrm{Var}(q_t), \tag{A.59}$$

and

$$\mathrm{Var}\left[FS_t^{\$,n}\right] = \left(F_1^{\$,n}\right)^2\mathrm{Var}(x_t)+\left(F_2^{\$,n}\right)^2\mathrm{Var}(\sigma_{g,t}^2)+\left(F_3^{\$,n}\right)^2\mathrm{Var}(q_t)+\left(F_4^{\$,n}\right)^2\mathrm{Var}(\pi_t). \tag{A.60}$$

Substituting equations (A.50), (A.59), and (A.60) into equations (A.58), we obtain the model-implied b and R^2 in terms of model parameters.

B. Appendix

Table B1
Multivariate predictive regressions: IRVRP, EVRP, and FS.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
1-month holding period												
IRVRP	0.830**	0.705**	0.598**	0.512**	0.443**	0.389**	0.345**	0.311**	0.284**	0.212*	0.166$^+$	0.090
	(14.647)	(11.472)	(8.177)	(6.103)	(4.869)	(4.134)	(3.741)	(3.605)	(3.523)	(2.433)	(1.904)	(0.976)
EVRP	−0.015	−0.021	−0.028	−0.040	−0.055	−0.073	−0.091$^+$	−0.109*	−0.126$^+$	−0.187**	−0.218**	0.264**
	(−0.557)	(−0.687)	(−0.815)	(−1.027)	(−1.295)	(−1.579)	(−1.813)	(−1.980)	(−1.842)	(−3.347)	(−3.775)	(5.911)
FS	0.291**	0.339**	0.369**	0.387**	0.396**	0.398**	0.394**	0.385**	0.373**	0.302**	0.259**	−0.040
	(4.621)	(5.535)	(5.532)	(5.347)	(5.205)	(5.105)	(5.109)	(5.144)	(5.008)	(3.769)	(3.127)	(−0.459)
R^2	0.842	0.796	0.729	0.659	0.595	0.536	0.485	0.439	0.399	0.263	0.195	0.074
3-month holding period												
IRVRP	0.749**	0.553**	0.409**	0.310**	0.241$^+$	0.193	0.158	0.133	0.114	0.045	−0.023	0.168
	(12.907)	(6.409)	(3.791)	(2.585)	(1.926)	(1.489)	(1.159)	(0.937)	(0.772)	(0.301)	(−0.157)	(1.459)
EVRP	−0.025	−0.042$^+$	−0.061*	−0.081*	−0.101**	−0.119**	−0.134**	−0.147**	−0.156**	−0.173**	−0.171**	0.351**
	(−1.012)	(−1.870)	(−2.357)	(−2.556)	(−2.779)	(−3.080)	(−3.275)	(−3.401)	(−3.499)	(−3.779)	(−3.839)	(8.461)
FS	0.302**	0.334**	0.350**	0.359**	0.362**	0.359**	0.352**	0.343*	0.331*	0.287*	0.279$^+$	−0.074
	(3.714)	(3.243)	(3.042)	(2.874)	(2.852)	(2.781)	(2.632)	(2.490)	(2.349)	(2.041)	(1.923)	(−0.564)
R^2	0.716	0.561	0.445	0.369	0.318	0.280	0.251	0.227	0.206	0.131	0.093	0.139
	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
6-month holding period												
IRVRP	0.582**	0.335**	0.191	0.100	0.041	0.002	−0.022	−0.036	−0.045	−0.077	−0.127	0.253*
	(6.705)	(2.907)	(1.345)	(0.617)	(0.231)	(0.013)	(−0.112)	(−0.184)	(−0.227)	(−0.389)	(−0.654)	(1.960)
EVRP	−0.016	−0.039	−0.059	−0.078$^+$	−0.096*	−0.112*	−0.126**	−0.138**	−0.147**	−0.157**	−0.141*	0.273**
	(−0.406)	(−0.871)	(−1.290)	(−1.712)	(−2.117)	(−2.480)	(−2.787)	(−3.025)	(−3.188)	(−3.003)	(−2.283)	(4.985)
FS	0.277*	0.311$^+$	0.336 $^+$	0.357*	0.370*	0.373$^+$	0.370$^+$	0.362$^+$	0.351$^+$	0.310	0.307	−0.113
	(1.961)	(1.934)	(1.951)	(1.970)	(1.967)	(1.933)	(1.872)	(1.798)	(1.723)	(1.519)	(1.487)	(−0.751)
R^2	0.461	0.290	0.220	0.188	0.169	0.155	0.144	0.134	0.125	0.089	0.070	0.111
12-month holding period												
IRVRP	−0.046	−0.024	−0.032	−0.055	−0.081	−0.101	−0.112	−0.117	−0.116	−0.105	−0.154	0.234$^+$
	(−0.376)	(−0.164)	(−0.178)	(−0.270)	(−0.356)	(−0.411)	(−0.434)	(−0.434)	(−0.420)	(−0.394)	(−0.614)	(1.668)
EVRP	0.030	0.013	0.004	−0.004	−0.012	−0.021	−0.030	−0.039	−0.048	−0.071	−0.071	0.148 $^+$
	(0.300)	(0.133)	(0.039)	(−0.051)	(−0.153)	(−0.272)	(−0.405)	(−0.541)	(−0.674)	(−1.089)	(−1.113)	(1.822)
FS	0.054	0.109	0.172	0.232	0.280	0.312	0.329	0.333	0.330	0.286	0.296	0.022
	(0.272)	(0.498)	(0.742)	(0.960)	(1.117)	(1.209)	(1.245)	(1.242)	(1.219)	(1.100)	(1.117)	(0.137)
R^2	0.005	0.011	0.025	0.041	0.055	0.063	0.068	0.068	0.066	0.052	0.049	0.087

Note: This table presents results of the multivariate regression.
$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)\cdot\mathrm{IRVRP}_t + \beta_2^{(\tau)}(h)\cdot\mathrm{EVRP}_t + \beta_3^{(\tau)}(h)\cdot\mathrm{FS}_t^{(\tau)} + \varepsilon_{t+h}^{(\tau)}$,
where $rx_{t+h}^{(\tau)}$ is the $h-$ period excess return for a Treasury security with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$ tenor, IRVRP_t is the measure of interest rate variance risk premium, EVRP_t is the equity variance risk premium measure constructed by Bollerslev et al[10] using S&P 500 index options, and $\mathrm{FS}_t^{(\tau)}$ is the forward spread between the one-year forward rate τ-year ahead and the on-year zero coupon yield. The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length determined according to Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^+$ for $p < 0.1$, where p is the p-value.

Table B2
Control for macro principal components.

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
A: 1-month holding period												
IRVRP	1.009**	0.958**	0.907**	0.862**	0.821**	0.784**	0.749**	0.715**	0.683**	0.541**	0.437**	0.086
	(10.278)	(10.919)	(13.423)	(11.641)	(11.100)	(10.461)	(9.968)	(9.688)	(9.409)	(7.875)	(6.061)	(1.157)
F1	−0.054	−0.038	−0.033	−0.034	−0.040	−0.047	−0.054	−0.061	−0.066	−0.079	−0.077	−0.187*
	(−0.892)	(−0.632)	(−0.617)	(−0.574)	(−0.662)	(−0.780)	(−0.914)	(−1.046)	(−1.159)	(−1.513)	(−1.554)	(−2.369)
F2	−0.126	−0.105	−0.086	−0.071	−0.058	−0.047	−0.038	−0.030	−0.023	0.002	0.010	0.114
	(−1.584)	(−1.436)	(−1.427)	(−1.037)	(−0.818)	(−0.639)	(−0.497)	(−0.387)	(−0.291)	(0.019)	(0.111)	(1.482)
F3	−0.047*	−0.066**	−0.085*	−0.101**	−0.115**	−0.128**	−0.138**	−0.146**	−0.153*	−0.172*	−0.181*	−0.005
	(−2.170)	(−2.699)	(−2.443)	(−2.678)	(−2.766)	(−2.783)	(−2.738)	(−2.666)	(−2.575)	(−2.366)	(−2.386)	(−0.118)
F4	−0.094	−0.078	−0.065	−0.056	−0.052	−0.049	−0.047	−0.045	−0.042	−0.014	0.027	−0.134
	(−1.472)	(−1.297)	(−1.401)	(−1.032)	(−0.898)	(−0.826)	(−0.769)	(−0.713)	(−0.649)	(−0.227)	(0.516)	(−1.238)
F5	0.041	0.021	0.001	−0.016	−0.028	−0.037	−0.041	−0.044	−0.044	−0.040	−0.046	0.031
	(1.343)	(0.638)	(0.027)	(−0.382)	(−0.665)	(−0.840)	(−0.927)	(−0.948)	(−0.924)	(−0.806)	(−0.890)	(0.411)
F6	0.081**	0.092**	0.095**	0.093*	0.088^{+}	0.081	0.074	0.067	0.062	0.055	0.076	−0.107*
	(3.314)	(2.913)	(2.813)	(2.175)	(1.724)	(1.367)	(1.095)	(0.899)	(0.757)	(0.531)	(0.740)	(−2.030)
F7	0.049^{+}	0.041	0.026	0.009	−0.008	−0.024	−0.038	−0.048	−0.056	−0.065	−0.054	0.073
	(1.814)	(1.326)	(0.743)	(0.228)	(−0.200)	(−0.562)	(−0.855)	(−1.075)	(−1.241)	(−1.386)	(−1.088)	(1.060)
F8	0.000	0.021	0.048	0.077^{+}	0.106*	0.133**	0.157**	0.176**	0.192**	0.226**	0.222**	0.017
	(0.009)	(0.581)	(1.207)	(1.857)	(2.491)	(2.958)	(3.311)	(3.507)	(3.617)	(3.814)	(4.189)	(0.242)
R^2	0.779	0.713	0.643	0.582	0.530	0.488	0.452	0.421	0.393	0.289	0.223	0.081
B: 3-month holding period												
IRVRP	0.841**	0.670**	0.551**	0.475**	0.425**	0.391**	0.365**	0.343**	0.323**	0.225**	0.134	0.147
	(8.286)	(6.362)	(5.472)	(4.742)	(4.087)	(3.922)	(3.744)	(3.498)	(3.216)	(2.595)	(1.368)	(1.178)
F1	0.030	0.069	0.077	0.071	0.059	0.046	0.031	0.018	0.006	−0.035	−0.056	−0.222
	(0.394)	(0.818)	(0.869)	(0.757)	(0.723)	(0.583)	(0.426)	(0.261)	(0.093)	(−0.649)	(−1.001)	(−1.523)
F2	−0.064	−0.006	0.034	0.059	0.073	0.082	0.088	0.092	0.095	0.108	0.114	0.108
	(−0.753)	(−0.063)	(0.412)	(0.666)	(0.811)	(0.934)	(1.012)	(1.053)	(1.072)	(1.408)	(1.489)	(1.109)
F3	-0.042^{+}	-0.044^{+}	−0.044	−0.043	−0.042	−0.042	−0.042	−0.043	−0.043	−0.050	-0.064^{+}	0.163^{+}
	(−1.752)	(−1.693)	(−1.206)	(−1.040)	(−1.244)	(−1.270)	(−1.260)	(−1.235)	(−1.172)	(−1.603)	(−1.959)	(1.824)
F4	−0.061	−0.010	0.031	0.059	0.077	0.090	0.098	0.104	0.109	0.134^{+}	0.170*	−0.177
	(−0.688)	(−0.103)	(0.375)	(0.713)	(0.895)	(1.036)	(1.160)	(1.271)	(1.379)	(1.719)	(2.086)	(−1.308)
F5	0.058	0.033	0.006	−0.019	−0.040	−0.056	−0.069	−0.078	−0.084	−0.091*	−0.085*	0.098
	(1.433)	(0.697)	(0.115)	(−0.351)	(−0.763)	(−1.078)	(−1.319)	(−1.484)	(−1.588)	(−2.218)	(−2.082)	(1.344)
F6	0.077**	0.082*	0.083*	0.084^{+}	0.084^{+}	0.085^{+}	0.086	0.087	0.089	0.101*	0.118*	-0.079^{+}
	(2.645)	(2.324)	(1.984)	(1.684)	(1.712)	(1.647)	(1.573)	(1.494)	(1.412)	(2.071)	(2.394)	(−1.815)
F7	0.055*	0.058*	0.055	0.048	0.042	0.036	0.032	0.029	0.029	0.042	0.059	0.002
	(2.423)	(2.028)	(1.475)	(1.155)	(0.971)	(0.793)	(0.676)	(0.617)	(0.606)	(0.942)	(1.578)	(0.035)
F8	−0.007	0.009	0.023	0.036	0.048	0.058	0.066	0.073	0.077	0.079	0.064	0.036
	(−0.140)	(0.150)	(0.372)	(0.563)	(0.705)	(0.854)	(0.976)	(1.065)	(1.131)	(1.111)	(0.818)	(0.591)
R^2	0.635	0.469	0.362	0.298	0.258	0.231	0.211	0.195	0.182	0.134	0.112	0.132
	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
C: 6-month holding period												
IRVRP	0.517**	0.276*	0.169	0.121	0.100	0.092	0.089	0.088	0.087	0.061	0.014	0.215
	(4.296)	(2.068)	(1.282)	(0.891)	(0.763)	(0.722)	(0.711)	(0.708)	(0.700)	(0.490)	(0.113)	(1.372)
F1	0.185	0.203	0.180	0.150	0.120	0.093	0.069	0.048	0.029	−0.035	−0.080	−0.194
	(1.498)	(1.633)	(1.554)	(1.400)	(1.220)	(1.030)	(0.832)	(0.625)	(0.413)	(−0.543)	(−1.172)	(−1.130)
F2	0.062	0.153	0.200^{+}	0.222*	0.232*	0.234*	0.232*	0.227*	0.221*	0.191^{+}	0.171^{+}	0.081
	(0.563)	(1.451)	(1.912)	(2.044)	(2.171)	(2.245)	(2.276)	(2.258)	(2.160)	(1.794)	(1.676)	(0.777)
F3	−0.031	−0.030	−0.030	−0.032	−0.035	−0.037	−0.038	−0.040	−0.040	−0.035	−0.026	0.110
	(−1.390)	(−0.899)	(−0.809)	(−0.912)	(−0.984)	(−1.020)	(−1.008)	(−1.017)	(−0.972)	(−0.837)	(−0.663)	(1.506)
F4	−0.080	−0.009	0.044	0.081	0.107	0.127	0.142	0.155	0.165	0.207*	0.242**	−0.164
	(−0.631)	(−0.077)	(0.406)	(0.671)	(0.901)	(1.095)	(1.272)	(1.430)	(1.598)	(2.179)	(2.742)	(−0.964)
F5	0.037	0.009	−0.009	−0.021	−0.030	−0.037	−0.040	−0.042	−0.042	−0.032	−0.020	0.055
	(0.842)	(0.179)	(−0.175)	(−0.415)	(−0.590)	(−0.704)	(−0.767)	(−0.796)	(−0.820)	(−0.642)	(−0.417)	(1.143)
F6	0.055	0.052	0.052	0.054	0.057	0.059	0.061	0.063	0.064	0.062	0.060	−0.093*
	(1.455)	(1.226)	(1.163)	(1.221)	(1.285)	(1.324)	(1.320)	(1.301)	(1.232)	(1.262)	(1.562)	(−2.103)

(*continued on next page*)

Table B2 (*continued*)

	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	Equity
F7	0.101**	0.116*	0.117*	0.112*	0.105*	0.099$^+$	0.093$^+$	0.089	0.086	0.086	0.094$^+$	−0.054
	(2.704)	(2.540)	(2.435)	(2.190)	(2.018)	(1.855)	(1.713)	(1.615)	(1.577)	(1.627)	(1.958)	(−0.844)
F8	−0.022	−0.029	−0.029	−0.024	−0.016	−0.007	0.002	0.011	0.020	0.043	0.044	0.092*
	(−0.439)	(−0.528)	(−0.495)	(−0.400)	(−0.263)	(−0.112)	(0.039)	(0.180)	(0.303)	(0.659)	(0.688)	(2.291)
R^2	0.421	0.256	0.195	0.170	0.158	0.152	0.147	0.144	0.140	0.125	0.121	0.123
D: 12-month holding period												
IRVRP	−0.411*	−0.365$^+$	−0.308$^+$	−0.251	−0.198	−0.150	−0.109	−0.075	−0.048	0.014	0.006	0.308
	(−2.065)	(−1.960)	(−1.772)	(−1.536)	(−1.259)	(−0.999)	(−0.743)	(−0.530)	(−0.344)	(0.099)	(0.040)	(1.596)
F1	0.376*	0.343*	0.289*	0.234$^+$	0.181	0.133	0.090	0.052	0.019	−0.105	−0.194*	−0.204
	(2.002)	(2.202)	(2.096)	(1.869)	(1.579)	(1.264)	(0.925)	(0.579)	(0.225)	(−1.322)	(−2.310)	(−1.026)
F2	0.305$^+$	0.347*	0.374*	0.385**	0.385**	0.379**	0.370**	0.358**	0.346**	0.291*	0.254*	0.062
	(1.766)	(2.218)	(2.561)	(2.805)	(2.979)	(3.044)	(3.066)	(3.039)	(2.985)	(2.507)	(2.080)	(0.510)
F3	0.013	0.013	0.014	0.012	0.008	0.003	−0.001	−0.005	−0.007	−0.009	−0.001	0.025
	(0.405)	(0.418)	(0.437)	(0.371)	(0.245)	(0.102)	(−0.033)	(−0.149)	(−0.236)	(−0.275)	(−0.019)	(0.629)
F4	−0.033	0.019	0.057	0.087	0.110	0.128	0.142	0.154	0.163	0.203	0.246*	−0.094
	(−0.213)	(0.123)	(0.362)	(0.555)	(0.718)	(0.845)	(0.956)	(1.042)	(1.121)	(1.542)	(2.051)	(−0.644)
F5	0.053	0.054	0.057	0.056	0.053	0.049	0.044	0.040	0.037	0.033	0.043	−0.029
	(0.890)	(0.969)	(1.039)	(1.034)	(0.974)	(0.901)	(0.816)	(0.753)	(0.705)	(0.631)	(0.854)	(−0.659)
F6	0.030	0.038	0.041	0.044	0.047	0.049	0.050	0.051	0.052	0.057	0.062$^+$	−0.059$^+$
	(0.561)	(0.769)	(0.888)	(0.991)	(1.075)	(1.163)	(1.228)	(1.285)	(1.350)	(1.557)	(1.817)	(−1.953)
F7	0.065	0.083	0.089	0.088	0.085	0.081	0.076	0.072	0.069	0.063	0.068	−0.059
	(1.018)	(1.319)	(1.428)	(1.443)	(1.409)	(1.340)	(1.267)	(1.189)	(1.125)	(1.030)	(1.167)	(−1.096)
F8	0.008	0.014	0.020	0.027	0.033	0.040	0.045	0.049	0.053	0.065	0.069	0.003
	(0.124)	(0.230)	(0.341)	(0.468)	(0.600)	(0.712)	(0.815)	(0.887)	(0.949)	(1.200)	(1.382)	(0.041)
R^2	0.120	0.118	0.120	0.127	0.135	0.144	0.152	0.158	0.163	0.176	0.196	0.102

Note: this table presents results of the multivariate regression.
$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)\cdot \text{IRVRP}_t + \beta_2^{(\tau)}(h)\cdot \text{F}_t + \varepsilon_{t+h}^{(\tau)}$,
where $rx_{t+h}^{(\tau)}$ is the $h-$period excess return for Treasuries with tenors of τ (=2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 20) years, and the equity market portfolio with $\tau = \infty$, IRVRP_t is the measure of interest rate variance risk premium, and F_t contains eight macro principal components of Ludvigson and Ng.[6] The t-statistics presented in parentheses are calculated using Newey and West[43] standard errors with the optimal lag length of Newey and West.[44] All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from March 1993 through February 2013. Significance levels: ** for $p < 0.01$, * for $p < 0.05$, and $^+$ for $p < 0.1$, where p is the p-value.

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