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Discussion

Behavioral heterogeneity and financial crisis: The role of sentiment

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ABSTRACT

Recent empirical works have confirmed the importance of sentiment in asset pricing. In this paper, we propose that sentiment may not affect everyone in a homogeneous way. We construct a sentiment indicator taking into consideration behavioral heterogeneity of interacting investors. We find that sentiment contributes to several financial anomalies such as fat tails and volatility clustering of returns. More importantly, investor sentiment could be a significant source of financial market volatility. Our model with sentiment is able to replicate different types of crises, in which the crisis severity is enhanced with rise of sentiment sensitivity of chartist traders.

1. Introduction

For a long time, Efficient Market Hypothesis (EMH) has been the cornerstone and mainstream belief of the modern asset pricing theory. In recent years, however, skepticism against the validity of EMH has grown in light of its failure to explain several ubiquitous financial regularities, such as excess volatility and systemic under- or over-valuation of stock prices relative to their intrinsic values. This gives rise to the alternative behavioral finance theory which aims to provide rationale behind the unexplained market anomalies.

Behavioral finance challenges the fundamental assumption of EMH, in that investors are assumed to be boundedly rational and human psychology plays a crucial role in investment decision-making. In fact, even before behavioral paradigm came into the limelight in finance and economics, investor sentiment has been perceived as a common phenomenon by financial analysts and investors. As quoted from [1], "One of the oldest theories about financial markets, expressed long ago in newspapers and magazines rather than scholarly journals, is, if translated into academic words, a price-to-price feedback theory". The feedback theory suggests that an increase in speculative prices is further propagated into bubble when financial successes of some investors are envied by the others and lead to public enthusiasm towards the speculative asset despite such upward spiral in prices is unsustainable. While conventional wisdom largely supports the idea that investor sentiment may overcome rational thoughts in trading behavior, the sentiment analysis has only started gaining recognition in financial academic research in the past two decades.

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One of the pioneering works formalizing the role of investor sentiment in financial market is the noise trader model proposed by De Long et al. [2]. In their model, uninformed noise traders are susceptible to the influence of sentiment that is in part unpredictable, while rational investors are wary of the noise trader risk and refrain from aggressive arbitrage, thus contributing to prolonged mispricing in financial markets. The subsequent related studies conduct more in-depth analyses on specific channels of investor sentiment. Lux [3,4] explicitly model market mood contagion through social interaction among agents to provide a behavioral explanation for bubbles and crashes. Daniel et al. [5] and Barberis et al. [6] construct models of investor sentiment based on psychological evidence to reconcile the empirical findings of over-reaction and under-reaction of stock prices to news. In particulars, [5] attribute sentiment to overconfidence and self-attribution, whereas [6] concentrate on conservatism and representativeness heuristic. More recently, researchers try to quantify the effect of sentiment on financial markets using empirical method. Baker and Wurgler [7,8] develop a "top down" approach to behavioral finance, by first forming a composite sentiment index and then empirically testing the effects of the sentiment index on different types of stocks. They find that low sentiment can predict higher returns for a subset of stocks. By investigating the interactions between daily media context from Wall Street Journal column and stock market from 1984–1994, [9] finds that media content is linked to the behavior of individual investors rather than serves as a proxy for new information about fundamental asset values or proxy for market volatility. Furthermore, he finds that high media pessimism predicts downward pressure on market price and increased market volatility. To date, there have been many studies, both theoretical and empirical, that prove evidence of investor sentiment effects in financial market (See for examples, [10–13]). In view of that, several experimental studies have brought this into laboratory environment and find that investor's psychology, specifically over-optimism [14], friendship network [15] and induced positive mood [16] can amplify market price oscillations. Most empirical and experimental studies focus on addressing what determines sentiment and how sentiment could be used in financial analysis. Very few, however, track the roots of sentiment formation in behavioral trading and model the sentiment formation process in dynamic systems.

Previous literature has the consensus that the investor sentiment can affect asset prices but the question of its importance on these prices remains. More specifically, can sentiment explain financial crisis wherein traditional financial models have failed to rationalize? How does sentiment work in the formation of financial crisis? These are particularly pertinent questions to ask given that we are living in the period with more frequent occurrence of financial crises. Comparing to the large volume of published works on investor sentiment, few studies have directly linked sentiment to market crises. Among these are [17,18] that focus on U.S. stock market crash of 1987 and [19] who use a panel data of international stock markets to find the contribution of investor sentiment in raising the probability of crises within one-year horizon.

Against such backdrop, we aim to investigate the role of investor sentiment on the dynamics of asset prices and market crises within the framework of a heterogeneous agent model (HAM). HAM is a burgeoning framework under behavioral finance which incorporates interacting agents with heterogeneous trading beliefs in the model. Instead of following standard representative agent assumption, HAM assumes the differentiation of traders, especially with respect to their expectations or beliefs on future price. A strand of HAM literature based on a dichotomy of fundamentalist and chartist beliefs has been proven useful in accommodating market features that seem not easily reconcilable under the traditional financial market theory. These features include fat tail, volatility clustering, bubbles and crises (see pioneering works by Day and Huang [20]; Lux [3]; Brock and Hommes [21]; Kaizoji [22]; Chiarella and He [23]; He and Westerhoff [24]). While HAM has gained increased popularity in recent years, only a small number of HAM studies have taken into account investor sentiment [25,26], let alone a rigorous research on the role of sentiment in financial market. To fill this gap in the literature, in this paper, we propose a HAM model with sentiment indicator that captures memory of sentiment, social interaction and sentiment shock. Our idea of social interaction is inspired by the work of Lux [3,4] which emphasize that agents are not isolated units. Speculators, also known as chartists, rely on both actual price movements as well as the behavior of their competitors in forming their expectations. As such, we suggest that sentiment may not affect everyone in a homogeneous way. This conjecture is founded on the abundant evidence of behavioral heterogeneity between individual traders and institutional traders, in which the former often lacks access to insider information and thus is more susceptible to market sentiment (see [2,27]). Under our HAM setting, fundamentalists represent rational arbitrageurs that possess information about fundamental asset value, whereas chartists are ill-informed speculators extrapolating on market trend. We thus discriminate between these two groups by assuming only chartists' expectations are liable to market sentiment. An endogenous mechanism between sentiment and agent's belief switching is developed as investors are allowed to switch between fundamentalist and chartist beliefs according to the past performance of each strategy, while the sentiment index is contingent on the fraction of adopted beliefs in the market.

We model heterogeneous responses to sentiment under a fundamentalist-chartist framework. Some highlights of our model simulation findings include: (1) sentiment contributes to stylized facts such as fat tails, volatility clustering and long memory dependence of daily returns that are commonly observed in actual stock markets; (2) we find that market volatility increases with the presence of sentiment and investor sentiment may help to explain the excess volatility puzzle; and (3) our model with sentiment is able to replicate different types of crises, including sudden crisis, disturbing crisis and smooth crisis as in [28] and show that sentiment sensitivity of investors is positively correlated with the frequency and the magnitude of crisis.

This paper is organized as follows. Section 2 defines investor sentiment and explains the formation of market sentiment. Section 3 presents the HAM model. Section 4 analyzes and discusses the simulation results with focus on stylized facts, market volatility and crises of the US stock market. Section 5 carries out several robustness checks. Section 6 concludes.

2. Sentiment

Classical finance theory neglects the role of investor sentiment as investors are supposed to be rational. Rational investor assumption is the key for many classical models, in which market is also assumed to be perfectly efficient. Even if some investors are not rational, arbitrageurs can exploit their irrational behavior, thus causing prices to reflect the fundamental information of the market. The assumption of the rational investor is generally quite useful in modeling, but it misses many important exceptions. John Maynard Keynes was one of the first economists to explore the relationship between mental processes and economic activity. He argued that human psychology played a crucial role in driving economic activity. As he stated, *Our decisions to do something positive…can only be taken as the result of animal spirits—a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities*. Keynes's notion of animal spirits highlight the role of psychological factors in fueling economic and investment activity. With the emergence of behavioral economics and finance, the role of psychological factors has been widely investigated in financial markets. As a common concept in psychology, sentiment has attracted much attention in behavioral finance. In psychology, sentiment is understood as human emotion. Human experience a broad range of emotions, including joy, fear, stress, anger, gloom and so forth. These emotions could influence the decision making of human. Hence, it is important to explore cognitive or emotional factors in decision making. There are so many emotions involved in investment behavior, to measure the aggregate sentiment for investors in financial markets, we focus on the sources of sentiment. In other words, we care about what factors cause the change of sentiment in financial market. By reviewing literature in psychology and behavioral finance, we identify three important sources of sentiment formation in financial markets, which are memory of sentiment, social interaction and sentiment shocks (news, policies and events etc.).

Memory of sentiment One important phenomenon documented by psychologists is sentiment persistence. Human brain stores information with different sentiments. This memory of sentiment may last for a few days or weeks. In terms of trading behavior, investors' sentiment is persistent if there is no arrival of new information. If investors are optimistic in the investment, they will keep this sentiment for periods until new information changes their mood. Many psychological experiments have shown that individuals are slow to change their beliefs even in the face of new information. The sentiment model developed in [6] is based on the persistence of sentiment and conservatism behavior of investors. Empirical evidences of sentiment persistence in financial markets are also found in [29–31].

Social interaction Humans are inherently social. The social interaction is an important way to form opinion, sentiment or beliefs. People who regularly interact with each other tend to think and behave similarly. As mentioned in [32], there are two main strands literature dealing with this herding behavior. The first group focuses on information cascades model proposed by Banerjee [33]. He argues that people acquire information in sequences by observing the actions of other individuals who precede them in the sequence. In financial markets, the trading activity of one investor conveys information to other investor that can cause latter to react, leading to a cascade of trading. Another strand of literature focuses on interpersonal communication of investors. The conversation and social interaction is more relevant to describe mass behavior that informational cascades. Conversation not only serves to exchange information, it also reinforces memories of pieces of information. With these social interactions, sentiment may also be contagious within the group. Suppose the majority in a social group hold the optimistic opinion on stock market, this bullish sentiment may spread in the group through social interaction. With the help of social media, the sentiment may spread even faster.

Sentiment shock Sentiment shocks refer to exogenous sources that can shift the sentiment. An example is pseudo signal which is also called noise. It may be news, policies, rumors or advice of a financial guru. The noise could shift the current sentiment of investors and lead to noise trading. As stated in [34], noise is often a causal factor much more powerful than a small number of large events can be. As news and noise information are so important in measuring market sentiment, many studies attempt to develop a sentiment index based on media news. Soo [35] finds that media sentiment has significant predictive power in the housing market.

3. Model

In this section, we set up an asset pricing model with a single risky asset to characterize time series momentum and investor sentiment in the financial market. The modeling approach follows closely the current HAM framework by incorporating bounded rationality, belief heterogeneity and adaptive learning process.

In the HAM literature, the market fractions of different types of traders play a pivotal role in determining the market price behavior. According to [3], the time-varying market fraction of investors is the source of market mood or market sentiment, which may introduce complex dynamics in the financial market. Based on both theoretical and empirical evidence, our model extends early models by introducing investor sentiment into the decision making process of agents. In each trading period, a population of agents is assumed to be distributed among three groups, each relying upon different behavioral rules. These include fundamentalists who trade according to fundamental analysis, as well as momentum traders and contrarian traders who trade differently based on historical price trends. In particular, we postulate that sentiment affects different types of agents in a heterogeneous way. We assume that fundamentalist group is more rational and is immune from market sentiment, while the chartists, both momentum traders and contrarian traders, are susceptible to market sentiment. Moreover, momentum and contrarian traders also react differently with respect to positive and negative sentiments. An endogenous mechanism between sentiment and agents' belief switching is developed, in that investors are allowed to switch their beliefs according to past performance while the sentiment index is contingent on the fraction of adopted beliefs in the market. As in [20], market price in each period is determined by a market maker who adjusts price as a function of excess demand.

3.1. Fundamentalists

We assume that fundamentalists have more knowledge about the economy and have an idea about the market fundamental value. Hence, fundamentalists make decision based on the market price deviation from fundamental price, μ_t. In particular, they believe that market price p_t is mean-reverting to its intrinsic value and hence will buy (sell) the stock when its current price is below (above) its fundamental value. They estimate the fundamental price based on various types of fundamental information, such as expected dividends, earnings, price-earnings ratios, economic growth and so forth. In each period, the fundamental price is updated with new information arrival which is accessible to the public. The prior of μ_t is governed by:

$$\mu_t = \mu + e_t \tag{1}$$

where μ is the mean of fundamental value and the noise term e_t is independently and normally distributed with mean 0 and standard deviation σ. Instead of deriving the demand functions from expected utility maximization, we adopt simple demand functions for all the three types of agents. As shown in some literature, such as [36,37], and [20], these seemingly 'ad hoc' assets demand functions can be reconciled with the underlying utility maximization. The excess demand of fundamentalists $D_{f,t}$ is based on the spread between the latest market price p_t and the fundamental price μ_t, which can be written as:

$$D_{f,t} = A(x_t)(\mu_t - p_t) \tag{2}$$

The reaction function $A\left(x_t\right)$ captures the behavior of the fundamentalists when price is around the fundamental price. It is assumed to be a nonlinear smooth and symmetric function of price deviation from fundamental value. Let $x_t = p_t - \mu_t$ denotes this price deviation, then:

$$A(x_t) = \frac{ax_t^2}{1 + bx_t^4} \tag{3}$$

$A(x_t)$ could mimic the change in confidence of fundamentalists. The similar capture of nonlinear confidence behavior for fundamentalist has also been discussed in [20,38,39], and [40]. We assume fundamentalists' confidence continuously increases with absolute price deviation $|x_t|$ in a reasonable zone $x_t \in (-Z_t, Z_t)$, and the range is determined by parameter b. Within this reasonable zone, fundamentalists firmly hold the fundamental strategy, and the rise of price misalignment makes them feel more confident that price will revert to fundamental value soon, so they should grasp the opportunity to buy or sell stocks to maximize their gain. As argued by Reitz and Westerhoff [39], the larger the deviation from fundamental value, the stronger the confidence in mean reversion of fundamentalists. However, if the misalignment further increases and exceeds the reasonable zone, with high uncertainty in the market, traders face a fundamental risk [40,41] which causes the fundamentalists' confidence to diminish and the traders thus become increasingly reluctant to submit orders [42]. The decreasing power of fundamentalist with increasing price deviation has been proven theoretically and empirically in [38,43,44], and [45]. The shape of $A\left(x_t\right)$ function is shown in Fig. 1. The nonlinear and non-monotonic reaction function is not a crucial assumption for the dynamics generated from the model, but it is more realistic assumption of fundamentalist behavior in financial market.

3.2. Chartist

There are two types of chartists in the financial market, namely momentum traders and contrarian traders. Unlike the fundamentalists, both momentum traders and contrarian traders focus only on their short-term estimated market value, v_t, albeit different strategies are adopted by both parties. Similar to fundamentalists, chartists also have time-varying extrapolation rate, but their trading behaviors or confidence levels are sensitive to market sentiment.

Momentum traders When the current market price is above the short-term value, momentum traders expect future market price to rise and choose to take a long position; conversely, they take a short position. Excess demand of momentum traders is assumed to evolve over time based on the current short-term value by:

$$D_{mo,t} = \beta_1 m_t \left(p_t - v_t\right) \tag{4}$$

where $\beta_1 m_t$ is the time-varying extrapolation rate of price trend. $\beta_1 > 0,$ represents the base extrapolation rate without sentiment effect. m_t is the time-varying sentiment factor, which is updated each period and will affect the trading decision of investors for next period. The sentiment factor is constructed as:

$$m_t = 1 + h_1 tanh\left[\kappa(p_t - v_t)\right] S_t \tag{5}$$

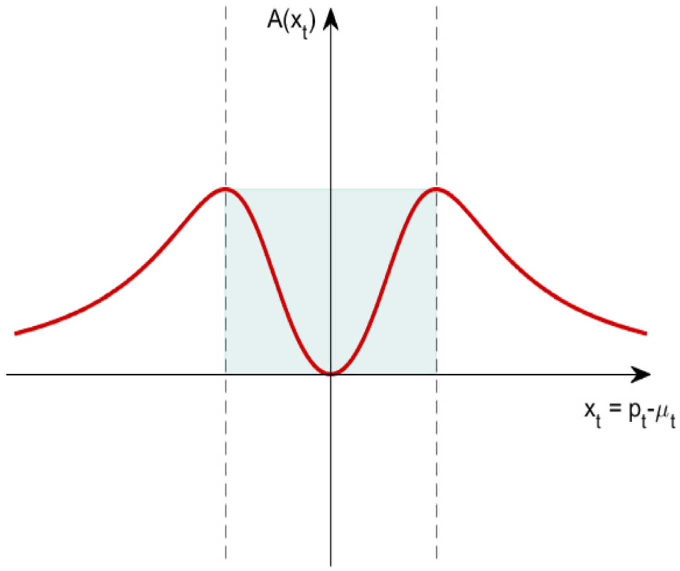

Fig. 1. Confidence function for fundamentalist.

where the sentiment index S_t is derived from social interaction of different types of agents and random sentiment-related information such as news and policies. $S_t \in [-1, 1]$ and the construction of sentiment index will be introduced later. $h_1 \in [0, 1]$ measures sensitivity of momentum traders to market sentiment. If $h_1 = 0$, it means investors are totally immune to sentiment, and the extrapolation rate is only determined by β_1 as $m_t = 1$. On the contrary, $h_1 = 1$ indicates that traders are very sensitive to sentiment that they perceive in financial markets.

Both positive and negative sentiments are expected to stimulate opposite impacts on chartists' long position and short position. More specifically, positive sentiment can reinforce the confidence of momentum traders in long position and raise their cautiousness in short position, while negative sentiment can bolster confidence of momentum traders on short position and weaken it on long position. If the price is above v_t while current market sentiment is positive, momentum traders will feel more confident to follow the price trend and buy in. Hence, their confidence level increases with further price deviation given positive market sentiment. On the other hand, if price trend is upward but market sentiment is negative, momentum traders will still follow the trend but with less confidence. This contradiction of momentum traders' prediction against market sentiment makes them more cautious and reduce their demand for the speculative asset. Besides the market sentiment, the price deviation from the short-term value $(p_t - v_t)$ also exerts an impact on the investor's sentiment. To standardize price deviation $(p_t - v_t)$ within the range of $(-1, 1)$, we introduce a tanh function with a scaling factor κ. Thus, the range of m_t is $[0, 2]$.

Contrarian traders Unlike momentum traders, contrarian investors trade stock based on the hypothesis of market overreaction. Specifically, when current price is higher than short-term value v_t, they believe that future market price will drop and therefore take a short position; conversely, they take the long position. We assume contrarian traders use the same method as momentum traders to calculate short-term value v_t, hence the demand function of contrarian traders can be expressed as:

$$D_{co,t} = \beta_2 c_t \left(p_t - v_t\right) \tag{6}$$

Similarly, $\beta_2 c_t$ is the time-varying extrapolation rate of price trend for contrarian traders. $\beta_2 < 0$, represents the base extrapolation rate without sentiment effect and c_t is the time-varying sentiment factor for contrarian traders, which can be written as:

$$c_t = 1 - h_2 tanh[\kappa(p_t - v_t)]S_t \tag{7}$$

where h_2 is sensitivity of contrarian traders to market sentiment with a range $[0, 1]$. Although contrarian traders adopt trading strategy opposite to that of momentum traders, they are affected by market sentiment in the same way. When market price is above v_t, contrarian traders expect price to decline. If the market sentiment is negative, contrarian traders will be more confident to take the short position. However, positive market sentiment will decrease the confidence level of contrarian investors. Besides the market sentiment, price deviation is another element that contributes to contrarian traders' sentiment factor c_t.

Short-term value v_t for chartists Let us look more closely at the short-term asset value, v_t. We assume that all chartists, both momentum traders and contrarian traders, hold on to an identical short-term asset value. As in [28,46], we assume that chartists adopt the adaptive belief mechanism where they update their expectations on short-term asset value according to different price regimes. They believe in support and resistance levels which are derived from common rules of technical analysis. Accordingly, chartists divide price domain $P = [P_{\min}, \ P_{\max}]$ into n regimes such that:

$$\mathbb{P} = \bigcup_{j=1}^{n} P_j = [\bar{p}_0, \bar{p}_1) \bigcup [\bar{p}_1, \bar{p}_2) \bigcup \cdots \bigcup [\bar{p}_{n-1}, \bar{p}_n] \tag{8}$$

where $\bar{p}_j$ for $j = 1, 2, \cdots, n$ represents the different support and resistance levels set by the chartists. The short-term asset value can be simply extrapolated as the average of the top and the bottom threshold prices:

$$v_t = \left(\bar{p}_{j-1} + \bar{p}_j\right)/2, \qquad \text{if} \ \ p_t \in \left[\bar{p}_{j-1}, \bar{p}_j\right) \tag{9}$$

When price fluctuates within the current regime, there are enough reasons for chartists to believe that the short-term asset value will remain unchanged for next period. However, once the price breaks through either the support or resistance lines, chartists will adjust their expectation on the short-term asset value according to Eq. (9). This regime dependent phenomenon is commonly found in stock market with chartist's beliefs evolve with regime switching. According to [28], the short-term asset value for each period is estimated as:

$$v_t = \left(\lfloor \frac{p_t}{\lambda} \rfloor + \lceil \frac{p_t}{\lambda} \rceil\right) \cdot \frac{\lambda}{2}, \qquad \text{if } p_t \in \left[\bar{p}_{j-1}, \bar{p}_j\right) \quad \text{and} \quad j = 1, 2 \cdots n \tag{10}$$

3.3. Belief switching regime

One of the important features underlying the model is the belief switching regime of agents, which has become widely adopted since it was first proposed by Brock and Hommes [47]. Experimental evidence of beliefs switching behavior has been found in recent studies by Anufriev and Hommes [48], Anufriev et al. [49], Anufriev et al. [50] and Anufriev et al. [51]. This regime switching behavior assumes that, at the end of each trading period, agents may switch their prediction strategies or beliefs conditional on the past performance of three rules. Specifically, the performance measure depends on realized profitability π_t, which is defined as:

$$\pi_{n,t} = (p_t - p_{t-1})\, D_{n,t-1} \tag{11}$$

Where n denotes different types of agents. For simplicity, we assume that the gross interest rate between period $t-1$ and period t is one and there is no cost for fundamentalists to acquire additional information. We further introduce additional memory into the measurement of performance. The performance U_t can be calculated as the geometrically declining weighted average of the realized profits, given by:

$$U_{n,t} = \varphi U_{n,t-1} + \pi_{n,t} \tag{12}$$

where $0 \leq \varphi \leq 1$ represents the strength of memory put into the last-period performance.

We use $\omega_{n,t}$ to denote market fraction of three different types of investors. The fractions of three groups vary endogenously over time according to the choice model introduced by Manski et al. [52] as well as [21,47]:

$$\omega_{n,t} = \frac{exp(\rho U_{n,t})}{\sum_{h=1}^{3} exp(\rho U_{n,t})} \tag{13}$$

Note that, the new fractions of traders are determined on the basis of the most recent performance measure $U_{n,t}$. The parameter $\rho > 0$ is the intensity of choice. It measures the sensitivity of agents to the difference of past performance. The higher is ρ, the quicker agents will respond to difference in performance by switching to the most profitable strategy. For finite ρ, $\omega_{h,t}$ is always positive which implies that not all agents are going for strategy that gives highest profit.

3.4. Sentiment index

As pointed out in Section 2, there are three important sources of sentiment, namely memory of sentiment(last-period sentiment), investor mood from social interaction, and sentiment shock such as news, polices, firm innovations and so forth. We develop the sentiment index by simply combining these three sources. Although it is only used to explain the formation of sentiment in our model. One can also use this method to construct sentiment indexes for the real financial markets. The index function can be written as:

$$S_t = \eta_1 S_{t-1} + \eta_2 SI_t + \eta_3 \epsilon_t \tag{14}$$

where η_1, η_2, η_3 are weights assigned to different factors, such that $\eta_1 + \eta_2 + \eta_3 = 1$. ϵ_t is the sentiment shock, which we assume to follow a uniform distribution with range $[-1, 1]$. SI_t is social interaction of different types of investors, which influences the current market mood. The idea is inspired by majority opinion formation in [3,53], and [54], in which the majority opinion index is computed as the difference between optimistic and pessimistic individuals. Different from [54], we assume that the opinion index not only contains the opinion of optimistic and pessimistic chartists, but also includes the opinion of fundamentalists. The social interaction should exist among all types of investors, including both fundamentalist and chartist groups. Fundamentalists are optimistic (pessimistic) investors when market price is below (above) their fundamental values. Two different types of chartist may change their opinion under different market states. For instance, when current market price is above the short-term value v_t, momentum traders believe price will continue to go up and thus they belong to the optimistic group. In contrast to momentum traders, contrarian traders believe price trend will reverse and hence they belong to the pessimistic group. However, in the next period, if the market price falls below the short-term value v_{t+1}, momentum traders will become pessimistic while contrarian traders will switch to optimistic opinion. Besides the fractions of different agents, the price deviations from fundamental value μ_t and short-term value v_t could also affect the opinion index. Large price deviation could make investor opinion more convincing in social interaction. By considering both fractions of agents and magnitude of price deviation, we construct the social interaction index as:

$$SI_t = tanh\left[\kappa\left(\mu_t - p_t\right)\right]\omega_{f,t} + tanh\left[\kappa\left(p_t - v_t\right)\right]\left(\omega_{mo,t} - \omega_{co,t}\right) \tag{15}$$

where $\omega_{f,t},\ \omega_{mo,t},\ \omega_{co,t}$ are fractions of fundamentalists, momentum traders and contrarian traders in financial market, respectively. tanh function and κ are used to scale the price deviation, so both the range of SI_t and the range of sentiment index S_t can be constrained to $[-1, 1]$.

3.5. Market clearing mechanism

Instead of using Walrasian auctioneer clearing mechanism, we adopt market maker mechanism, which is akin to the role of specialists in New York Stock Exchange. We assume net zero supply of the risky asset, and market price is set by a market maker who take into consideration excess demand of all agents. The aggregate market's excess demand is weighted by population fraction. Hence, for a three-agent model (including fundamentalists, momentum traders and contrarian traders), the price p_{t+1} is set by market maker according to the aggregate excess demand, that is:

$$p_{t+1} = p_t + \gamma\left(\omega_{f,t}D_{f,t} + \omega_{mo,t}D_{mo,t} + \omega_{co,t}D_{co,t}\right) \tag{16}$$

where γ measures the speed of price adjustment by the market maker.

4. Numerical simulation with stochastic model

4.1. Stylized facts

We calibrate the sentiment model such that it mimics some well-documented stylized facts of financial markets following the practice of existing literature (See for example, [55]). As summarized by Westerhoff and Dieci [56], the five salient characteristics of real-world speculative prices include (1) price distortions in the forms of bubbles and crashes; (2) excess price volatility; (3) leptokurtic distribution of returns (characterized by kurtosis exceeding 3); (4) negligible autocorrelation of daily returns; and (5) strong autocorrelation of absolute daily returns. These patterns can be seen in Fig. 2, which depicts the dynamics of US stock prices and returns based on daily S&P500 index from Jan 3, 2000 to Feb 12, 2019. As seen from the top panel, the evolution of daily stock prices shows both strong price appreciations and crashes in some periods. The second panel displays daily log returns which show evidence of volatility clustering. Distribution of returns in the third panel indicates the presence of fat tails. The last panel plots the autocorrelation functions (ACF) using both raw returns and absolute returns. We can observe the absence of autocorrelations in raw returns and the slow-dampening autocorrelations in absolute returns, implying long memory of daily returns. We conduct simulation analysis on the basis of the standard parameter setting as shown in Table 1. Although parameter setting does affect the simulation results, they are not the key. After a tedious trial-and-error calibration exercise, we find sentiment model with this base group parameters could consistently generate most of stylized facts in the stock market. Further experiments reveal that the model performance does not react overly sensitively with respect to changes in the parameter setting. With the following in-depth Monte Carlo simulations, we can conclude that this calibrated model systematically replicates the main statistical properties in financial markets. However, future work could try to estimate the parameters for such a complicated nonlinear model. For the explanatory purposes, the model's performance already appears to be sufficient.

To examine the effect of sentiment on price movements, we compare model with sentiment to model without sentiment by setting the sentiment sensitivity parameter, h, from 1 to 0, while holding all other parameter values constant. Simulation results of 17,000 periods (daily) price trajectories are shown in Fig. 3. Table 2 summarizes the descriptive statistics of both actual and artificial market returns. Overall, we find that the simulated series with investor sentiment exhibiting much more realistic statistical properties as compared to series without investor sentiment. As shown by the top left panel of Fig. 3, the model with sentiment is capable to produce more volatile prices (as indicated by dark line) relative to rather stable fundamental values (as indicated by blue line). Unlike the case without sentiment where prices mostly fluctuate closely around the fundamental value, prolonged bubbles and crashes can be generated when investor sentiment is included in the framework. Meanwhile, by allowing for sentiment, the return trajectories show volatility clustering similar to the actual S&P 500 returns, while the distribution of returns from the model with sentiment demonstrates leptokurtic behavior given the presence of fat tails. From Table 2, the kurtosis and skewness for model

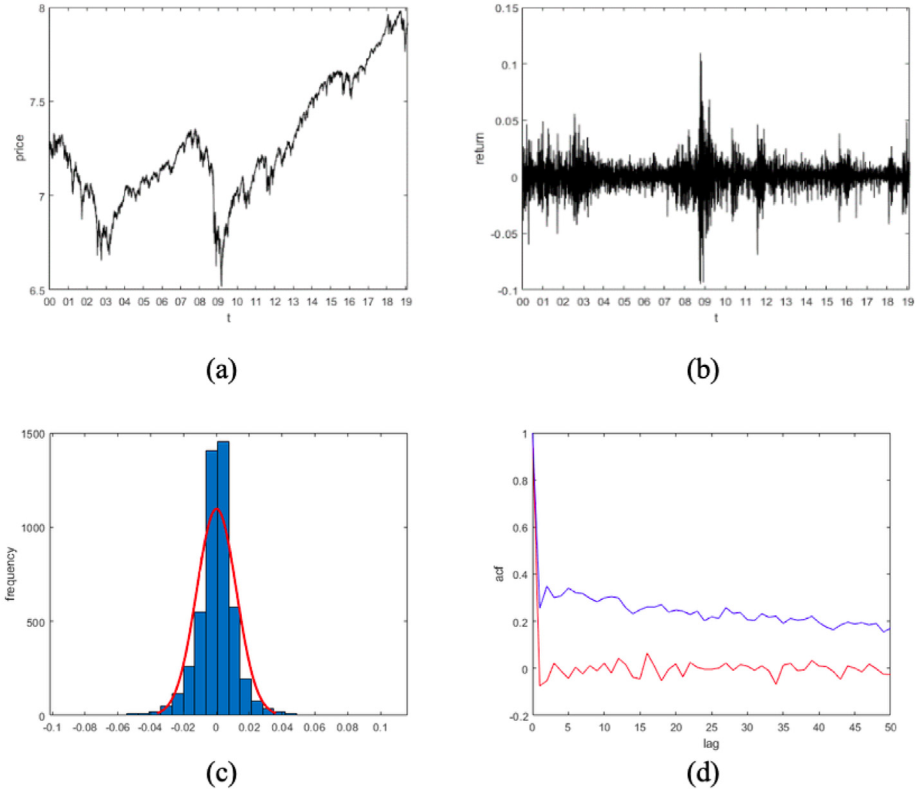

Fig. 2. The dynamics of daily S&P500 index between Jan 3, 2000 and Feb 12, 2019. The panel shows (a) the evolution of the stock price index (b) the returns, (c) the histogram of returns overlaid by normal curve and (d) the autocorrelation function of raw returns (red line) together with the autocorrelation function of absolute returns (blue line).

Table 1
Standard parameter setting.

Parameter	Value	Definition
μ	1014	Mean of fundamental prices
σ	1	SD of fundamental prices
β_1	1.75	Momentum extrapolation rate
β_2	-1.25	Contrarian extrapolation rate
φ	0.1	Performance memory strength
ρ	0.5	Intensity of choice
γ	0.845	Speed of price adjustment
η_1	0.4	Last-period sentiment weight
η_2	0.5	Social interaction weight
η_3	0.1	Sentiment shock weight
λ	12	Support and resistance level
a	1.11×10^{-5}	Confident function factor
b	1×10^{-8}	Confident function factor
κ	1000	Scaling factor
$h = h_1 = h_2$	0/1	Without sentiment/with sentiment

Table 2
Summary statistics of returns.

Variable	Kurtosis	Skewness	Mean	SD	Min	Max
Actual market						
S&P 500	11.481	-0.215	0.000	0.012	-0.095	0.110
Artificial market						
With sentiment	6.677	-0.099	0.000	0.010	-0.095	0.064
Without sentiment	2.140	-0.017	0.000	0.005	-0.010	0.010

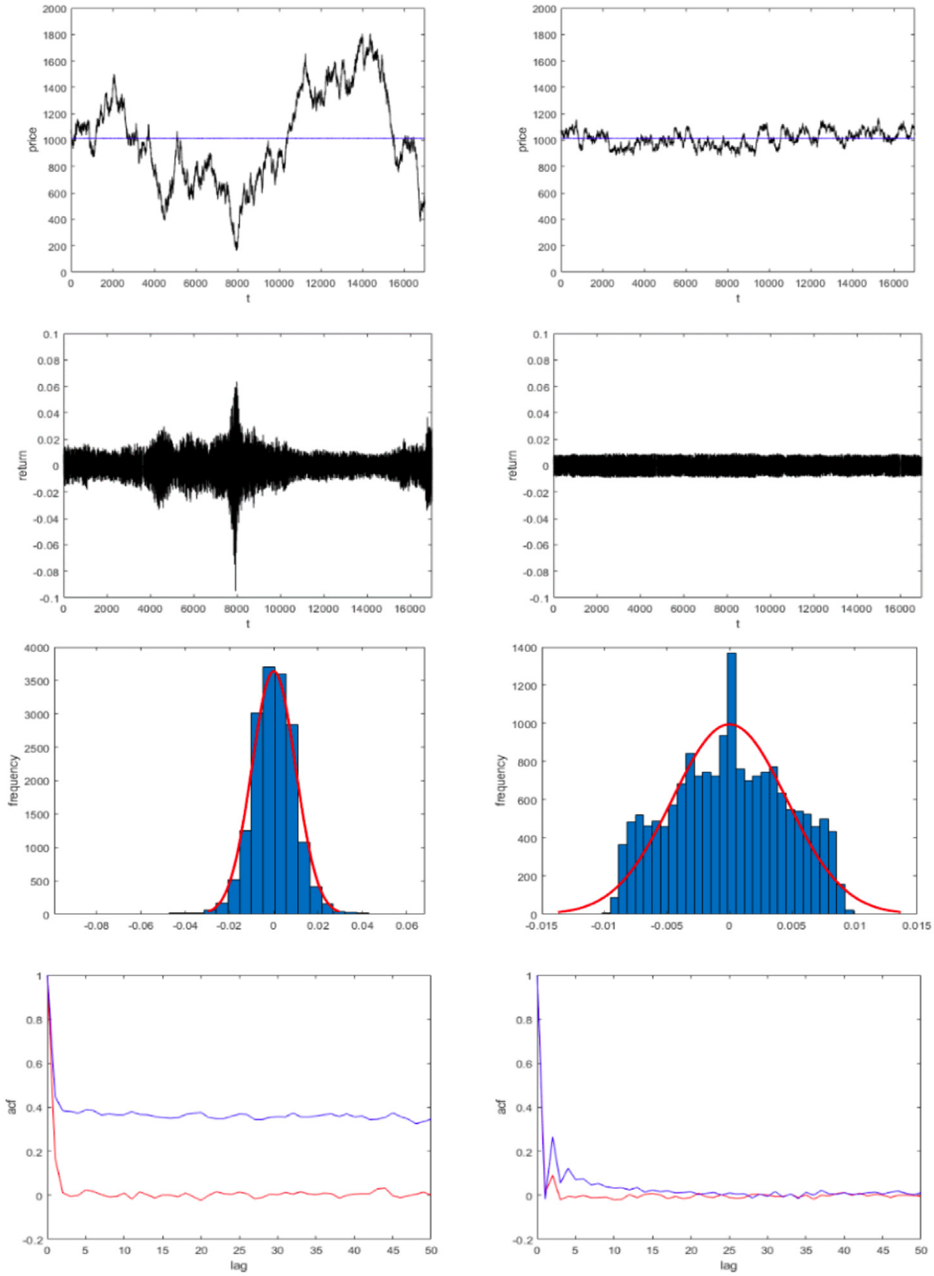

Fig. 3. The dynamics of the models with sentiment (left) and without sentiment (right). The panels show, from top to bottom, the evolution of the stock prices, the returns, the histogram of returns overlaid by normal curve and the ACF of raw returns (red line) together with the ACF of absolute returns (blue line), respectively. The simulation run is based on 17,000 observations.

with sentiment are 6.68 and −0.099, respectively; whereas model without sentiment have underestimated kurtosis and skewness of 2.14 and −0.017, respectively.

To check whether there exists long memory dependence in daily returns, we plot ACFs for both raw daily returns (blue line) and absolute daily returns (red line). Without sentiment, the absolute returns have fast-decaying ACF, suggesting there is no long-range dependence for daily returns. However, with sentiment, ACF of absolute returns is strong and persistent even after 50 lags. This finding matches the stylized facts of real stock returns, that cross-correlation should be weak for raw returns but strong for absolute returns. This qualitative finding holds for simulation based on deterministic model where we further exclude the contribution of stochastic noise to these statistical properties. We also conduct

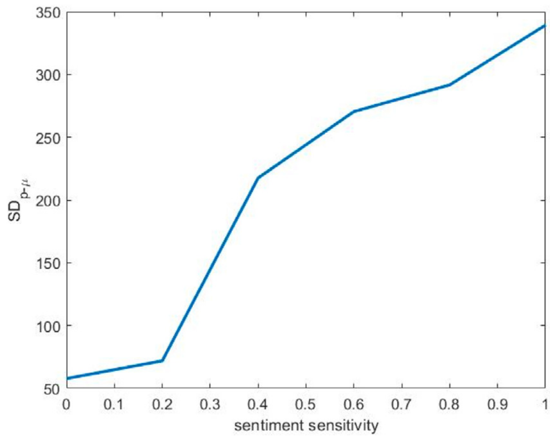

Fig. 4. The effect of sentiment sensitivity on market volatility with 1,000 simulations.

robustness checks by evaluating several statistical properties of simulated prices and returns for 1000 simulation runs across a range of sentiment sensitivity parameter values. For more details, please refer to Table 3.

4.2. Sentiment and excess volatility

Prices of financial assets are typically more volatile than can be justified within standard asset pricing model. This result, known as excess volatility, has been documented in many studies, such as [57,58], and [59]. Recently, some studies attempt to use behavioral approach to tackle the excess volatility puzzle (See for examples, [60–63]). In order to study the effect of sentiment on excess volatility, we use the standard deviation of the market prices from the fundamental values $SD_{p-\mu}$ as a quantitative measure of market volatility. The standard deviation value, $SD_{p-\mu}$, is computed as follows:

$$SD_{p-\mu} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(p_t - \mu_t)^2} \tag{17}$$

Keeping the same standard parameter set, the deviation of the market prices from the fundamental values $SD_{p-\mu}$ is 358.95 for the model with sentiment and 58.46 for the model without sentiment. This implies that the market is more volatile when sentiment effect is included. In general, we find that $SD_{p-\mu}$ becomes larger as the sentiment sensitivity parameter h increases from 0 to 1.

To check the result robustness, we perform Monte Carlo simulation by running the stochastic model 1,000 times using the standard parameter values with different levels of sentiment sensitivity h from 0 to 1 with an interval of 0.2. Then, we calculate the standard deviation value using Eq. (17) for each simulated series. Lastly, the average standard deviation value is calculated based on 1000 simulations for each sentiment sensitivity level. Fig. 4 shows the average standard deviation values $SD_{p-\mu}$ for 1,000 simulation runs at different h values. As we can see, the average value of $SD_{p-\mu}$ increases with the sentiment sensitivity, suggesting that the market volatility is positively correlated to the investors' sensitivity to market sentiment.

4.3. Sentiment and crises

Financial crisis is the main aspect we are interested to investigate under the HAM framework. Many researchers have proven and indicated that the standard financial theory fails to envisage the occurrence of financial crises, and heterogeneous beliefs and interaction of heterogeneous agents should be taken into account to understand financial crises. Some studies have substantiated that models with heterogeneous beliefs have extraordinary power in explaining financial crises (See [28,63–65]). In this section, we investigate different financial crises under the HAM framework as in [28] with the primary focus on the role of sentiment on the formation of crises.

Huang et al. [28] have successfully replicated three typical types of financial crises, namely sudden crisis, smooth crisis and disturbing crisis, by using HAM. They find that financial crises are attributable to the endogenous price dynamic from agents' interaction and both fundamentalists and chartists could potentially contribute to the crises. Inspired by their study, we aim to generate the three typical types of financial crises using our three-agent model with fundamentalists, momentum traders and contrarian traders. For more accurate replication of crisis patterns, we set $\mu = 200$, $\rho = 0.7$, $\gamma = 1.8$, $\lambda = 10$, $b = (1/150)^4$, while keeping other parameters in line with the standard values.

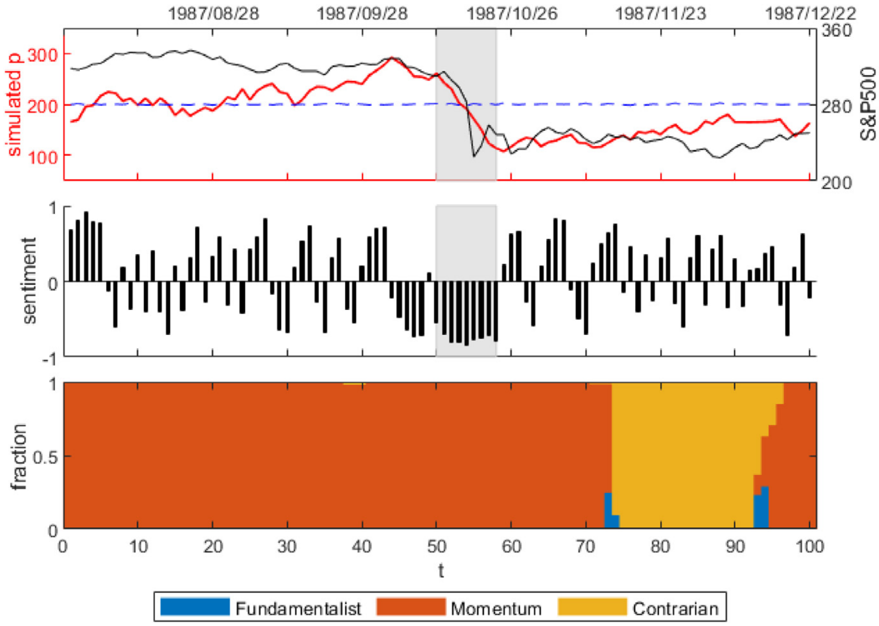

Fig. 5. Sudden crisis modeling. The panels show, from top to bottom (a) comparison between simulated price with S&P 500 from 1987/8/3 to 1987/12/22, (b) simulated sentiment, and (c) fractions of 3 types of investors.

In a sudden crisis, the stock price drops abruptly from the peak (or near peak) to the bottom within a short time frame. According to [28], when the price is at the peak, it is highly overvalued. Observing opportunities for profit, investors switch to fundamental strategy excessively and execute great selling forces that cause the steep fall in price. Contrary to their results, we find no contribution of fundamentalists before and during the sudden crisis. As illustrated in Fig. 5, there is a dramatic price fall between t=50 and t=60, during which, the market is dominated by momentum traders without any switching to fundamental strategy. Our results suggest that it is the strong negative sentiment that drives the market crash. Due to either an exogenous cause or regime switching, the stock price starts to go down at around t=45, which makes the momentum traders change their strategies from buy to sell and create a bearish sentiment. The strong negative sentiment further accelerates selling forces of the momentum traders and subsequently, leads to market panic and causes a sharp decline in price.

In a smooth crisis, the price declines moderately but persistently over a period of time without a visible crash. As shown from Fig. 6, the downward trend starts somewhere between $t = 30$ and $t = 40$ with an increase in the fraction of fundamentalists. After the first sign of selling from fundamentalists, there are some counter-movements in the price caused by contrarian traders. When the downward trend becomes more observable, investors cluster to momentum trading strategy and execute more selling forces to push the price further down. During the period of price decline, the market sentiment switches between bullish and bearish with more negative values overall. From $t = 30$ to $t = 70$, the average value of sentiment is -0.3751.

A disturbing crisis is characterized by volatile fluctuations with a downward trend and possible moderate crashes in price. The period of disturbing crisis is somewhere between sudden crisis and smooth crisis. Fig. 7 shows a simulated disturbing crisis based on our model. Price fluctuates disturbingly before starting to drop sharply at around $t = 37$ due to the significant and negative sentiment. From $t = 45$ to $t = 55$, the price becomes volatile again as the sentiment fluctuates between positive and negative values. Subsequently, the downward trend continues with another phase of strong bearish sentiment. Similar to the other types of crisis, momentum traders are responsible for propagating the downward trend. However, during the crisis, some investors shift to contrarian strategy because of high market volatility.

The common feature of these three figures is that crises are always accompanied by negative sentiment. During the crises, bearish sentiment dominates the market which in turn amplifies the crises. To explore the correlation between sentiment and asset return, we calculate the correlation coefficients between these two variables for the whole simulated sample as well as each crisis samples. All the results are positive, which indicate that investor sentiment is positively correlated with return. The correlation coefficient is 0.41 for the whole sample, while coefficients for three crises samples are all above 0.8. The scatter plots for sentiment-return correlation for three crises are shown in Fig. 8. The sentiment index seems to have stronger predictive power on asset prices during the crisis periods.

To further investigate the effect of sentiment on crises, we compare the frequency and magnitude of crises in the simulated series from models with and without sentiment effect in 1000 simulations. Following [19,66], we define the magnitude of a crisis as the percentage drop from the peak to the trough. The date of the peak is the month when the

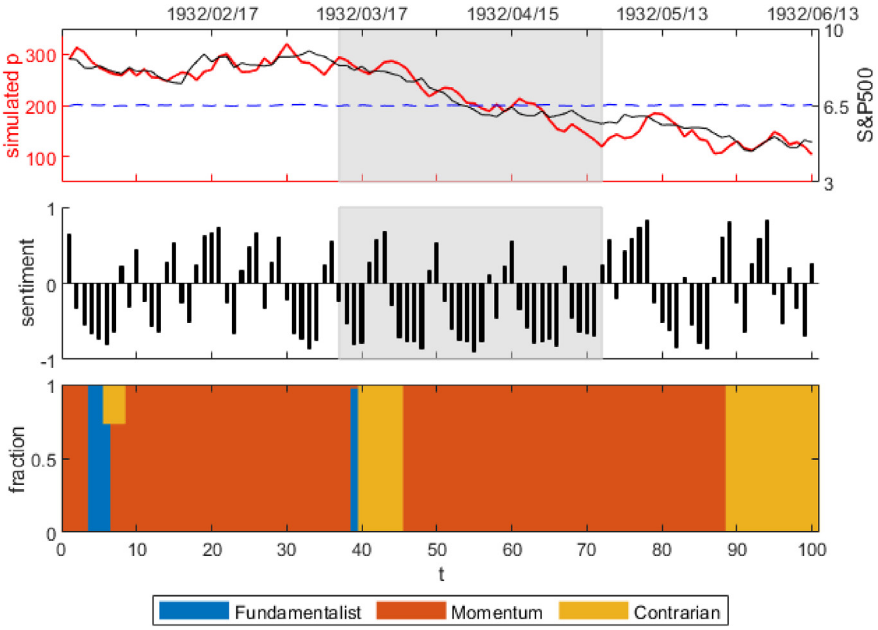

Fig. 6. Smooth crisis modeling. The panels show, from top to bottom, (a) comparison between simulated price with S&P 500 from 1932/1/20 to 1932/6/13, (b) simulated sentiment and (c) fractions of 3 types of investors.

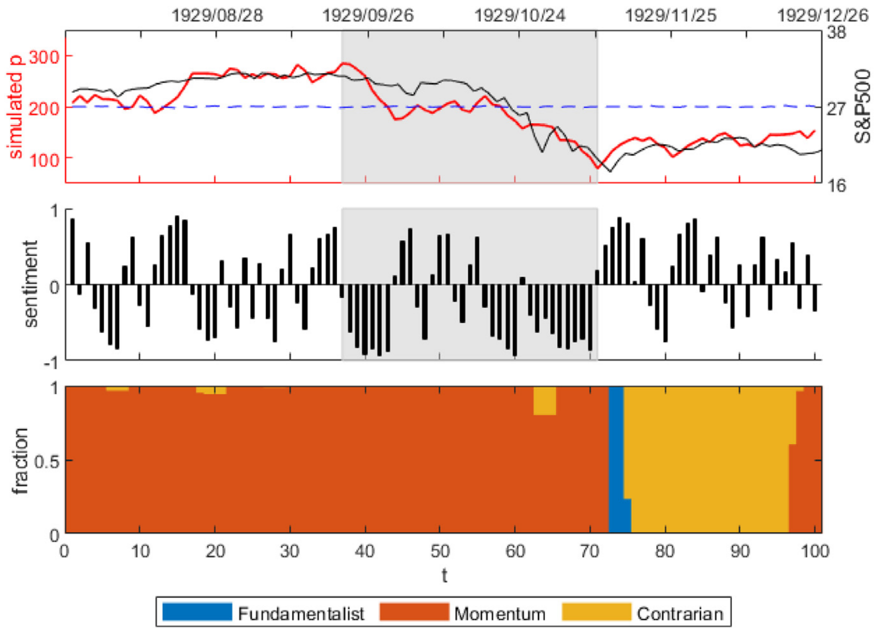

Fig. 7. Disturbing crisis modeling. The panels show, from top to bottom, (a) comparison between simulated price with S&P 500 from 1929/8/1 to 1929/12/26 (b) simulated sentiment and (c) fractions of 3 types of investors.

price reaches its maximum value over a T-period window prior to the crisis identification, and the date of the trough is the month when the price reaches its minimum during the crisis. We adopt a crisis indicator called CMAX used in [19] with some adjustments. In this method, CMAX is a ratio calculated by dividing the current value of asset by the maximum price over the previous T periods, in which T is usually set as one to two years.

$$CMAX_t = \frac{P_t}{max(P_{t-T} \cdots P_t)} \tag{18}$$

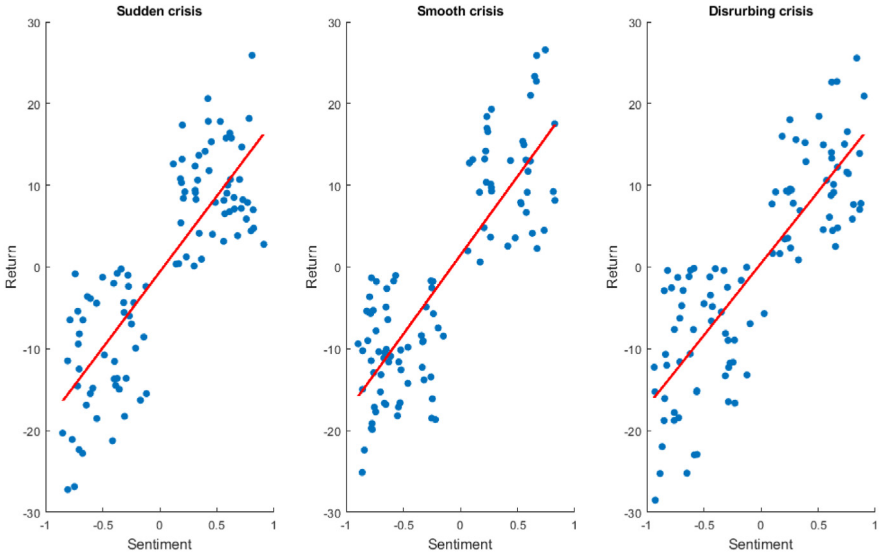

Fig. 8. Scatter plots of sentiment and asset return for different types of crises.

where P_t is the stock market index at time t. CMAX equals one if the price rises over the period considered, indicating a bullish market. Conversely, if the price declines over the period, CMAX is less than one. A crisis is detected each time CMAX drops below a threshold value set at two standard deviations below the mean of CMAX. Both mean and standard deviation values are calculated on the whole sample. However, this method may mistakenly identify the bubble correction as a crisis. To fix this problem and make it more suitable for detecting crises in simulated data, we add one compulsory condition, in which the current price must be lower than the fundamental price for a certain threshold value. Therefore, the crisis indicator C_t is defined as following:

$$C_t = \begin{cases} 1, & \text{if} \quad CMAX_t < \overline{CMAX} - 2\sigma \quad \text{and} \quad P_t < \tau * \mu_t \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

where σ is the standard deviation value of CMAX for the whole sample, and $0 < \tau < 1$ is the threshold to determine the minimum magnitude to be defined as a crisis. To check the efficiency of this method, we use the S&P 500 monthly data from 1950m1 and 2018m12 and the corresponding fundamental values constructed from historical monthly dividend data to detect the occurrence of actual crisis periods in the US stock market. Both the real S&P 500 prices and dividends on monthly frequency are provided by Shiller [67]. In accordance to [68], we calculate fundamental stock price value using static Gordon growth model with constant discount rate r and growth rate g of the dividend flows D_t, for which

$$P_t^* = \frac{1+g}{r-g} D_t \tag{20}$$

where P_t^* denotes the fundamental price of the stock index. We set $T = 12$ *months* and $\tau = 0.9$. As shown in Fig. 9, five periods of crises are identified during the period 1950–2018. The first crash detected is the Kennedy Slide occurred in 1962, followed by the second tech-stock crash in 1970 and the third crash in 1973–1974 due to the end of Bretton Woods monetary system and the oil crisis. The most recent two market crashes detected are the 1987 Black Monday crash and the 2008–2009 global financial crisis.

As our adapted CMAX indicator has been proven reliable in identifying crisis in the real financial market, we attempt to use it in the simulated data. To be consistent with case of real financial market, we convert the simulated daily data to monthly data and choose $T = 12$ *months* and $\tau = 0.9$ to detect crisis. To obtain a visual impression on the effect of sentiment on crisis, we try to detect the crisis in the previous two simulated time series with and without sentiment, respectively. From Fig. 10, we find more crises in the case with sentiment than the case without. To check the robustness of the result and investigate the magnitude of crisis in these two cases, we further conduct Monte Carlo simulations with the stochastic model using the standard parameter values across different levels of sentiment sensitivity with an interval of 0.2. As shown in Fig. 11, the number of crisis increases with sentiment sensitivity level h, which illustrates the significant positive relationship between the sentiment sensitivity and the frequency of crisis occurrence. We further investigate whether sentiment affects the magnitude of crisis. As expected, the average magnitude of crisis rises with higher h which corroborates the effect of sentiment on the depth of crisis. Hence, we conclude that in the simulations, the market sentiment has a significant effect on crisis in terms of frequency and magnitude. The result is consistent

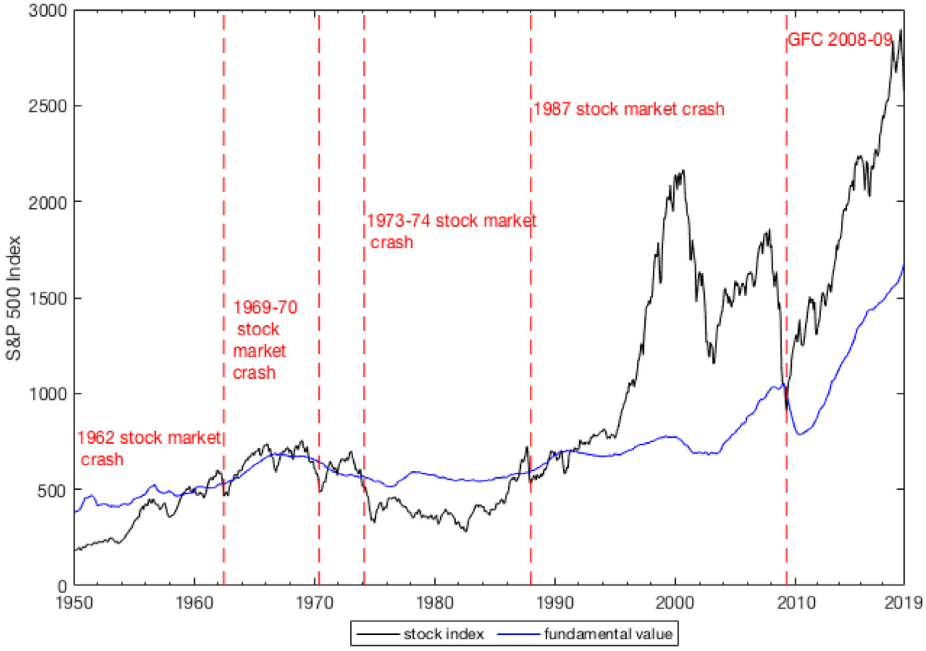

Fig. 9. Crises detected from 1950 using real S&P 500 data.

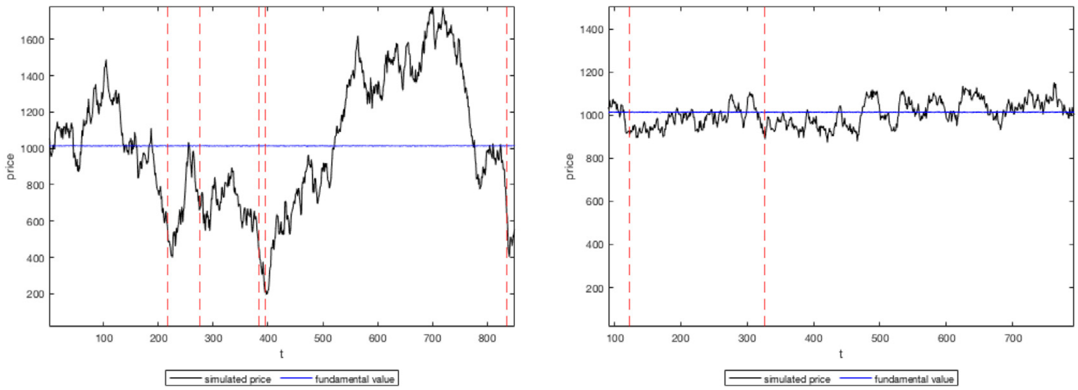

Fig. 10. Crisis identified in simulated monthly data with sentiment (left) and without sentiment (right), τ = 0.9.

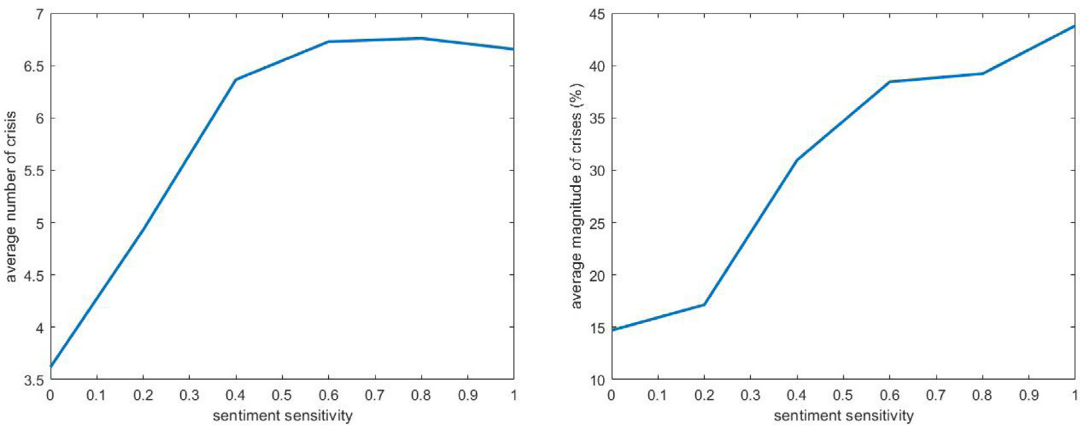

Fig. 11. The effect of sentiment sensitivity on frequency of crisis (left) and average magnitude of crises (right) with 1000 simulations.

Table 3
Simulated market dynamics with different sentiment sensitivities (average for 1000 simulations).

	h = 0	h = 0.2	h = 0.4	h = 0.6	h = 0.8	h = 1
Kurtosis	2.155	2.075	3.396	4.615	4.73	5.804
Skewness	−0.01	−0.015	−0.026	−0.036	−0.037	−0.04
AC r_1	0.01	0.041	0.166	0.219	0.189	0.166
AC r_5	−0.006	−0.004	0.004	0.001	0.007	0.004
AC r_{10}	−0.003	−0.004	−0.002	−0.003	−0.002	−0.003
AC r_{20}	−0.003	−0.002	−0.002	−0.001	−0.001	−0.001
AC $\lvert r_1\rvert$	−0.014	−0.003	0.172	0.344	0.372	0.412
AC $\lvert r_5\rvert$	0.07	0.075	0.179	0.252	0.253	0.298
AC $\lvert r_{10}\rvert$	0.039	0.046	0.167	0.241	0.243	0.29
AC $\lvert r_{20}\rvert$	0.015	0.025	0.158	0.231	0.235	0.282
$SD_{p-\mu}$	57.796	71.944	217.539	270.436	291.671	339.284
# of crisis	3.618	4.929	6.365	6.728	6.759	6.655
Magnitude of crisis (%)	14.703	17.141	30.961	38.443	39.213	43.804

with findings in [19], which show that investor sentiment positively influences the probability of the occurrence of stock market crises using panel data of 16 countries.

In summary, we have demonstrated that investor sentiment contributes to more realistic stylized facts. The simulation results in Table 3 also show that the market will be more volatile if investors are very sensitive to market sentiment. More importantly, we successfully replicate different crises, and find that sentiment could be an important source of crisis formation. Through 1000 Monte Carlo simulations, we find the evidence that sentiment could amplify both the frequency and magnitude of financial crises.

5. Robustness analysis

The goal of this section is to perform robustness checks with alternative models. Firstly, our model employs many assumptions, which could make it sensitive to changes in parameters. To check the robustness of our model to changes in parameter settings, we adjust the value of each parameter by ±20% separately while holding other parameters constant, and run 1000 simulations for each adjustment. We find that our main results still hold, which implies our model is not particularly sensitive to any individual parameter changes. Secondly, for simplicity, the two sensitivity parameters h_1 and h_2 are assumed to be identical in the benchmark model. In Section 5.1, we relax this assumption and conduct more analysis by allowing for the heterogeneity of sentiment sensitivity between momentum trader and contrarian trader. Thirdly, we reduce the level of nonlinearity of our model by relaxing several nonlinear assumptions that we previously imposed on the investor behavior. By excluding possible influences from other factors, such as nonlinear confidence of fundamentalist and regime-switching trading rules of chartist, we further confirm the pivotal role of sentiment in contributing to the formations of stylized facts and crises in the model. In Section 5.2, we consider different setups for fundamentalist's demand function, including the linear form as in [56] and nonlinear form as in [20]. In Section 5.3, we set different trading rules for chartists to examine the model's robustness to chartist's strategy. The main outcomes of the sentiment model still hold even when we simplify these assumptions albeit the benchmark model is still the best in terms of replicating the stylized facts and crises.

5.1. Heterogeneity in sentiment sensitivity

We run 1000 Monte Carlo simulations using different values for h_1 and h_2. The simulated results are shown in Table 4. Overall, as h_1 increases, the model generates more realistic statistical properties even if h_2 decreases. It is interesting to note that the average fraction of momentum traders is generally very high (86.1%–94.5%). Most of the time, the market is dominated by momentum traders. Hence, it seems the model is mostly governed by h_1. On the contrary, h_2 has minimal effect due to small fraction of contrarian traders (1.2%–4.5%). In addition, we also find that there is a positive correlation between sentiment sensitivity and fraction of traders for both types of chartists. Accordingly, the participation of chartists in the market depends on their respective sensitivity to sentiment.

5.2. Linear and nonlinear behaviors of fundamentalist

Sentiment model is robust to the alternative assumptions of fundamentalist behavior. Specifically, the main results hold even if we change the confidence function in the benchmark model into two commonly used forms in the extant HAM literature, which are the linear function as in [56] and the nonlinear monotonous function as in [20]. The linear and nonlinear functions are shown in Eq. (21) and Eq. (22), respectively.

$$D_{f,t} = a_f(\mu_t - p_t) \tag{21}$$

$$D_{f,t} = (p_t - a_1\mu_t)^d(a_2\mu_t - p_t)^d(\mu_t - p_t) \tag{22}$$

Table 4
Simulated market dynamics with $h_1 \neq h_2$ (average for 1000 simulations).

	$h_1 = 0, h_2 = 1$	$h_1 = 0.2, h_2 = 0.8$	$h_1 = 0.4, h_2 = 0.6$	$h_1 = 0.6, h_2 = 0.4$	$h_1 = 0.8, h_2 = 0.2$	$h_1 = 1, h_2 = 0$
Kurtosis	2.194	2.082	3.537	4.415	4.795	5.664
Skewness	−0.009	−0.0014	−0.029	−0.035	−0.035	−0.038
AC r_1	0.009	0.04	0.168	0.218	0.189	0.166
AC r_5	−0.002	−0.001	0.005	0.001	0.007	0.005
AC r_{10}	−0.003	−0.003	−0.002	−0.003	−0.003	−0.004
AC r_{20}	−0.003	−0.003	−0.002	−0.001	−0.001	−0.001
AC $\|r_1\|$	0.01	0.021	0.189	0.331	0.378	0.409
AC $\|r_5\|$	0.088	0.09	0.193	0.238	0.259	0.296
AC $\|r_{10}\|$	0.058	0.06	0.18	0.227	0.251	0.288
AC $\|r_{20}\|$	0.032	0.037	0.17	0.219	0.243	0.279
$SD_{p-\mu}$	57.164	68.815	225.516	264.671	295.907	338.424
# of crisis	3.719	4.8	6.388	6.631	6.614	6.655
Magnitude of crisis (%)	14.608	16.743	31.562	37.117	39.597	42.976
Average fraction of fundamentalist (%)	9.2	8.6	8.2	6.8	5.5	4.3
Average fraction of momentum (%)	86.1	87.7	89.1	91.3	92.9	94.5
Average fraction of contrarian (%)	4.6	3.7	2.6	1.9	1.5	1.2

In Eq. (21), a_f is a positive constant and fundamentalists are assumed to react with the same speed of mean reversion given any price deviation. Eq. (22) is the generalized version of demand function for fundamentalist following [20], in which $d < 0, 0 \leq a_1 < 1, a_2 > 1$. The function implies that the more price deviates from its fundamental value, the more quickly the price trend is going to reverse within the range $(a_1\mu_t, a_2\mu_t)$. As can be seen from Table 5, after replacing the chance function with linear or nonlinear monotonous function, the sentiment model can still replicate most of the stylized facts, and the amplification effect of sentiment on volatility and crises remains. It suggests that the exact specification of fundamental demand component is not crucial for the model's ability to explain the stylized facts and crises in the financial markets. However, the nonlinear confidence function of fundamentalist can generate more realistic stylized facts with a wider range of sentiment sensitivity. For instance, the tail fatness appears from $h = 0.4$ under the benchmark model, whereas under the linear chance function, tail fatness only becomes evident at $h = 1$. In addition, it also helps to explain why the boom and bust cycles in the actual financial markets last longer than expected. Fundamentalists may lose confidence and fail to drive price back to fundamental value if the price deviation is big enough .

5.3. Different technical rules of chartists

Finally, we check the robustness of the sentiment model with regards to the technical trading rule of chartists. According to [69], moving average is one of the simplest and most popular trading rules in technical analysis. Furthermore, it is well known that different trading strategies utilize different lengths of past periods (see [70–72]). To account for different time horizons in forming strategies for chartists, we use moving average with different large lengths of historical prices. The short-run reference value for chartists can be written as a simple moving average of L lag prices:

$$v_t = \frac{1}{L}\sum_{i=1}^{L} p_{t-i} \tag{23}$$

where L is a positive integer. If $L = 1$, the simple moving average is equivalent to trend extrapolative rule as in [56], which assumes the chartists base their orders on the most recent price change. In this case, The larger the price trend, the stronger the demand. The excess demand of momentum traders can be computed as:

$$D_{mo,t} = \beta_1 m_t (p_t - p_{t-1}) \tag{24}$$

Following [25], we also test an exponential moving average rule for chartists, in which the short-run can be written as:

$$v_t = kp_t + (1-k)v_{t-1} \tag{25}$$

Table 6 tests the robustness in our model using simple moving average rule with different values of L. It shows that the value of L does not affect the main results, and our model can still duplicate stylized facts and crisis. Table 7 compares results for the two alternative trading strategies — exponential moving average and extrapolative trend. All the results are based on 1000 simulations with a linear demand function of the fundamentalists. Overall, most parts of stylized facts can still be replicated. More importantly, the impact of sentiment on excess volatility, leptokurtosis of returns, volatility clustering and magnitude of crisis still persists.

Table 5
Simulated market dynamics with different sentiment sensitivities using linear and [20] setups for fundamentalist demand function (average for 1000 simulations).

	Linear demand function						Day and Huang (1990) demand function					
	$h = 0$	$h = 0.2$	$h = 0.4$	$h = 0.6$	$h = 0.8$	$h = 1$	$h = 0$	$h = 0.2$	$h = 0.4$	$h = 0.6$	$h = 0.8$	$h = 1$
Kurtosis	2.378	2.312	2.657	2.931	3.053	3.377	2.537	2.551	3.442	4.397	4.807	5.309
Skewness	−0.018	−0.02	−0.057	−0.091	−0.088	−0.109	−0.009	−0.012	−0.043	−0.077	−0.086	−0.106
AC r_1	0.013	0.041	0.159	0.218	0.188	0.165	0.022	0.05	0.183	0.225	0.191	0.166
AC r_5	−0.004	0.000	0.006	0.002	0.007	0.004	−0.003	0.000	0.003	−0.002	0.002	−0.002
AC r_{10}	−0.001	0	−0.001	−0.001	−0.001	−0.002	0.000	0.000	−0.002	−0.002	−0.002	−0.003
AC r_{20}	0.000	0.000	−0.001	−0.002	−0.001	−0.001	0.000	0.000	−0.001	−0.001	−0.002	−0.001
AC $\lvert r_1 \rvert$	0.035	0.063	0.172	0.299	0.332	0.352	0.087	0.123	0.248	0.37	0.403	0.418
AC $\lvert r_5 \rvert$	0.103	0.126	0.183	0.209	0.214	0.238	0.134	0.165	0.231	0.276	0.291	0.315
AC $\lvert r_{10} \rvert$	0.074	0.097	0.166	0.195	0.202	0.227	0.086	0.116	0.194	0.25	0.27	0.298
AC $\lvert r_{20} \rvert$	0.058	0.084	0.158	0.188	0.196	0.223	0.058	0.088	0.175	0.235	0.259	0.29
$SD_{p-\mu}$	141.601	164.813	239.047	266.797	277.417	293.9	145.606	164.613	234.051	281.749	304.278	336.214
# of crisis	5.56	5.872	6.774	7.376	7.268	7.353	5.452	5.837	6.59	6.76	6.842	6.8
Magnitude of crisis (%)	20.166	22.44	30.059	35.723	36.732	38.883	20.927	23.511	33.006	38.715	40.851	42.916

Note: We set $a_f = 0.01$ for linear demand function; $d = -0.4$, $a_1 = 0$ and $a_2 = 2.4$ for [20] demand function.

Table 6
Simulated market dynamics with different period length L (average for 1000 simulations).

Without sentiment ($h = 0$)				
	$L = 1$	$L = 5$	$L = 10$	$L = 20$
Kurtosis	3.557	3.507	3.513	3.421
Skewness	−0.006	−0.001	−0.001	0
AC r_1	0.095	0.114	0.111	0.089
AC r_5	0.002	0.028	0.026	0.012
AC r_{10}	−0.002	0	0.008	0.008
AC r_{20}	−0.002	−0.003	−0.002	0
AC $\|r_1\|$	0.106	0.065	0.061	0.05
AC $\|r_5\|$	0.059	0.056	0.058	0.048
AC $\|r_{10}\|$	0.057	0.054	0.054	0.047
AC $\|r_{20}\|$	0.056	0.052	0.053	0.045
$SD_{p-\mu}$	176.808	169.885	169.963	158.062
# of crisis	9.078	9.096	9.076	8.953
Magnitude of crisis (%)	40.234	39.708	39.86	37.884
With sentiment ($h = 1$)				
	$L = 1$	$L = 5$	$L = 10$	$L = 20$
Kurtosis	4.375	4.451	6.198	4.945
Skewness	−0.033	−0.017	−0.034	−0.018
AC r_1	0.221	0.224	0.231	0.164
AC r_5	−0.049	0.099	0.107	0.05
AC r_{10}	0.001	0.02	0.056	0.036
AC r_{20}	−0.006	−0.005	0.004	0.011
AC $\|r_1\|$	0.232	0.141	0.168	0.111
AC $\|r_5\|$	0.09	0.109	0.153	0.105
AC $\|r_{10}\|$	0.082	0.094	0.137	0.1
AC $\|r_{20}\|$	0.078	0.087	0.122	0.92
$SD_{p-\mu}$	197.421	206.542	230.592	203.714
# of crisis	9.564	9.561	9.277	8.871
Magnitude of crisis (%)	49.301	50.39	55.511	49.493

6. Conclusion

In this paper, we develop a sentiment model with heterogeneous agents in a financial market. We conjecture that investor sentiment has heterogeneous influence on different types of investors and the sentiment effect is transmittable through interaction among agents. We then carry out an in-depth analysis on the role of sentiment with model simulations.

The key findings of this paper are in two respects. Firstly, reconciling our model with the actual US stock market, we discover that by supplementing the standard model with investor sentiment, it dramatically improves the model's ability to replicate stylized facts in financial markets. In particular, numerous key indicators, such as negative skewness, leptokurtosis and long memory of returns, market volatility, as well as the number of crisis detected increase with higher level of sentiment sensitivity. Secondly, in many HAM studies, the generation of market volatility and financial crisis is related to mean-reverting action of fundamentalist (see [28,68,73]). Fundamentalist gradually accumulates market power in bull market to pull the price down to fundamental value or even lower value, which could transit the market state from bull to bear and trigger the crashes or crises. However, we find that financial crisis can be triggered even without mean-reverting action of fundamentalist. With the presence of investor sentiment, just the momentum traders alone can initiate sudden crisis when there is abrupt downward pressure in the market sentiment. In other words, the sentiment channel provides an additional explanation underlying different types of financial crises.

While our sentiment HAM model is capable of replicating and even explaining several important features of financial market, it is not without limitations. One constraint is our model is based on a single market framework. Nonetheless, given that international financial markets are becoming more and more integrated, it is imperative to account for sentiment spillover between countries. Our future research may involve extending the sentiment model to a multi-market setup, so that we are able to develop deeper understanding on the role of investor sentiment as well as the causes of financial crisis on an international setting.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Table 7
Simulated market dynamics with different sentiment sensitivities using exponential moving average and extrapolative trend strategy for chartists demand function (average for 1000 simulations).

	Exponential moving ($k = 0.5$) average						Extrapolative trend strategy					
	$h = 0$	$h = 0.2$	$h = 0.4$	$h = 0.6$	$h = 0.8$	$h = 1$	$h = 0$	$h = 0.2$	$h = 0.4$	$h = 0.6$	$h = 0.8$	$h = 1$
Kurtosis	3.5	3.549	3.617	3.723	3.913	4.259	3.557	3.593	3.651	3.748	3.939	4.375
Skewness	−0.002	−0.003	−0.004	−0.006	−0.01	−0.017	−0.006	−0.007	−0.01	−0.014	−0.02	−0.033
AC r_1	0.113	0.129	0.148	0.17	0.196	0.226	0.095	0.111	0.131	0.155	0.185	0.221
AC r_5	0.021	0.029	0.039	0.051	0.066	0.086	0.002	0.007	0.014	0.022	0.034	0.049
AC r_{10}	0	0.001	0.003	0.006	0.01	0.017	−0.002	−0.002	−0.002	−0.002	−0.001	0.001
AC r_{20}	−0.003	−0.003	−0.004	−0.004	−0.005	−0.006	−0.002	−0.003	−0.003	−0.004	−0.005	−0.006
AC $\|r_1\|$	0.07	0.079	0.09	0.105	0.124	0.15	0.106	0.121	0.14	0.164	0.194	0.232
AC $\|r_5\|$	0.055	0.059	0.063	0.07	0.08	0.096	0.059	0.061	0.064	0.069	0.077	0.09
AC $\|r_{10}\|$	0.053	0.056	0.06	0.065	0.072	0.084	0.057	0.059	0.062	0.066	0.072	0.082
AC $\|r_{20}\|$	0.051	0.054	0.058	0.062	0.069	0.079	0.056	0.058	0.06	0.064	0.069	0.078
$SD_{p-\mu}$	169.035	172.429	176.441	181.499	188.128	196.864	176.808	178.924	181.32	184.624	189.657	197.421
# of crisis	9.162	9.286	9.364	9.413	9.494	9.596	9.078	9.203	9.287	9.403	9.487	9.564
Magnitude of crisis (%)	39.815	41.13	42.697	44.513	46.699	49.425	40.234	41.41	42.838	44.543	46.671	49.301

Note: We set $a_f = 0.01$ for linear demand function and change the parameter of chartists as $\beta_1 = 0.35$, $\beta_2 = -0.25$.

References

[1] R.J. Shiller, From efficient markets theory to behavioral finance, J. Econ. Perspect. 17 (1) (2003) 83–104.
[2] J.B. De Long, A. Shleifer, L.H. Summers, R.J. Waldmann, Noise trader risk in financial markets, J. Polit. Econ. 98 (4) (1990) 703–738.
[3] T. Lux, Herd behaviour, bubbles and crashes, Econ. J. 105 (431) (1995) 881–896.
[4] T. Lux, The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions, J. Econ. Behav. Organ. 33 (2) (1998) 143–165.
[5] K. Daniel, D. Hirshleifer, A. Subrahmanyam, Investor psychology and security market under-and overreactions, J. Finance 53 (6) (1998) 1839–1885.
[6] N. Barberis, A. Shleifer, R. Vishny, A model of investor sentiment, J. Financ. Econ. 49 (3) (1998) 307–343.
[7] M. Baker, J. Wurgler, Investor sentiment and the cross-section of stock returns, J. Finance 61 (4) (2006) 1645–1680.
[8] M. Baker, J. Wurgler, Investor sentiment in the stock market, J. Econ. Perspect. 21 (2) (2007) 129–152.
[9] P.C. Tetlock, Giving content to investor sentiment: The role of media in the stock market, J. Finance 62 (3) (2007) 1139–1168.
[10] G.W. Brown, M.T. Cliff, Investor sentiment and the near-term stock market, J. Empir. Financ. 11 (1) (2004) 1–27.
[11] Z. Da, J. Engelberg, P. Gao, The sum of all FEARS investor sentiment and asset prices, Rev. Financ. Stud. 28 (1) (2015) 1–32.
[12] W.Y. Lee, C.X. Jiang, D.C. Indro, Stock market volatility, excess returns, and the role of investor sentiment, J. Bank. Financ. 26 (12) (2002) 2277–2299.
[13] R.F. Stambaugh, J. Yu, Y. Yuan, The short of it: Investor sentiment and anomalies, J. Financ. Econ. 104 (2) (2012) 288–302.
[14] A. Hüsler, D. Sornette, C.H. Hommes, Super-exponential bubbles in lab experiments: evidence for anchoring over-optimistic expectations on price, J. Econ. Behav. Organ. 92 (2013) 304–316.
[15] T. Makarewicz, Contrarian behavior, information networks and heterogeneous expectations in an asset pricing model, Comput. Econ. 50 (2) (2017) 231–279.
[16] Y. Lahav, S. Meer, The effect of induced mood on prices in asset markets-experimental evidence, 2012, Available At SSRN 2050299.
[17] J.J. Siegel, Equity risk premia, corporate profit forecasts, and investor sentiment around the stock crash of October 1987, J. Bus. 65 (4) (1992) 557–570.
[18] M.N. Baur, S. Quintero, E. Stevens, The 1986–88 stock market: Investor sentiment or fundamentals? Manag. Decis. Econ. 17 (3) (1996) 319–329.
[19] M. Zouaoui, G. Nouyrigat, F. Beer, How does investor sentiment affect stock market crises? Evidence from panel data, Financial Rev. 46 (4) (2011) 723–747.
[20] R.H. Day, W. Huang, Bulls, bears and market sheep, J. Econ. Behav. Organ. 14 (3) (1990) 299–329.
[21] W.A. Brock, C.H. Hommes, Heterogeneous beliefs and routes to chaos in a simple asset pricing model, J. Econom. Dynam. Control 22 (8–9) (1998) 1235–1274.
[22] T. Kaizoji, Speculative bubbles and crashes in stock markets: an interacting-agent model of speculative activity, Physica A 287 (3–4) (2000) 493–506.
[23] C. Chiarella, X.-Z. He, Heterogeneous beliefs, risk, and learning in a simple asset-pricing model with a market maker, Macroecon. Dyn. 7 (4) (2003) 503–536.
[24] X.-Z. He, F.H. Westerhoff, Commodity markets, price limiters and speculative price dynamics, J. Econom. Dynam. Control 29 (9) (2005) 1577–1596.
[25] C. Chiarella, X.-Z. He, L. Shi, L. Wei, A behavioural model of investor sentiment in limit order markets, Quant. Finance 17 (1) (2017) 71–86.
[26] T. Lux, Estimation of an agent-based model of investor sentiment formation in financial markets, J. Econom. Dynam. Control 36 (8) (2012) 1284–1302.
[27] A. Kumar, C.M. Lee, Retail investor sentiment and return comovements, J. Finance 61 (5) (2006) 2451–2486.
[28] W. Huang, H. Zheng, W.-M. Chia, Financial crises and interacting heterogeneous agents, J. Econom. Dynam. Control 34 (6) (2010) 1105–1122.
[29] G.W. Brown, M.T. Cliff, Investor sentiment and asset valuation, J. Bus. 78 (2) (2005) 405–440.
[30] M. Schmeling, Institutional and individual sentiment: Smart money and noise trader risk? Int. J. Forecast. 23 (1) (2007) 127–145.
[31] S. Heiden, C. Klein, B. Zwergel, Beyond fundamentals: investor sentiment and exchange rate forecasting, Eur. Financial Manag. 19 (3) (2013) 558–578.
[32] R.J. Shiller, Conversation, information, and herd behavior, Am. Econ. Rev. 85 (2) (1995) 181–185.
[33] A.V. Banerjee, A simple model of herd behavior, Q. J. Econ. 107 (3) (1992) 797–817.
[34] F. Black, Noise, J. Finance 41 (3) (1986) 528–543.
[35] C.K. Soo, Quantifying sentiment with news media across local housing markets, Rev. Financ. Stud. 31 (10) (2018) 3689–3719.
[36] A. Beja, M.B. Goldman, On the dynamic behavior of prices in disequilibrium, J. Finance 35 (2) (1980) 235–248.
[37] C. Chiarella, X.-Z. He, C. Hommes, A dynamic analysis of moving average rules, J. Econom. Dynam. Control 30 (9–10) (2006) 1729–1753.
[38] F.H. Westerhoff, S. Reitz, Nonlinearities and cyclical behavior: the role of chartists and fundamentalists, Stud. Nonlinear Dyn. Econ. 7 (4) (2003) 3.
[39] S. Reitz, F. Westerhoff, Commodity price cycles and heterogeneous speculators: a STAR–GARCHSTA model, Empir. Econ. 33 (2) (2007) 231–244.
[40] S. Reitz, M.P. Taylor, The coordination channel of foreign exchange intervention: A nonlinear microstructural analysis, Euro. Econ. Rev. 52 (1) (2008) 55–76.
[41] S. Figlewski, Subjective information and market efficiency in a betting market, in: Efficiency of Racetrack Betting Markets, World Scientific, 2008, pp. 285–298.
[42] A. Shleifer, R.W. Vishny, The limits of arbitrage, J. Finance 52 (1) (1997) 35–55.
[43] J.A. Frankel, K.A. Froot, et al., Understanding the US dollar in the eighties: the expectations of chartists and fundamentalists, Econ. Rec. 62 (1) (1986) 24–38.
[44] J. Frankel, K. Froot, Chartists, fundamentalists and the demand for dollars, in: A. Courakis, M. Taylor (Eds.), Private Behavior and Government Policy in Interdependent Economies, Oxford University Press, Oxford, 1990.
[45] S. Manzan, Nonlinear mean reversion in stock prices, Quant. Qual. Anal. Soc. Sci. 1 (3) (2007) 1–20.
[46] W. Huang, H. Zheng, Financial crises and regime-dependent dynamics, J. Econ. Behav. Organ. 82 (2–3) (2012) 445–461.
[47] W.A. Brock, C.H. Hommes, A rational route to randomness, Econometrica 65 (5) (1997) 1059–1095.
[48] M. Anufriev, C. Hommes, Evolutionary selection of individual expectations and aggregate outcomes in asset pricing experiments, Am. Econ. J. Microecon. 4 (4) (2012) 35–64, http://dx.doi.org/10.1257/mic.4.4.35.
[49] M. Anufriev, C.H. Hommes, R.H.S. Philipse, Evolutionary selection of expectations in positive and negative feedback markets, J. Evol. Econ. 23 (3) (2013) 663–688, http://dx.doi.org/10.1007/s00191-011-0242-4.
[50] M. Anufriev, T. Bao, J. Tuinstra, Microfoundations for switching behavior in heterogeneous agent models: An experiment, J. Econ. Behav. Organ. 129 (2016) 74–99, http://dx.doi.org/10.1016/j.jebo.2016.06.002.
[51] M. Anufriev, T. Bao, A. Sutan, J. Tuinstra, Fee structure and mutual fund choice: An experiment, J. Econ. Behav. Organ. 158 (2019) 449–474, http://dx.doi.org/10.1016/j.jebo.2018.12.013.

[52] C.F. Manski, D. McFadden, et al., Structural Analysis of Discrete Data with Econometric Applications, MIT press Cambridge, MA, 1981.
[53] A. Kirman, Ants, rationality, and recruitment, Q. J. Econ. 108 (1) (1993) 137–156.
[54] T. Lux, M. Marchesi, Scaling and criticality in a stochastic multi-agent model of a financial market, Nature 397 (6719) (1999) 498–500.
[55] N. Schmitt, F. Westerhoff, Heterogeneity, spontaneous coordination and extreme events within large-scale and small-scale agent-based financial market models, J. Evol. Econ. 27 (5) (2017) 1041–1070.
[56] F.H. Westerhoff, R. Dieci, The effectiveness of Keynes–Tobin transaction taxes when heterogeneous agents can trade in different markets: a behavioral finance approach, J. Econom. Dynam. Control 30 (2) (2006) 293–322.
[57] J.Y. Campbell, R.J. Shiller, Cointegration and tests of present value models, J. Polit. Econ. 95 (5) (1987) 1062–1088.
[58] S.F. LeRoy, R.D. Porter, The present-value relation: Tests based on implied variance bounds, Econometrica 49 (3) (1981) 555–574.
[59] R.J. Shiller, Do stock prices move too much to be justified by subsequent changes in dividends? Technical Report, National Bureau of Economic Research, 1980.
[60] B. Dumas, A. Kurshev, R. Uppal, Equilibrium portfolio strategies in the presence of sentiment risk and excess volatility, J. Finance 64 (2) (2009) 579–629.
[61] T. Li, Heterogeneous beliefs, asset prices, and volatility in a pure exchange economy, J. Econom. Dynam. Control 31 (5) (2007) 1697–1727.
[62] M. Lof, Rational speculators, contrarians, and excess volatility, Manage. Sci. 61 (8) (2015) 1889–1901.
[63] W. Xiong, Bubbles, crises, and heterogeneous beliefs, Technical Report, National Bureau of Economic Research, 2013.
[64] E. De Jong, W.F. Verschoor, R.C. Zwinkels, Behavioural heterogeneity and shift-contagion: Evidence from the Asian crisis, J. Econom. Dynam. Control 33 (11) (2009) 1929–1944.
[65] T. Lux, F. Westerhoff, Economics crisis, Nat. Phys. 5 (1) (2009) 2–3.
[66] S.A. Patel, A. Sarkar, Crises in developed and emerging stock markets, Financ. Anal. J. 54 (6) (1998) 50–61.
[67] R.J. Shiller, Irrational Exuberance: Revised and Expanded Third Edition, Princeton University Press, 2015.
[68] C. Hommes, et al., Booms, busts and behavioural heterogeneity in stock prices, J. Econom. Dynam. Control 80 (2017) 101–124.
[69] W. Brock, J. Lakonishok, B. LeBaron, Simple technical trading rules and the stochastic properties of stock returns, J. Finance 47 (5) (1992) 1731–1764.
[70] W.F. De Bondt, R. Thaler, Does the stock market overreact? J. Finance 40 (3) (1985) 793–805.
[71] N. Jegadeesh, Evidence of predictable behavior of security returns, J. Finance 45 (3) (1990) 881–898.
[72] N. Jegadeesh, S. Titman, Returns to buying winners and selling losers: Implications for stock market efficiency, J. Finance 48 (1) (1993) 65–91.
[73] A. Szafarz, Financial crises in efficient markets: How fundamentalists fuel volatility, J. Bank. Financ. 36 (1) (2012) 105–111.