Corporate Investment and Liquidity Management under Aggregate Uncertainty Shocks

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Abstract

This paper investigates how firms manage their investment and liquidity when aggregate uncertainty is stochastic. I develop and structurally estimate a dynamic model where firms face stochastic aggregate uncertainties, financial frictions, and time-varying risk premia. In my model, firms have a precautionary-savings motive and real options to wait. They interact with each other and reduce investment and financing activities when aggregate uncertainty is high. Such reduction is further amplified by higher discount rates in high uncertainty states. My model predicts that (1) large positive aggregate uncertainty shocks depress investment, equity issuance, and payout; (2) the impacts of uncertainty shocks on real investment and output depend on firms’ liquidity positions; and (3) more productive firms’ equity issuance and investment are more vulnerable to aggregate uncertainty shocks but not their payout. Finally, counterfactual experiments show that (1) a model without dynamic uncertainties cannot explain the observed firm behaviors in high uncertainty states, and (2) time-varying risk premia have quantitatively important effects.

JEL Classification: C23, D92, E22, G32, G35.

Keywords: Cash holdings; Costly external financing; Financial frictions; Investment; Liquidity management; Real options; Structural estimation; Aggregate uncertainty shocks; Variance premium.
1 Introduction

Economic uncertainty has been a pervasive concern among businesses and policy makers, especially during and after the recent 2009 crisis. For example, in a survey of small business CEOs in December 2012, CEOs of all industries, regions, and genders named economic uncertainty as the most significant issue they currently faced.\footnote{WSJ VISTAGE Small Business CEO Survey, December 2012, available at \url{http://www.vistageindex.com/?ym=201212#1}.} Also, central bankers have worried that increase in uncertainty can aggravate recession and slow recovery by “inhibiting spending”.\footnote{Federal Open Market Committee (FOMC) Minutes, June 2009. More similar statements are made in April 2008, September 2010, and etc.} In macroeconomics, a large literature following Bloom (2009) justifies such concerns.\footnote{See Bloom (2014) for a survey of this literature.} They find that uncertainty shocks have substantial real impacts on output, investment, and employment and can potentially induce short recessions. Stock and Watson (2012) have even identified heightened uncertainty as one of the two culprits that produced the recent crisis in 2009.\footnote{The other one is financial disruption.}

Aggregate uncertainty shocks not only affect real investment and output but should also affect liquidity and financing decisions. Conceptually, in anticipation of uncertainty shocks, firms could build liquidity buffer by hoarding more cash in advance. Empirically, using the CBOE S&P 500 Volatility Index (VIX) as a proxy for aggregate uncertainty, I find that firms substantially cut their equity issuance, payout, and investment, at the same time hoard more cash when aggregate uncertainty is high. Quantitatively, such reduction is large. In states where VIX is above its 90th percentile, firms cut their investment and financing activities by at least 10%, and hoard 9% more cash relative to their normal levels. However, despite such conceptually relevance and empirical importance, few theoretical or quantitative research has addressed the following questions.\footnote{A few empirical papers have studied the impacts of uncertainty shocks on firm cash holdings, like Chen, Wang, and Zhou (2014) and Gao and Grinstein (2014).}

How should firms optimally manage liquidity when uncertainty is high? How should firms prepare themselves for potential uncertainty shocks? And how do such optimal risk management policies in turn affect real investment?

In this paper, I address these questions both empirically and theoretically. I propose and structurally estimate a dynamic model to investigate firms’ behaviors around uncertainty shocks. My model builds on the recent development in both dynamic corporate finance, like Hennessy and Whited (2005) and Bolton, Chen, and Wang (2011, 2013), and macroeconomics, such as Bloom (2009). The model has the following four building blocks: (1) regime-switching states with different volatilities of productivity growth; (2) investment irreversibility with both fixed and convex adjust-
ment costs; (3) financial frictions, including both external financing costs and cash-carrying costs; and (4) risk premia for productivity risk and uncertainty risks. A firm in this economy optimally manages its cash and capital stocks by conditioning its equity issuance, payout, and investment decisions on changing uncertainty states.

My model explains why aggregate uncertainty shocks simultaneously depress firm investment and financing activities. It works through three channels. First, irreversibilities in investment and issuance create the real options to wait, whose values increase with the level of uncertainty. So higher uncertainty levels induce a firm to postpone both investment and issuance decisions and delay of one reinforces the other. Second, higher uncertainty means higher precautionary demand for cash, which delays payout. Finally, higher uncertainty inflates discount rates on capital stock, making real investment less attractive. Higher discount rates further increase cash holdings and reduce capital investment, reinforcing the effects of the previous two channels.

Furthermore, I also study how heterogeneous firms respond differently to aggregate uncertainty shocks. Firm heterogeneity is considered in two dimensions: productivity and cash stock, which are also the state variables of the model. The model predicts that high productivity firms’ investment and equity issuance are more affected by aggregate uncertainty shocks; but this is not true for their payout. The model also predicts that low cash firms are more likely to be affected in their issuance and payout but not their investment. Cross-sectional empirical evidence overall supports the model predictions.

To evaluate the quantitative importance of the model ingredients, I structurally estimate the model and conduct counterfactual experiments. The results in counterfactual experiments show that (1) financial frictions are necessary to explain the observed comovement of investment and financing activities; (2) a model without dynamic uncertainties cannot explain the firm behaviors in high uncertainty states; and (3) risk premia are quantitatively important to match the impacts of uncertainty shocks.

The primary contribution of this paper is to provide a structural framework to evaluate the financing and investment activities under uncertainty shocks. It achieves this goal by combining two important literature in corporate finance and macroeconomics: financial frictions and uncertainty shocks. On the one hand, financial frictions require firms to optimally manage its composition of liquid (cash) and illiquid (capital) assets. Optimal cash holding problem naturally emerges, as a result of financial friction and uncertainty, according to Opler, Pinkowitz, Stulz, and Williamson (1999), Bates, Kahle, and Stulz (2009), Riddick and Whited (2009), Bolton, Chen, and Wang (2011), Asvanunt, Broadie, and Sundaresan (2009), Nikolov and Whited (2013), and Nikolov, Schmid,

Aggregate uncertainty can have different meanings in different papers. Here, I consider aggregate uncertainty as the volatility of aggregate cash flow. A shock to aggregate uncertainty will also increase the associated risk premia.
and Steri (2013). Such optimal cash holding problem ought to be more relevant when aggregate uncertainty and risk are changing over time. Therefore, I build on this literature and address how optimal liquidity management is affected by uncertainty shocks and how firms achieve such management via financing and investment policies.

On the other hand, a large macroeconomics and finance literature starting from Bloom (2009) has investigated the impact of uncertainty shocks on real investment and employment. But most of them either assume a world without financial frictions or focus on the debt versus equity tradeoff assuming collateral constraints or possibility of strategic default. The former line of research includes Bloom, Bond, and Reenen (2007), Bloom (2009), Bloom et al. (2014), Julio and Yook (2012), Doshi, Kumar, and Yerramilli (2014), and Gulen and Ion (2015). And the latter consists of Gilchrist, Sim, and Zakrajsek (2010), Arellano, Bai, and Kehoe (2011), Khan and Thomas (2011), Christiano, Motto, and Rostagno (2014), and McQuade (2014). While the latter has incorporated financial frictions from the liability side of the firms, it does not model the liquidity decisions on the left hand side. Both streams of literature speak little about the dynamics on optimal composition of liquid versus illiquid assets under uncertainty shocks and how dynamics of such composition feedback to the real side like corporate investment.

In the intersection of these two literature, some papers investigate how cash holdings and other financing activities are affected by uncertainty shocks. For example, Gao and Grinstein (2014) find that cash holdings are affected by systematic uncertainty shocks but not idiosyncratic uncertainty shocks. Chen, Wang, and Zhou (2014) explore the effects of uncertainty on cash holdings and capital structure. My paper differs from them in that this paper provides a theoretical framework for firm behaviors under dynamic uncertainties, emphasizing the real option effects due to irreversibilities and uncertainty shocks. Moreover, I investigate the joint dynamics of issuance and payout. It turns out that the separation of the two financing decisions has important implications.

The second contribution is that this paper allows me to evaluate the conventional view that payout is negative issuance. Conventionally, people tend to think that a firm that is less likely to issue is more likely to distribute. This view is valid if a firm is mainly subject to first-moment (productivity) shocks or shocks to cash balance. These shocks increase the likelihood of one type of financing activities but depress the other. However, this conventional view is, false in an environment with uncertainty shocks. Positive uncertainty shocks reduce issuance for high productivity firms and payout for low productivity firms. As a result, the average issuance and payout are both lower in high uncertainty states. This comovement emphasizes the divorce of issuance and payout and overturns the conventional belief. The divorce is a result of financial frictions and becomes more evident under uncertainty shocks.

Joint consideration of issuance and payout are also explored in Dittmar and Dittmar (2008) and
Warusawitharana and Whited (2014). The former explore the effects of aggregate productivity on repurchase, issuance, and merger waves. And the latter looks at misvaluation. Both papers focus on first moment effects, which drive payout and issuance in opposite directions. In this sense, this paper complements theirs by focusing on a different and importance force, uncertainty shocks, and emphasizes the separation.

Uncertainty shocks are not the only force that pushes issuance and payout in the same direction. Financing cost shock is another force. For example, Bolton, Chen, and Wang (2013) and Eisfeldt and Muir (2014) show that adverse financing cost shocks also depress payout and issuance simultaneously. My paper complements their findings in two ways. On the one hand, uncertainty shocks affect the financing and investment activities through both the real and financing side, and feedback to each other. But financing costs affect firm behaviors mainly through financing side, and feedback to real investment through an indirect channel. Therefore, uncertainty shocks tend to have larger impact on real investment than financing cost shocks, holding their impacts to the equity issuance constant. On the other hand, I show that uncertainty shocks and financing costs are positively correlated. This is consistent with Stock and Watson (2012), who find that both financing and uncertainty shocks are major contributor to the recent crisis. Studying both kind of shocks thus help us to better understand the underlying mechanism of the recent crisis and its aftermath.

Finally, my paper also contributes to the literature on how risk premia can affect firms’ real and financing policies. While there are extensive literature on both asset pricing and corporate decisions in finance, few has explored the asset pricing implications on corporate finance, despite of its prevalence and importance. Almeida and Philippon (2007), Li, Livdan, and Zhang (2009), Bhamra and Strebulaev (2011), Ai and Kiku (2012), Bolton, Chen, and Wang (2013), and Alti and Tetlock (2014) are examples of such exceptions. My model incorporates a relatively simple risk premia structure, motivated by Capital Asset Pricing Model and variance premium from Bekaert and Hoerova (2014). By counterfactual experiment, I show that both market and variance risk premia play important roles in corporate financing and investment decisions.

The rest of the paper proceeds as follows. Section 2 presents preliminary reduced form evidence to motivate the model. The next four sections estimate a dynamic model and interpret the results. Section 3 proposes the dynamic model. Section 4 presents the solution and estimation methods. Section 5 discusses the quantitative implications of the model. The model not only explains the

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7In Dittmar and Dittmar (2008), the issuance and repurchase waves are positively correlated. But notice that their measures of issuance and repurchase are different from mine. They use the logarithm of the total amount of issuance and repurchase, while my key variables are the issuance and repurchase denominated by a firm’s capital or illiquid assets. While their results can be contaminated by the increasing number of firms over time, my measures here focus on the average corporate policies relative to the firm size.
underlying mechanism of the impacts of uncertainty shocks, but also predicts how heterogeneous firms respond to uncertainty shocks differently. Those heterogeneous responses are tested both in model and empirically in Section 6. Counterfactual experiments are conducted to evaluate the importance of the modeling features in Section 7. Section 8 concludes.

2 Do Uncertainty Shocks Matter?

This section provides preliminary empirical evidence on the impacts of aggregate uncertainty shocks on investment and liquidity management. Here I describe data and present reduced form evidence of the comovement of investment, equity issuance, payout, and cash ratios. The results highlight a novel finding, higher uncertainty predicts simultaneous reduction of issuance, payout, and investment in aggregate. This motivates a model to explain the financing and investment decisions under uncertainty shocks.

2.1 Data Description

I obtain quarterly financial data of public firms in the United States from the Compustat database. Cash is measured by cash and equivalents (CHE). Capital is total assets (AT) net of cash.\(^8\) Cash flow is operating income before depreciation (OIBDP) plus Research and Development expenditures (XRD); the latter are considered as a component of investment rather than costs, as in Alti and Tetlock (2014). Investment is capital expenditure (CAPX) net of sale of property, plant and equipment (SPPE), plus Research and Development expenditures (XRD) and acquisitions (AQC). Equity payout is the sum of equity dividends payout (DVC) and the repurchase of common stocks. The secondary equity offerings (SEOs) and repurchase data are extracted from Securities Data Corporation (SDC) Platinum database.\(^9\) These two variables span from 1994Q1 through 2014Q2 because repurchase data became available in SDC starting in year 1994. Repurchase data are matched into each completed deal and repurchase amount.\(^10\) Among the deals of SEOs, rights issues

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\(^8\)I also alternate the definitions of capital in this empirical session, finding similar results. I choose this variable as a benchmark for capital because most existing empirical researches uses total assets as the denominator, and total assets are the sum of cash and capital in the model.

\(^9\)Those data in Compustat can be problematic proxies for active financing policies because Compustat data include employee stock compensation in both issuance and repurchase, as suggested in McKeon (2014) and Warusawitharana and Whited (2014).

\(^10\)Repurchase data in SDC Platinum is reported in the unit of plan. Within each plan, there are multiple authorization and completed deals. Warusawitharana and Whited (2014) apportion the total value repurchased within a plan to each authorization date. Instead, I match the date and amount of each completed deals because these are more likely the actual repurchase realized. If the numbers of completed date are different from the number of completed amounts, I supplement with the Compustat repurchase data to supply the missing information.
and unit issues are excluded. Finally, both repurchase and SEO data are matched to Compustat data by CUSIP number. The construction of these key variables is summarized in Appendix A1.

Throughout the analysis, I exclude utilities (SIC 4900–4999), financial firms (SIC 6000–6999), and government institutions (SIC 9000–9999), because they have different business models from all other public firms. Firm-quarter observations are excluded if they have missing or non-positive values in the book assets, book value of equity, sales, and capital. Firms with missing values in cash or investment are also excluded as they are the key variables I want to explain. To minimize the outlier effects, firms with real book assets less than 10 million dollars, or firms with real sales lower than 2.5 million dollars are excluded. In order to reduce the effects coming from new entry, I also require a firm to have a two-year history to be included in the sample. The final sample consists of 230,801 firm-quarter observations and 8,258 firms. Finally, I winsorize all the firm-level ratio variables at their 1st and 99th percentiles.

There are many measures that try to quantify macroeconomic uncertainties. The primary proxy used in this paper is the S&P 500 Volatility Index (VIX) from the Chicago Board Options Exchange (CBOE). The VIX is computed as the implied volatilities from the option trading prices in the S&P 500 indexes, and thus they can serve as a forward-looking measure of stock price volatilities. The VIX is used as the primary measure for time-varying uncertainties throughout the whole paper because it is the most widely used proxy for uncertainties in both researches and practices.

I also extract macroeconomic data from St. Louis Federal Reserve Economic Data (FRED), including the Consumer Price Index (CPI), quarterly GDP growth and NBER business cycle dates. The CPI is used to adjust all dollar variables to real values. All my variables are in year 2000 dollar values. Quarterly GDP growth and NBER business cycle dates serve as control variables in my analysis.

2.2 Summary Statistics

Table 1 summarizes the key variables describing the VIX, average firm financing, and investment policies. The first variable is the logarithm of the VIX in the last quarter. I use the last quarter value instead of the current one because the VIX and the aggregate uncertainty in my model are forward-looking. When a firm expects a high aggregate uncertainty at the beginning of a given period, it may adjust its liquidity and capital stocks immediately, but the effects of such adjustments

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11Bloom et al. (2014) show that stock return volatilities are tightly linked to plant-level productivity shocks.

12In addition to market-based measures of uncertainties, there are also other proxies for uncertainties. For example, Baker, Bloom, and Davis (2012) implement an Economic Policy Uncertainty Index (EPU), which is a weighted average of frequencies of the words “uncertain” and “uncertainties” in the newspaper, the possible expiration of the tax code provisions, and the forecast dispersion of monetary policy and government spending from surveyed professionals. I also conduct analysis on EPU measures. The results are very similar but the statistical significance varies.
Table 1: Summary Statistics

The sample includes a total of 240,682 firm-quarter observations. Aggregate uncertainty is measured by the logarithm of the VIX in the last quarter. Cash is denominated by current period end total assets. All the remaining variables are denominated by last quarter end assets net of cash, which are denoted by $K_{t-1}$. At the top row, 25th, 50th, 75th, and 90th denote the percentile value of the payout for the corresponding variables.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>90th</th>
</tr>
</thead>
<tbody>
<tr>
<td>log[VIX(t-1)]</td>
<td>2.978</td>
<td>0.328</td>
<td>2.701</td>
<td>2.992</td>
<td>3.189</td>
<td>3.375</td>
</tr>
<tr>
<td>Investment / $K_{t-1}$</td>
<td>5.122%</td>
<td>10.493%</td>
<td>0.921%</td>
<td>2.255%</td>
<td>5.354%</td>
<td>11.781%</td>
</tr>
<tr>
<td>Issuance/$K_{t-1}$</td>
<td>0.930%</td>
<td>15.684%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Payout/$K_{t-1}$</td>
<td>0.569%</td>
<td>1.331%</td>
<td>0</td>
<td>0</td>
<td>0.423%</td>
<td>1.743%</td>
</tr>
<tr>
<td>Cash / asset</td>
<td>16.393%</td>
<td>19.201%</td>
<td>2.339%</td>
<td>8.442%</td>
<td>23.765%</td>
<td>45.474%</td>
</tr>
</tbody>
</table>

The investment-to-capital ratio has the following characteristics. First, quarterly investment is highly skewed to the right. The mean is even higher than the 75th percentile. This high skewness of investment is consistent with Cooper and Haltiwanger (2006). Furthermore, about 24% of firm-quarter observations have an absolute investment ratio of less than 1%. This implies a large inaction region on investment. As Cooper and Haltiwanger (2006) point out, both the skewness and inaction region tend to suggest the existence of substantial non-convex adjustment costs on capital. Finally, the high investment ratio compared to previous literature is due to the inclusion of R&D expenditures in investment.

The next two rows describe firm equity financing policies. Issuance and payout are small and infrequent events. Only 1.50% of firm-quarter observations have positive new issuance in the sample, and only 39.23% of observations pay out. This implies a large inaction region of liquidities, where firms neither issue nor distribute. This is consistent with Almeida, Campello, and Weisbach (2004); Riddick and Whited (2009); and Bolton, Chen, and Wang (2011).

The last row reports cash-to-asset ratios. Public firms in the United States hoard substantial cash, with cash-to-asset ratios of 16.4%. This is consistent with a lot of empirical research including

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13 For example, November 2011 is considered as a high uncertainty month. This will be matched with the fiscal quarter including the whole month of December 2011, which are quarterly statements disclosed in the fiscal quarter end, i.e., December 2011, January and February 2012. Quarterly statements at the end of November 2011 is not considered a match to November 2011 because a firm observing high uncertainty at the end of November did not have time to adjust its decisions yet. This is also consistent with Gulen and Ion (2015).

14 About 1% of firm-quarter observations have negative values lower than -1%. So investment ratios less than 1% accounts for (25%-1%)=24% of all firm-quarter observations.
2.3 Defining Aggregate Uncertainty States

I first investigate the behavior of aggregate uncertainty, which is proxied by the VIX. Figure 1 plots the monthly VIX index from 1994 January through 2015 August. Large jumps in the VIX are labeled with the corresponding events. Shaded areas indicate NBER economic recessions in United States.

One can see that the VIX is time-varying. It is very volatile. In some periods, the VIX can be as low as 11, suggesting that the implied volatility for the stock index one-month ahead is only 11%. Occasionally it jumped after some extreme events. Such extreme events include both economic ones, such as the recent Lehman collapse and the 2013 US debt ceiling crisis, and political ones, such as 9/11 and the second Gulf War. The Lehman collapse has the highest VIX value, about 61 and 62, in 2008 October and November. These spikes in the VIX seem to suggest that fluctuations in the VIX track potentially a regime-switching process., which is also shown in Bloom (2009).

This figure plots the monthly VIX index from 1994 January through 2015 August. The shaded areas are the NBER recession periods. The value of the VIX index is the implied volatility of S&P 500 in the following month period, stated in terms of percentage point. For example, When Lehman collapsed in 2008 October, the VIX has a value of 61. This means that the implied volatility of S&P 500 for 2008 November is 61%.

Figure 1: VIX Index 1994–2015
Table 2: Moments Conditional on Uncertainty States

L denotes low uncertainty states where the VIX index in the last quarter is below its 90th percentile value; H denotes the high uncertainty states where the VIX is above its 90th percentile value. The heading t-stat denotes the two-sided t-test statistics for the null hypothesis that H state variables minus L state variables have the same mean. The t-test allows different variance across subsamples. ***, **, and * denote statistical significance levels at 1%, 5%, and 10%, respectively.

<table>
<thead>
<tr>
<th>#obs</th>
<th>L</th>
<th>H</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>209,072</td>
<td>25,988</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log[VIX(t-1)]</td>
<td>2.942</td>
<td>3.600</td>
<td>211***</td>
</tr>
<tr>
<td>Investment / K</td>
<td>5.168%</td>
<td>4.753%</td>
<td>-6.561***</td>
</tr>
<tr>
<td>Issuance / K</td>
<td>0.986%</td>
<td>0.476%</td>
<td>-6.075***</td>
</tr>
<tr>
<td>Issuance probability</td>
<td>1.497%</td>
<td>0.974%</td>
<td>-7.873***</td>
</tr>
<tr>
<td>Payout / K</td>
<td>0.577%</td>
<td>0.504%</td>
<td>-8.984***</td>
</tr>
<tr>
<td>Payout probability</td>
<td>40.239%</td>
<td>38.033%</td>
<td>-6.900***</td>
</tr>
<tr>
<td>Cash-to-asset</td>
<td>16.229%</td>
<td>17.712%</td>
<td>11.291***</td>
</tr>
</tbody>
</table>

In order to determine the different regimes of uncertainty states, I divide the whole sample periods into low and high uncertainty states (hereafter, L and H states) using a simple cutoff rule. The cutoff is the 90th percentile of the VIX. If the last quarter VIX is above the cutoff, this quarter is classified as a high uncertainty state. Otherwise, it is classified as a low uncertainty state. I also apply the Forward-Filtering-Backward-Smoothing (FFBS) method to estimate the latent states and the probabilities of switching states. For details on the classifications of low versus high uncertainty states and the related statistics, please refer to Appendix A2.

2.4 Corporate Behaviors under Low and High Uncertainty States

I then look at corporate investment and liquidity management behaviors under different uncertainty states. Table 2 reports the moments that are conditional on different states. L and H denote the low and high uncertainty states, respectively. The last column reports the t-test statistics on whether two conditional moments are equal.16

15 Bloom (2009) uses a cutoff rule of 1.65 standard deviation above the Hodrick and Prescott (1997) trend, which is equivalently 95th percentile of the filtered VIX if logVIX is assumed to be normally distributed. I do not filter the VIX process because both financing and investment behaviors depend on the uncertainty level, not the detrended residual. When I use filtered VIX, the timing of the uncertainty states are almost identical and the conditional moments on financing and investment activities are almost the same. I also tried 75th and 90th percentile of VIX as cutoff rules. The overall results on aggregate uncertainty shocks become weaker or stronger but the qualitative implications are the same.

16 The t-test assumes that the variances in two states are different.
The first row of Table 2 suggests that investment shrinks significantly in a high uncertainty state.\textsuperscript{17} This is consistent with Bloom, Bond, and Reenen (2007); Bloom (2009); Gulen and Ion (2015); and Doshi, Kumar, and Yerramilli (2014). This result could be due to the aggregate uncertainty shocks as well as the accompanying economic downturn.\textsuperscript{18}

In the next four rows, issuance and payout both fall significantly in a high uncertainty state, in terms of both magnitude and probabilities. Issuance level and probabilities in state H are less than two-third of those in state L. Relative to state L, the payout level is 13% lower and the payout probability is 6% lower in state H. The evidence here suggests that uncertainty shocks may have a large impact on financing decisions.

The simultaneous reduction in issuance and payout also challenges the conventional belief that they are two opposite transactions and are likely to be negatively correlated. In later sections, I will show that these reductions are the results from the interaction of the real options effects, a precautionary-savings motive, and higher risk premia in the high uncertainty states. And the positive comovement between issuance and payout is a signature of the interaction of uncertainty shocks and financing costs.

The last row is the cash-to-asset ratio. Consistent with Gao and Grinstein (2014), cash holdings are higher in high aggregate uncertainty states. The table seems to suggest that firms become more cautious and demand more money when aggregate uncertainty is high.

My results here add to the existing literature on financing policies, and suggest that positive aggregate uncertainty shocks not only reduce investment, but also predict lower financing activities including both equity issuance and payout.

\subsection*{2.5 Panel Data Regressions}

In this subsection I present the empirical evidence of the differential firm policies under different uncertainty states with fixed effect regressions. These fixed effect regressions not only control for firm-level time-invariant heterogeneities, but also for additional control variables that might affect financing and investment policies. The results are very similar to conditional moment results: higher VIX predicts lower equity issuance, payout, and investment. It also predicts higher cash holdings in the next quarter.

\textsuperscript{17}If one separates the investment into positive and negative accounts, they decrease simultaneously, with magnitudes both economically and statistically significant. However, I do not push these results here because the Compustat measures investment in firm level, which already aggregates the plant-level investments. As mentioned in Bloom, Bond, and Reenen (2007), it is difficult to detect investment inaction region on the firm level. In this sense, the positive and negative accounts of investment may not be rightly accounted.

\textsuperscript{18}Bloom et al. (2014) reject the hypothesis that high uncertainties are caused by lower economic growth using natural experiment. But they cannot reject the hypothesis that heightened uncertainty causes recessions.
In this analysis, control variables include the following: lag cash-to-capital ratio, lag cash-flow-to-capital ratio, logarithm of firm’s size at the end of last quarter, firm age, lag book leverage, last quarterly real GDP growth, dummy of time after 2003 May, and lag average Q. Lag of cash to non-cash asset ratio is included because in a world with financial frictions, the cash balance at the beginning of the period may affect the end of period balance. Cash flows are included to control for productivity or demand shocks. Firm size and age are included to control for financing costs, following Hadlock and Pierce (2010). Book leverage is included to control for potential debt overhang problems. Last quarter real GDP growth is to isolate the aggregate first moment effects. A dummy that indicates time after 2003 May is also included, to take into account the impact of the Jobs and Growth Tax Relief Reconciliation Act of 2003 on payout and liquidity policies. Lag average Q is included in the investment equation because in traditional $q$ theory it is a proxy for investment opportunities. Finally, quarterly time trends, seasonal dummies, and firm fixed effects are included; coefficient standard errors are two-way clustered by firm and quarters.

Table 3 presents the regression results.$^{19}$ The first column suggests that average firm investment is lower in state H than in state L, by 0.47% of non-cash assets. This is a more than 8% decrease of investment from its L-state level of 5.76%. This finding corroborates the results in Bloom (2009); Bloom et al. (2014); Chen, Wang, and Zhou (2014); and Gulen and Ion (2015).

In column (2), the difference of issuance-to-capital ratio between two states is 0.54%, which is about 44% of the issuance-to-capital ratio in state L. Column (3) confirms this effect, by looking at the response in the probability of equity issuance. Moving from a low to high uncertainty state, a firm reduces issuance probability by 0.39 percentage point, about one-third of its L-state value.

The next two columns explore the impact of uncertainty on amount and probability of equity payout, including both dividend payout and repurchase. The payout is lower in state H than in state L, by 0.05% of non-cash assets. Although this number seems small, it accounts for more than 9% of the payout in low uncertainty quarters. So it is economically significant. Payout probabilities are 1.4 percentage points lower in state H than in state L, where are both economically and statistically significant.

Column (6) reports the result for net issuance, which is equity issuance net of payout. Both issuance and payout shrink, and issuance reduces even more than payout. As a result, the overall effects of uncertainty shocks are negative and significant. However, these results prompt a caveat on the usage of net issuance to summarize two different financing activities.$^{20}$ Although conventional

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$^{19}$For levels of issuance and payout, I also try Tobit with lower bound 0. These regression methods include control variables but not fixed effects as Tobit has no fixed-effect model. The effects of aggregate uncertainty are much stronger. For issuance and payout probabilities, I try fixed-effect Logit. The effects of aggregate uncertainty are more statistically and economically significant.

$^{20}$Recent usage of net issuance includes Eisfeldt and Muir (2014) and Belo, Lin, and Yang (2014). Both papers try
Table 3: Panel Data Regressions

H is a dummy variable that is 1 if the average VIX index in the last 3 months exceeds the 90th percentile of the sample. Net issuance is equity issuance net of payout. Control variables include lag of cash-to-capital ratios, lag of sales-to-capital ratios, log book size of assets, firm age, lag of book leverage, lag quarterly real GDP growth, dummy of time after 2003 May, lag average Q, time trend, and seasonal dummies. All regressions are fixed effect regressions with two-way clustered standard errors over firm and quarter level. Standard errors are in round parentheses below the coefficient estimates. All coefficient estimates and standard errors are multiplied by 100. ***, **, and * denote significance levels at 1%, 5%, and 10%, respectively.

<table>
<thead>
<tr>
<th>[coef×100]</th>
<th>(1) Invest.</th>
<th>(2) Iss.</th>
<th>(3) Iss. prob.</th>
<th>(4) Payout</th>
<th>(5) Payout prob.</th>
<th>(6) Net issuance</th>
<th>(7) Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi Uncertainty</td>
<td>-0.472***</td>
<td>-0.540***</td>
<td>-0.387***</td>
<td>-0.053***</td>
<td>-1.356***</td>
<td>-0.487***</td>
<td>0.670***</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.099)</td>
<td>(0.084)</td>
<td>(0.009)</td>
<td>(0.294)</td>
<td>(0.099)</td>
<td>(0.071)</td>
</tr>
<tr>
<td>Control</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.135</td>
<td>0.004</td>
<td>0.004</td>
<td>0.018</td>
<td>0.023</td>
<td>0.005</td>
<td>0.368</td>
</tr>
<tr>
<td># Obs</td>
<td>228,175</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
</tr>
<tr>
<td># Firms</td>
<td>8,235</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
</tr>
</tbody>
</table>

views suggest that issuance and payout have different directions of cash flows in the cash account, they are two financing decisions at different margins. The fact that a firm reduces issuance may not suggest that it is more likely to distribute. This becomes evident in an environment with different uncertainty states. One can see that by juxtaposing regression results (1) through (5): aggregate issuance and payout can both shrink on positive uncertainty shocks simultaneously. This challenges the conventional view that payout is negative issuance.

The last column describes the response of cash holdings (cash-to-asset ratios) to uncertainty shocks. Switching from a low to high uncertainty state, US public firms see a 0.67% increase in cash holdings. This is consistent with Gao and Grinstein (2014), which shows that an increase in realized aggregate volatility induces firms to save more.

Overall, both evidence in conditional moments and panel data regressions suggest that time-varying aggregate uncertainty has significant effects on firm financing and investment activities. High uncertainty depresses equity issuance, payout, and investment simultaneously. Further evidence using continuous VIX confirms these results and is presented in Table A.4 in the Appendix. However, no theory has taken account of this comovement of financing and investment policies. The mechanism of this observed response is not investigated, and the potential impacts are not to use net issuance to infer aggregate financing costs or activities.
quantified. Therefore, I propose a dynamic model and structurally estimate it in the following sections.

3 A Model with Financial Frictions and Uncertainty Shocks

This section develops a discrete-time dynamic model of a representative firm. I add three new ingredients onto an otherwise standard \( q \)-theory model of investment: (1) two regime-switching states with different volatilities; (2) financial frictions, which consist of both the external financing costs and the cash-carrying costs; and (3) risk premia for productivity risk and regime-switching risk. This model provides a framework for analyzing firm financing and investment under uncertainty shocks. I first describe a representative firm’s production technology and aggregate uncertainty states. I then present its balance sheet and statement of cash flows. Financial frictions and the pricing of risk are next. Lastly, I put them all together and describe the firm’s objective function.

3.1 Production technology and uncertainty states

The representative firm generates operating revenue in terms of final product by combining technology and productive capital stock,

\[
\Pi = ZK^\alpha.
\]

Here \( \Pi \) denotes the operating revenue, \( Z \) is technology or productivity of capital.\(^{21}\) \( K \) is capital stock. \( \alpha \) captures the curvature of production function.

The logarithm of productivity \( Z \) follows a random walk with drift and time-varying volatilities,

\[
\log Z' = \log Z + \mu_s - \frac{1}{2} \sigma_s^2 + \sigma_s \varepsilon'.
\]  

In the above equation and throughout the whole paper, the superscript \( t \) is used to denote the next period variables. Here, subscript \( s \) indicates the uncertainty states: \( s = H \) denotes high uncertainty state and \( s = L \) means low uncertainty state. \( \mu_s \) is the conditional productivity growth rate. \( \sigma_s \) is the state-dependent total volatility of productivity growth, \( \sigma_H > \sigma_L \). \( \varepsilon' \) denotes the total shocks to firm productivity growth and is assumed to be identically and independently distributed with normal distribution over time. It can have arbitrary correlation with the aggregate cash flow shocks \( \varepsilon'_a \). The Jensen correction term, \(-\frac{1}{2} \sigma_s^2\), keeps the mean of marginal product of capital \( \alpha ZK^{\alpha-1} \) invariant to the levels of uncertainty except the conditional growth rate \( \mu_s \).

\(^{21}\)In a more general sense, it can incorporate both the supply side shocks (like productivity shocks) and demand side shocks, as in Bloom (2009) and Bloom et al. (2014). In this case, a more appropriate name is business condition. But technology is a more familiar term in the literature.
The uncertainty states \( s \in \{L, H\} \) are the first new ingredient of my model. They follow a two-state Markov regime-switching process. The transition probabilities are,

\[
P \equiv \begin{bmatrix} \Pr (s'|s) \end{bmatrix}_{s',s \in \{L,H\}} = \begin{bmatrix} p_L & 1 - p_L \\ 1 - p_H & p_H \end{bmatrix}.
\]  

(2)

Here \( p_s \) denotes the probability of staying in the same state starting from state \( s \). As in Bloom (2009) and Bloom et al. (2014), I assume that both aggregate and idiosyncratic uncertainty rise and fall at the same time, so a single regime-switching process determines both the aggregate and idiosyncratic regimes.

This two-state Markov process is a simple but flexible way to model a time-varying uncertainties. On the one hand, one can interpret these regime-switching states as jumps in volatilities. This interpretation is consistent with Bloom (2009), who finds strong evidence that stock-market volatility has jumps.\(^{22}\) A typical example is the Lehman Brother bankruptcy raised the value of VIX index from around 20 in August to 60-80 in October 2008. On the other hand, this method also accommodates other stationary processes like autoregressive processes, by the approximation technique introduced by Tauchen (1986). For example, Eisfeldt and Muir (2014) use two-state Markov chain to capture an autoregressive process of linear financing costs. In the context of uncertainty shocks, both jump and autoregressive components exist and I use this simple specification to capture both. Nonetheless, it is flexible to extend the two-state process to include more states to capture more complicated dynamics.

### 3.2 Balance sheet

My model relates tightly to the actual accounting system of a firm. Table 4 presents a representative firm’s balance sheet in book values. On the asset side of the balance sheet, a firm has two types of assets, cash \( W \) and capital stock \( K \). The differences between these two assets are their liquidity and return. Cash is liquid as it can be freely converted from/to final products and readily distributed to shareholders. Capital stock is illiquid, because it demands an adjustment costs to convert to/from final products, which are also called technological irreversibilities. Cash earns a constant return \( r_W \) and capital stock generates real output per quarter.

The evolutions of cash and capital stock are as follows. Cash is accumulated through the net cash flows generated by the firm. The latter can be classified as cash flows from operating, investing,
Table 4: Model Balance Sheet - Book Value

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash ( W )</td>
<td>Equity ( W + K )</td>
</tr>
<tr>
<td>Capital stock ( K )</td>
<td></td>
</tr>
</tbody>
</table>

and financing activities, which will be detailed in the statement of cash flows. The cash evolution is

\[ W' - W = CF_{OP} + CF_{INV} + CF_{FIN}. \] (3)

Capital stock depreciates over time and is replenished by new investment. It evolves according to

\[ K' = (1 - \delta) K + I. \] (4)

Here \( \delta \geq 0 \) is the depreciation rate of capital. \( I \) is gross investment rate of capital.

On the liability side of the balance sheet, a firm has book equity \( W + K \). A firm in my model is not allowed to finance through debt. This assumption not only simplifies my model and but also narrows the focus on firm financing decisions on the equity issuance and payout around uncertainty shocks. The latter is the emphasis of this paper and is relatively under-explored compared to capital structure decisions.\(^\text{23}\)

### 3.3 Statement of cash flows

As in a standard statement of cash flows, a firm’s cash flows are classified into three sources, net cash flows from operating, investing, and financing. They are discussed separately in the following paragraphs.

**Cash flows from operating activities** Cash flows from operating activities after tax are assumed to be

\[ CF_{OP} = ZK^{1-\alpha} + r_W W. \] (5)

This is the sum of after-tax profits from production and return on cash. Here, \( \tau \) is tax rate. \( r_W \) is return on cash.

\(^\text{23}\)This assumption narrows my model application as it cannot explain what happen to capital structure around uncertainty shocks. Readers interested in capital structure decisions around uncertainty shocks can refer to McQuade (2014) and Chen, Wang, and Zhou (2014).
**Cash flows from investing activities** Cash flows from investing activities are a standard one from q-theory model, It has a fixed cost component and a quadratic cost component,

\[ CF_{INV} = -\left( i + 1_{i \neq 0} \xi_0 + \frac{\xi_2}{2} i^2 \right) K. \] (6)

Here, \( i \) is the investment-to-capital ratio. \( \xi_0 \) is the fixed cost parameter, which is paid when investment is not zero. \( \xi_2 > 0 \) is the convex adjustment cost parameter of investment. All those costs are assumed to be proportional to \( K \) for tractability.\(^24\)

**Cash flows from financing activities** Cash flows from financing activities is the firm’s net issuance \( F \) minus payout \( D \),

\[ CF_{FIN} = F - D. \] (7)

Payout \( D \) here includes both dividend payment and repurchase, as my model does not distinguish between them. The \( CF_{FIN} \) itself will be the net issuance measure commonly used in the empirical literature. A positive number of net issuance means a firm is issuing new equities; while a negative number indicates payout to the existing shareholders, either through dividends or stock repurchases. In my model, a firm never pays out and issues simultaneously.

### 3.4 Financial frictions

The financial frictions in this economy consist of two parts. One is the external financing costs, which is imposed on existing shareholders when raising new equities; the other is cash-carrying costs.

When a firm issues new equities, i.e, \( F > 0 \), the existing shareholders have to give up stake in book value of \( F \) to the new shareholders. In addition to that, the issuance itself imposes costs to the existing shareholders. Such financing costs in reality are captured by underwriting fees and adverse response of share prices after new issuance announcement, with the latter being a majority part. I assume that the financing costs are\(^25\)

\[ 1_{\{F > 0\}} (\lambda_0 K + \lambda_1 s F) \]

\(^24\)I also try including a fire-sale discount in the adjustment cost structure. It does not affect the estimation results.

\(^25\)The financing costs are directly imposed on the value of the existing shareholders, instead of a subtraction from the financing cash flows. The former is adopted in Riddick and Whited (2009), Bolton, Chen, and Wang (2011), Bolton, Chen, and Wang (2013), and Eisfeldt and Muir (2014); while the latter is used in Warusawitharana and Whited (2014). I choose this structure of financing costs because the indirect costs, consisting of the negative response of the share prices following the issuance announcement, are much more substantial for the existing shareholders than the direct costs, which are gross spread, or underwriting fees.
Here $\lambda_0 > 0$ are the fixed cost parameters of equity issuance; and $\lambda_{1s} > 0$ are the linear financing cost parameters. $s$ here denotes the uncertainty states. I allow the linear financing costs to be dependent on the aggregate uncertainty state. This is motivated by the observation that during high uncertainty states, financing costs tend to be high. For empirical evidence of this dependency, please refer to Appendix A6.

The cash-carrying costs capture the idea that cash has a lower return than risk-free rate, i.e.,

$$r_W < r.$$  

Because of this lower return of cash, a firm will have incentive to distribute its cash back to the shareholders when the cash balance is sufficiently high. There are three observations that motivate the carry costs of cash. The first are tax disadvantages on interest earned on cash, which reduce the interest earned on cash by some tax rate, usually lower than the tax rate on firm revenues (See Riddick and Whited (2009)). The second one are the convenience yield, a wedges between return on the cash holdings and the cost of capital, motivated by Azar, Kagy, and Schmalz (2014). Finally, holding cash may have agency costs. Such agency costs include moral hazard problems, like free cash flow pointed out by Jensen (1986), or the adverse selection problems identified by Myers and Majluf (1984). All these suggest that cash inside firm has an inferior return than risk-free assets.

### 3.5 The pricing of risk

The pricing of risk in this economy is captured by an exogenously given stochastic discount factor (SDF hereafter). The SDF has the following structure

$$M_{s,s'} = \exp \left( -\ln (1 + r_s) - \frac{1}{2} \eta^2 - \eta \epsilon_a' - \Gamma_{s,s'} \right).$$  

(8)

where $\eta$ is the constant market price of risk. $\epsilon_a'$ is the same aggregate cash flow shock as in the business condition equation. These terms are standard in many asset pricing models including Bansal and Yaron (2004). $\Gamma_{s,s'}$ is the price of regime shift risk, with the following properties$^{26}$:

$$\Gamma_{LL}, \Gamma_{HL} \geq 0,$$

$$\Gamma_{LH}, \Gamma_{HH} \leq 0.$$  

These assumptions basically say that agents in this economy dislike the high uncertainty state $H$, so marginal value of 1 dollar in the low uncertainty state $L$ is higher than that in the $H$ state,

$^{26}\Gamma$ is also demeaned so that conditional expectation of the SDF is $\frac{1}{1 + r_s}$.  

17
whichever their current state is. For example, if one expects the economy will switch from $L$ to $H$, one dollar now is worth $\exp(-\Gamma_{LH}) > 1$ in the future. Such specification is consistent with Dai, Singleton, and Yang (2007) and Chen (2010).

The addition of variance risk premium is new to the uncertainty shock literature in Macroeconomics. The inclusion of the variance risk premium is based on two consideration. First of all, this paper is about an aggregate state variable, uncertainty shocks, which has to be shown to affect both aggregate investment and employment separately from the aggregate cash flow channel, as in Bloom (2009) and Bloom et al. (2014). This implies that the uncertainty shocks are likely to affect aggregate consumption and thus the marginal utility of consumption. So aggregate uncertainty shocks are priced. Second, Bekaert and Hoerova (2014) document that variance risk premium consists of a significant portion of the value of VIX. Therefore, to fully understand the impact of change in VIX to firm financing and investment decisions, variance risk premium cannot be assumed away. For more details of this motivation see Appendix A5.

### 3.6 A Firm’s Problem and Simplification

An all equity firm maximizes the risk-adjusted present value for existing shareholders. The recursive form of this problem can be written as

$$V(K, W, Z, s) = \max_{K', W'} \left\{ -F - 1_{\{F > 0\}} (\lambda_0 s K + \lambda_1 s F) + \mathbb{E}_{Z', s'} | Z, s [M_{s'} | s V(K', W', Z', s')] \right\}.$$  \hspace{1cm} (9)

This equation simply says that given the current states $(K, Z, W, s)$, a firm chooses optimal capital and cash for the next period, $K'$ and $W'$, respectively, to maximize the sum of the current period cash flows and the discounted and risk-adjusted next period firm value.

The whole problem can be simplified using the constant-return-to-scale property. I can reduce the number of state variables by dividing $K$ to the whole problem. I denote the sales-to-capital stock ratio by $z \equiv \Pi / K$. The other variables denominated by capital stock $K$ are all denoted by their corresponding lower cases. The problem is thereby simplified to

\textsuperscript{27} For example, variance risk premium is not in Bloom (2009), Bloom, Bond, and Reenen (2007), Bloom et al. (2014), Arellano, Bai, and Kehoe (2011), Christiano, Motto, and Rostagno (2014), and Gilchrist, Sim, and Zakrajsek (2010).
\[ v(w, z, s) = \max_{i, c'} \left\{ -f - 1_{\{f > 0\}} (\lambda_0s + \lambda_1sf) + (1 - \delta + i) \mathbb{E}_{(z', s')|(z, s)} [M_{s, s'} v(w', z', s')] \right\} \]

s.t.
\[
\log z' = \log z - (1 - \alpha) \log (1 - \delta + i) + \left( \mu_s - \frac{1}{2} \sigma_s^2 \right) + \sigma_s \epsilon'
\]
\[
(1 - \delta + i) w' = (1 + r_W) w + z - \left( i + 1_{i \neq 0} \xi_0 + \frac{\xi_2}{2} i^2 \right) + \epsilon
\]
\[
\Pr(s'|s) = P \quad w \geq 0
\]

Further simplification of this problem under risk-neutral measure and proof of the existence of solution are in Appendix B1.

Finally, this model assumes no firm entry or exit, primarily for tractability reason. This may underestimate or overestimate the impact of uncertainty shocks. On the one hand, entry is usually costly, in both pecuniary and non pecuniary sense For example, going public costs a firm some substantial underwriting fee and forces it to disclose its financial and operational information. Therefore, allowing entry adds another level of real options, which will enhance real options effects from financial and technological frictions. On the other hand, firms going entry or exit usually have more extreme performance than the existing incumbents. And according to the real option logic, the extreme cases are more likely to happen in the low uncertainty state, when entry and exit are more frequent. As a result, one may observe more extreme cases in the low uncertainty state. If one does not take this into account, he/she might find the realized volatility level in the low uncertainty state even higher than that in the high uncertainty state.  

4 Solution and Estimation of the Model

This section describes the solution method and the structural estimation results. I first explain the solution and estimation method, then discuss the identification strategy. Finally, I present the estimation results. The resulted parameters are necessary to derive model implications in Section 3 and the counterfactual experiments in next section.

---

28This is why in Bloom et al. (2014) and Jurado, Ludvigson, and Ng (2015), only the plants and firms with substantial presence in the data periods are included in the sample. For this reason, I only use firms with more than 20 quarters of observations in the sample when I construct the comparable data moments for the structural estimation purpose. However, for the time series and panel data session, I use the full sample except the screening criteria mentioned in the data section.
Table 5: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Risk-free rate</td>
<td>0.007</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Curvature of profits with respect to capital</td>
<td>0.75</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.0371</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Price of market risk</td>
<td>0.24</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Carry cost of cash</td>
<td>0.007</td>
</tr>
<tr>
<td>$p_L$</td>
<td>Physical probability to stay in state $L$</td>
<td>0.9125</td>
</tr>
<tr>
<td>$p_H$</td>
<td>Physical probability to stay in state $H$</td>
<td>0.3580</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Fixed financing cost coefficient</td>
<td>0.0258</td>
</tr>
<tr>
<td>$\lambda_{1L}$</td>
<td>Linear financing cost coefficient in state $L$</td>
<td>0.2427</td>
</tr>
</tbody>
</table>

4.1 Solution and Estimation Method

I estimate most of the model parameters using numerical generalized method of moments (GMM). Some parameters are calibrated outside the structural estimation, because they can either be directly inferred from data or are already well-established in the literature. Table 5.

First are some parameters independent of uncertainty states. The risk-free rate $r$ is 0.7% quarterly, which is the average 3-month T-bill rate during the sample period. I set the curvature of production function $\alpha$ to 0.75, which is the same as Bloom (2009). Quarterly depreciation rate $\delta$ is 3.71%, which is the average depreciation rate (DP/PPEGT) in the sample. The market price of risk $\eta$ is 0.25 quarterly or 0.5 annually, which is about the Sharpe ratio of market returns in the United States during my sample period. The carry cost of cash, $\gamma$, is assumed to be the same as the risk-free rate. That is, a firm does not earn return on the cash it saves. When I structurally estimate $\gamma$, it has a corner solution that is the same as the risk-free rate.

The transition probabilities in the low and high uncertainty states are estimated from data using Bayesian estimation. The physical probabilities to stay in the same state from States $L$ and $H$ are, respectively, $p_L = 0.9125$ and $p_H = 0.3580$. The first number means that if current quarter is in low uncertainty state, there are probability of 0.91 that next quarter the economy will stay in the low uncertainty state. Interpretation for the latter is similar. For more details about the Bayesian estimation of the transition probabilities, please refer to Appendix A2.

The empirical average of the fixed and linear financing costs in the low uncertainty state are $\lambda_0 = 0.0258$ and $\lambda_{1L} = 0.2427$, respectively, estimated from the data. These financing costs seem to be high. But they reflect not only the underwriting costs but also the dilution costs imposed on the
market value of existing shares of common stocks.\footnote{For example, a firm has 100 existing shares, market price is 1.5 per share. If it issues 20 new shares, my current financing cost structure means that the new market value per share is now 1.45. The dilution cost is about 3.25%, which is consistent with the magnitude of dilution costs.} According to Bolton, Chen, and Wang (2013) and Eisfeldt and Muir (2014), I assume the fixed cost of financing is fixed but allow the linear financing costs to be state dependent. For more details about magnitudes of the financing cost parameters and the correlation between the uncertainties and financing costs, please see Appendix A6.

After calibration, 9 parameters are left to be estimated: the expected volatilities of productivities in the two states, $\sigma_s$; the state-dependent mean productivity growth, $\mu_s$; the fixed and convex adjustment cost coefficients of investment, $\xi_0$ and $\xi_2$; the change in linear financing cost coefficients from low to high uncertainty state, $\Delta\lambda_1$; and the prices of regime shift risk, $\Gamma_{LH}$ and $\Gamma_{HL}$.\footnote{There are totally four prices of regime shift risks. However, there are two restrictions on those risk prices. The two restrictions are that the two rows of risk-neutral probability transition matrix must sum to 1 in either row. See Dai, Singleton, and Yang (2007) for more details about those restrictions.}

The GMM structural estimation proceeds as follows. I first guess a set of parameters $\theta$, and solve the model numerically using policy function iteration. Then I numerically compute the stationary distribution of the state variables $(z, w, s)$. With this stationary distribution, the model moments can be directly computed without simulated artificial data. Next, GMM will choose the parameters that minimized the weighted difference between the model and data moments. The weighting matrix is the inverse of square of the corresponding moments unconditional on uncertainty states. The economic intuition behind this weighting matrix is that I am minimizing the squared sum of percentage deviations of model moments from corresponding data moments.\footnote{This weighting matrix is chosen over optimal weighting matrix because high uncertainty states have relatively less observations and thus higher standard errors. Using optimal weighting matrix will put too much weight on the low uncertainty states. This weighting matrix is also chosen over identity weighting matrix because scale of moments are very different. Identity weighting matrix tends to overweight moments with large magnitudes.} Appendix B2 provides the details of the procedure and the general idea of this GMM structural estimation.

4.2 Identification

Identification of the parameters relies on choosing moments sensitive to the parameter variations. I choose 16 state-dependent moments to estimate the 7 parameters. Below I describe and rationalize the choice of these 16 moments.

First eight moments are cash flow variables related to financing activities in different uncertainty states, including state-dependent distribution and issuance probabilities, average issuance, and the standard deviation of net issuance.\footnote{Average distribution is not included because the average distribution in data is very low relative to the model. This is also observed in Hennessy and Whited (2005, 2007), and Warusawitharana and Whited (2014). If I want} They are selected because the main interest of this paper is
to understand the financing and investment decisions of firms in different uncertainty states. All these variables are affected by the conditional mean of productivity growth $\mu_s$ and $\sigma_s$.

I then choose the moments related to investment. By matching the mean and standard deviation of investment, I am able to estimate the fixed and convex cost parameters of investment, $\xi_0$ and $\xi_2$. Investment is also informative about the return versus risk tradeoff of the firm. Both the financing and investment variables are also helpful to pin down the price of regime shift risk $\Gamma_{HL}$ because it affects all capital versus cash positions.

The next four moments are the conditional mean and standard deviation of cash-to-asset ratio. These variables inform us about the risk management on the asset side of the balance sheet. They are related to the conditional volatilities $\sigma_s$ and the price of regime shift risks $\Gamma_{HL}$. They are connected because cash holdings are the key of risk management, which respond to both the changing volatilities and risk premia.

### 4.3 Estimation Results

This subsection reports the estimation results of the moments and the parameters. Table 6 compares data moments and the estimated moments from model. It shows that the model fits the data reasonably well. Only four out of sixteen estimated moments are statistically different from the data moments at 5% level. Since the estimated system has many more moments than the parameters to estimate, it means the 9 parameters can produce good fits to the 16 moments of financing and investment.

Comparing the data versus fitted moments, I find that my model tends to have higher issuance probability than the data but lower mean issuance than data in the low uncertainty state. One potential explanation is that the fixed financing cost coefficient in the low uncertainty state is low but the linear financing cost coefficient is too high. Since the two parameters are estimated and fixed outside the estimation, the estimation itself cannot fix this problem. However, other than these two moments, other financing moments are still matched reasonably well as their deviation from the corresponding data moments are not statistically significant from zero. My model has higher standard deviation of cash flows than data. But notice that the standard deviation of investment and cash-to-asset ratio is still below the data ones. Therefore, the estimation automatically chooses a higher volatility of cash flow in order to match the other four standard deviations.

Table 7 reports the parameter estimates of the model. The most important parameters here are the two levels of expected volatilities. They are 0.05 and 0.10 in the low and high uncertainty states, respectively. The low uncertainty state volatility is similar to the unconditional volatility
Table 6: Data and Fitted Moments

$t$-statistic is the test for whether the model moment is significant different from the data moments. SD. is acronym for standard deviation. Cash ratio is cash over assets; other flow variable ratios are denominated by last quarter assets net of cash. In data, all variables are winsorized at its 1th and 99th percentile values each quarter. Only firms survive at least 8 quarters are included. ***, **, and * denotes the corresponding model moment is different from data moment at statistical significance of 1%, 5%, and 10%, respectively.

<table>
<thead>
<tr>
<th>Conditional Moments</th>
<th>State L</th>
<th>State H</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Mean investment</td>
<td>0.0517</td>
<td>0.0481</td>
</tr>
<tr>
<td>SD. investment</td>
<td>0.1063</td>
<td>0.1307</td>
</tr>
<tr>
<td>Issuance probability</td>
<td>0.0150</td>
<td>0.0165</td>
</tr>
<tr>
<td>Mean issuance</td>
<td>0.0099</td>
<td>0.0086</td>
</tr>
<tr>
<td>Distribution probability</td>
<td>0.4020</td>
<td>0.4432</td>
</tr>
<tr>
<td>SD. Net Issuance</td>
<td>0.1613</td>
<td>0.1353</td>
</tr>
<tr>
<td>Mean cash ratio</td>
<td>0.1623</td>
<td>0.1580</td>
</tr>
<tr>
<td>SD. cash ratio</td>
<td>0.1909</td>
<td>0.1616</td>
</tr>
</tbody>
</table>

$\chi^2 = 33.14$

estimated in Hennessy and Whited (2005, 2007), which have annualized volatility around 0.10-0.12, or quarterly volatility of 0.05-0.06. The high uncertainty state volatility is slightly higher than twice of the low uncertainty state one, which is consistent with the relative VIX ratios in the high versus low uncertainty states. It is also consistent with Bloom (2009) and Bloom et al. (2014). Finally, notice that even though the underlying cash flow volatility is high, it does not necessarily transmit to higher standard deviation in the net issuance, investment, or cash. Just as in Table 6, those standard deviations in the high uncertainty state are actually lower than those in the low uncertainty state, both in the data and in the model. This is because a firm endogenously chooses optimal financing and investment policies to smooth them out. This point will become clearer when I discuss stationary distribution of productivities in the next section.

Estimated mean productivity growth are 0.9% and -0.3% in the low and high uncertainty state, respectively. The mean productivity in the low state seems to be higher than the risk-free rate, which is 0.7%. But one has to take into account the capital depreciation, Jensen’s correction term for variance, and risk-adjustment from both market risk and regime-switching risk. For example, after Jensen’s correction, the mean productivity growth is only 0.39%. The mean productivity in the high uncertainty state seems to be low. But it is still much higher than the estimates in Warusawitharana and Whited (2014).

Fixed investment adjustment cost is about 0.0071 in my model. This low fixed investment
Table 7: Estimated Parameters

All parameters are in quarterly frequency, not annualized. ***, **, and * denotes the corresponding parameter is different from zero at statistical significance of 1%, 5%, and 10%, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Coefficient</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_L$</td>
<td>Expected volatility of productivity in state $L$</td>
<td>0.0461</td>
<td>0.0109***</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>Expected volatility of productivity in state $H$</td>
<td>0.1049</td>
<td>0.0336***</td>
</tr>
<tr>
<td>$\mu_L$</td>
<td>Mean of productivity growth, state $L$</td>
<td>0.0086</td>
<td>0.0050*</td>
</tr>
<tr>
<td>$\mu_H$</td>
<td>Mean of productivity growth, state $H$</td>
<td>-0.0033</td>
<td>0.0291</td>
</tr>
<tr>
<td>$\xi_0$</td>
<td>Fixed cost parameter of investment</td>
<td>0.0784</td>
<td>0.0074***</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>Convex cost parameter of investment</td>
<td>3.9332</td>
<td>0.4019***</td>
</tr>
<tr>
<td>$\Gamma_{LH}$</td>
<td>Price of regime shift risk, from state $L$ to $H$</td>
<td>-1.3522</td>
<td>0.9103</td>
</tr>
<tr>
<td>$\Gamma_{HL}$</td>
<td>Price of regime shift risk, from state $H$ to $L$</td>
<td>1.2522</td>
<td>0.3403***</td>
</tr>
<tr>
<td>$\Delta\lambda_1$</td>
<td>Change in linear financing costs</td>
<td>0.0071</td>
<td>0.0078***</td>
</tr>
</tbody>
</table>

Adjustment cost is consistent to the literature, including Cooper and Haltiwanger (2006), Bloom (2009), Riddick and Whited (2009), and Lee (2013). The quadratic investment adjustment cost is high in my model. This high estimate is mainly because with a higher frequency data, the adjustment cost parameters becomes higher. To be specific, quarterly investment is averagely only a quarter of the annual one. To make the aggregation of the convex adjustment costs of capital the same, the quarterly quadratic adjustment cost coefficient must be 4 times of its annual counterpart. For example, an annual quadratic adjustment cost of 1 implies an average quarterly quadratic adjustment cost of 4. In Warusawitharana and Whited (2014), they have the convex adjustment costs of capital with very similar magnitudes to mine.

Risk prices of regime shift from low to high uncertainty are $\Gamma_{LH} = -1.35$ and $\Gamma_{HL} = 1.25$. The former implies that a risk-averse agent perceives that the chance of switching from low to high uncertainty states is 2.86 times more likely to happen than the physical probability. The latter means that a risk-averse agent perceives the risk-adjusted probability (or risk-neutral probability) of switching from high to low uncertainty state is 0.19, much lower than the physical probability of 0.64. Both two suggest that people are seriously averse to the high uncertainty state. Both numbers have their magnitudes close to the switching risk premium in Bolton, Chen, and Wang (2013), which is ln 3 = 1.10.

---

*exp (1.35) − 1 = 2.86.
5 Model Implications

In this section, I present the implications of the model. To illustrate the effects and the mechanism, I first discuss the intuition of real option effects. Then I present the model implication in three perspectives: (1) when to invest or finance, which is the most important and illustrative results of this paper; (2) how much to invest or finance; and (3) stationary distributions of capital and cash. Between the second and third perspectives, I insert a subsection to discuss the separation of issuance and distribution. And I discuss the real impacts of aggregate uncertainties and liquidities at the last subsection. To single out the aggregate uncertainty effects and to better understand the mechanism, in the section I assume that the mean growth rates of productivities and financing costs are fixed to their low uncertainty state levels. That is, the mean growth rates and financing cost parameters are not state-dependent in this section.

5.1 Real Option, Irreversibilities, and Uncertainty Shocks

The key mechanism of this model hinges on the real option effects, as a result of the investment and financing irreversibilities. Real option values respond to change in uncertainties, generating interesting dynamics on corporate investment and financing.

There are three important implications of irreversibilities. First of all, because of irreversibilities, a firm has incentive to hoard resources inside firm. For example, because of investment adjustment costs, capital inside the firm and capital outside are not perfect substitutes. Therefore, inside capital is an important state variable of a firm. In the same sense, financing irreversibilities distinguish cash inside the firm from outside cash. This is why a firm has incentive to hoard cash inside firm. This intuition is consistent with Hennessy and Whited (2005), Riddick and Whited (2009), Bolton, Chen, and Wang (2011) and Nikolov, Schmid, and Steri (2013).\footnote{In Gomes (2001), there is also financing costs. But the financing costs depend on the difference between generated cash flows and desired investment. So cash is not a state variable in his paper.}

Second, in a dynamic setting, irreversibilities induce a real option to wait, or an inaction region. Take equity issuance as an example. Equity issuance is irreversible due to financing costs. A firm therefore tradeoffs whether to issue now or later. If it invests now, it pays the financing costs now can finance its investment. If it invests later, it saves the time value of the financing costs. And more importantly, a firm may find that it not longer needs so many cash or it may need even more. In this sense, the ability to wait is an option and has positive value. For this reason, a firm making issuance decision considers not only the financing cost paid, but also the loss of the option to wait. As a result, a firm will become inactive in issuance for considerable amount of cases. This option to wait is widely considered in the literature, as in McDonald and Siegel (1986), Dixit and Pindyck.
(1994), and Stokey (2008).

It is worth to point out that not all types of irreversibilities create an inaction region. For example, a model with only convex adjustment costs on investment and no financing costs does not have an inaction region on capital. In this case, a firm has smooth marginal value of investment. It can always a small amount of investment, with adjustment costs close to zero. In this sense, there is not a substantial region of zero investment. However, a firm still has the option to invest more or less, to time the investment. In more productive time, a firm invests more; and in less productive scenarios, a firm invests less, waiting for better investment opportunities in the future.

Finally, the value of option changes with time-varying uncertainties. An option has convex payoff over its underlying. So it increases value when the volatilities of underlying become higher. Therefore, when uncertainty is higher, so is the value of option. As a result, a firm averse of losing the more valuable option will postpone the investment or financing activities. It will require higher marginal benefits of action in order to compensate for the higher value of option to wait. This will enlarge the inaction regions.

In sum, irreversibilities in investment and financing create options to wait, which generate interesting dynamics under uncertainty shocks. Because of these real options, a firm’s decisions can be classified into two margins: (1) when to invest or finance; and (2) how much. The following two sections present the model results and their implications.

5.2 When to Invest or Finance in L States

This subsection describes when does a firm finance or invest in different uncertainty states. I represent the policy functions in action or inaction regions in financing or investment activities, which are separated by some boundaries. The boundaries here are the cutoff lines where a firm is indifferent between proactively managing its cash or capital and passively sitting on them. Those regions and boundaries are presented in Figure 2 and 3.

Figure 2 shows the action or inaction regions in the low uncertainty state (L). Panel (2a) plots capital investment. Cash-capital ratio is the horizontal axis while the log earnings-capital ratio is the vertical axis. On the vertical axis, the median log earnings-capital ratio in the L states is normalized to 0. So the numbers on the vertical axis can be interpreted as the log-deviation of earnings-capital ratio from median. The vertical axis measures the log marginal product of capital. Because the marginal product of capital is a monotonic function of productivity $Z$, I will use these two terms interchangeably.

In this chart, the region above the solid blue line is where a firm is actively investing, while
the region below is where a firm is neither investing or disinvesting.\textsuperscript{35} There is an inaction region between investment and disinvestment because of the fixed cost of capital. Fixed cost of capital makes investment or disinvestment an real option, which a firm will only exercise if the marginal benefits are higher than the cost of losing this option. This inaction region is consistent with Dixit and Pindyck (1994) and Stokey (2008).

The line between investing and inaction regions is an investment boundary, where a firm is indifferent between investment or inaction. This line is downward sloping. It means that a firm will invest either because the productivity is high or cash is high. The intuition will be more obvious if one looks at the Euler equation for real investment,\textsuperscript{36}

\begin{equation}
1 + \xi i = \frac{1}{1 + \eta} \frac{EV'}{W}, \text{ if } i \neq 0.
\end{equation}

This equation says that investment (on the left-hand side) is increasing with the marginal value of capital (the right-hand side numerator) but decreasing in the marginal value of cash (the right-hand

\textsuperscript{35}If the marginal product of capital is further lower, a firm may disinvest (when log earnings-capital ratio is about -2). Because in both data and model, the probability of disinvesting is very low, I do not plot it here. The disinvestment boundary is a flat line independent of cash because a firm does not need to hoard cash when disinvestment is optimal than investment or inaction.

\textsuperscript{36}Note that because of fixed costs of investment, this formula only applies when a firm invests or disinvest. It is not valid within the inaction region.
And generally speaking, marginal value of capital is higher when productivity is higher, and marginal value of cash is lower if cash holdings are higher. This explains why investment boundary is downward sloping. It is also consistent with Riddick and Whited (2009), Asvanunt, Broadie, and Sundaresan (2009), and Bolton, Chen, and Wang (2011).

Panel (2b) plots the financing regions and boundaries. The uppermost region is issuance region, the bottom one is distribution region, while the middle one is liquidity inaction. Again, the liquidity inaction region exists because the financing costs make equity issuance a costly real option. A firm will only exercise this costly option if the marginal benefits of issuance are higher than losing the option value of the financing option. On the other hand, because issuance is costly, distributing money also becomes irreversible, because the cost of collecting the same amount of cash back is higher than that of paying it out. So a firm does not immediately distribute even if it does not need cash immediately. In sum, the financing cost creates a wedge, or inaction region, between distribution and issuance.

In Panel (2b) of Figure 2, the upper red line is called the issuance boundary, where a firm is indifferent between issuance and inaction. It is upward sloping. This suggests that a firm is more likely to issue equity if current cash balance is low or current productivity is high. A firm with low cash balance may not have sufficient liquidity to fund its investment and thus has need for issuance. If a firm has high current productivity, it will have larger investment outflows but also higher operating inflows. In the estimated parameters from structural estimation, the effects from investment outflow dominate that from operating inflow. As a result, a firm with higher productivity has a low net inflow and is thus more likely to resort to outside financing.

The lower magenta line is the distribution boundary, where a firm is indifferent between distribution and inaction. Similar to the issuance boundary, it is upward sloping. This implies that a firm is less likely to distribute cash if its cash balance is low or productivity is high. The slope of distribution boundary is flatter than that of issuance boundary, because the irreversibility of distribution is not a direct one, but indirectly coming from the issuance costs.

5.3 When to Invest or Finance in H States

This subsection explores the corporate policies in H states and compares them with L states to interpret the effects of uncertainty shocks. These are the central results of this paper. The action/inaction regions and boundaries in both states are plotted in Figure 3. Solid lines denote the boundaries in the low uncertainty state, and dash lines denote the boundaries in the high uncertainty state. Overall, the increased uncertainty moves the boundaries away from inaction regions, due to the real option effects.

Panel (3a) of Figure 3 plots the investment boundary. The investment boundary in the high
uncertainty state (dash line) is higher than the one in the low state (solid line). This implies that firm now requires a higher productivity level to invest in high uncertainty state. So the investment region is now smaller and inaction region is larger. It can be explained by the real option effects. Investment is an real option because of irreversibility of investment and also financing. This convex real option has higher value when uncertainty is high. As a result, a firm now postpones the exercise of this real option, requiring a higher benefits of investment to compensate the loss of the more valuable investment option. This investment real option effect is consistent with McDonald and Siegel (1986) and Bloom (2009). Sometimes this is also called the wait-and-see option, where a firm chooses to wait longer when uncertainty is high.

Panel (3b) of Figure 3 plots the financing boundaries. Both issuance and distribution boundaries move away from the liquidity inaction regions, suggesting that firms postpone both issuance and distribution. Focus on the issuance first. The high uncertainty issuance boundary (red dash line) is higher than low uncertainty one (red solid line) due to both postponed investment and real option effects of financing. On the one hand, because investment is postponed, the need to issue to fund investment is also postponed. On the other hand, similar to investment costs, financing costs make issuance a real option. As a result, a firm will delay issuance if uncertainty is high, requiring a higher productivity and thus higher marginal benefits to issue. The increased financing costs during high uncertainty state aggravate this real option effect as it means now exercising real option is more costly.
The distribution is also postponed in high uncertainty state. This is evident in the figure as the
distribution boundary in the high uncertainty state (magenta dash line) is lower than that in the
low uncertainty state. The procrastination of distribution when uncertainty is high is an indirect
consequence of the real option effects on issuance. Because issuance means loss of the more valuable
real option and paying higher exercise costs, a firm wants to avoid issuance in the high uncertainty
state. In order to reduce the probability of future issuance, a firm will hoard more cash and delay
distribution. This explains why distribution is also postponed.

Notice that the delay of issuance also feedback to investment. When a firm postpones issuance,
the marginal value of cash is likely higher, conditional on the same productivity and cash. As
a result, a firm may invest less or even not invest. So the real option effects of investment and
financing interact and reinforce each other.

Moreover, discount rates in the high uncertainty state are likely to be higher than in the low
uncertainty state. As a result, a firm will invest less in risky capital and hoard more cash. This also
reinforces the delay of firm investment and financing activities.

The conclusion that firms are more likely to delay issuance and distribution does not mean
that one firm will cut issuance and distribution simultaneously. The model here allows firms to be
heterogeneous on the dimension of productivity and cash ratios. Firms with different state variables
correspond to different points on the figures. Depending on a firm’s current state, one will observe
different reaction of the firm to increased uncertainty. For example, one will observe that a firm
with high productivity and low cash level, i.e., near the issuance boundary, is more likely to cut
issuance. While a firm with low productivity and high cash is more likely to be detected that it
is cutting distribution. But if one aggregates all those heterogeneous firms on productivity and
cash ratio dimensions, one will observed simultaneous reduction on issuance and distribution, as
suggested in the data.

Finally, the conclusion that high uncertainty delays both issuance and distribution challenges the
conventional understanding of the relationship between issuance and distribution. Conventionally,
people think that issuance and distribution are negatively correlated, a firm that is more likely to
issue is less likely to distribute. This is true if a firm moves from one state to another state along
with the change in productivity or cash. But once we also allow the uncertainty state to be time-
varying, issuance and distribution can now be positively correlated, along with changing volatilities.
This phenomenon emphasizes to treat issuance and distribution separately into two accounts, not
netting them into one as the net issuance measure, especially in the context with uncertainty shocks.
The net of two decreasing accounts may blur the total effects of uncertainty shocks on the financing
decisions.
5.4 How Much to Invest or Finance

In this subsection, I look at how much does a firm invest or finance in different uncertainty states. I have described when do firms invest, issue, and distribute in the last subsection, which presents the investment and financing policies in the sense of extensive margin. In the sense of intensive margin, I need to describe the optimal policy functions. The results confirm those of last subsection and present more quantitative information about firm behaviors. The key results are presented in Figure 4 and 5.

Figure 4 plots the case when cash-capital ratio is fixed at 10%. Here I focus on how policies react to productivity and aggregate uncertainty shocks with the cash ratio fixed. Panel 4a shows that in both states investment is increasing in production. However, investment is relatively lower in State H, in both the extensive and intensive margin. In the sense of extensive margin, a firm starts to invest when log earnings-capital ratio is higher than 0.36 from its median in State L. In State H, a firm only starts to invest if log earnings-capital ratio is higher than 0.37. This extensive margin results are consistent with those in Panel 3a of Figure 3. On the other hand, conditioning on the same level of productivity, the high uncertainty state investment is no more than the low uncertainty state investment, sometimes much lower. Therefore, the investment in State H is less than that in State L even in the intensive margin sense.
Panel 4b plots the equity issuance and payout with the same cash-capital ratio. In both states, a firm is paying out cash to existing shareholders when productivity is very low, so net issuance is negative. When log earnings-capital ratio is higher than -0.06, a firm becomes inactive in liquidity in the low uncertainty state. But in State H, a firm enters into inaction region soon, at a lower cutoff value of log earnings-capital ratio, -0.10. This is consistent with the results in last subsection that a firm is less likely to payout in high uncertainty state. It also shows that the payout is lower in State H than in State L.

The right end of Panel 4b shows that the issuance is unambiguously lower in the high uncertainty state than in the low uncertainty state. In the sense of extensive margin, a firm requires a higher marginal product of capital to start to issue in State H. While in the sense of intensive margin, the amount of issuance is always in State H. Again, this is due to the joint effects of investment irreversibility, financing costs, and time-varying discount rates.

These two figures generate cross section implications of firm behaviors under uncertainty shocks. They predict that conditioning on the same level of cash-capital ratio, a firm with high productivity will reduce investment and issuance more after a positive uncertainty shock. While a firm with relatively low productivity will cut distribution more in response to more uncertain environment.

Figure 5 presents policy functions over cash with two different productivity levels. Panel (5a) and (5b) illustrate a case where earnings-capital ratio is 0.088, near the issuance boundaries of a firm. And Panel (5c) and (5d) illustrate the case where earnings-to-capital ratio is 0.25, near the financing boundaries.

In Panel 5a, one can see that investment is weakly increasing in cash-capital ratio. In State L, investment is relatively flat, so the firm investment does not depend on cash holding so much. But in the State H, when cash is too low, a firm chooses not to invest because of the real option effects. These effects are corroborated in Panel 5b. When cash is low, a firm chooses to fund the investment by issuing new equities in State L. While in State H, a firm chooses not to issue and not to invest when cash is low.

The remaining two panels show the case when a firm is near its distribution boundary. In State L, a firm will invest when cash-capital ratio is above 0.063 (Panel 5c blue solid line) and distribute when cash-ratio is more than 0.4 (Panel 5d blue solid line). In contrast, in State H, a firm stays inactive in both investment (Panel 5c red dash line) and distribution (Panel 5d red dash line).

Note that Figure 5 also has rich cross-section implications of firm response to uncertainty shocks with different cash-capital ratio. They predict that, conditioning on the same level of productivity,

\[37\] This policy function is flat along all levels of cash-capital ratio. When cash-capital ratio is high enough, this firm starts to invest more. The flatness here is a product of the discretization of the cash state. But it also tells the reader that investment is more sensitive to productivity than to cash.

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Figure 5: Policy Functions along Cash
a firm with high cash ratio will be more likely to cut distribution but less likely to cut issuance. The investment response to uncertainty shocks is however ambiguous, as it depends on whether a firm finances its investment by issuing new equities (Panel 5a) or by cutting distributions (Panel 5c).

Overall, the policy functions show that positive uncertainty shock not only postpones investment and financing activities, but also lower their magnitudes. They also generate rich cross-section implications of firm behaviors which will be tested later.

5.5 Separation of Issuance and Distribution

In the previous two subsections, I have shown that both issuance and distribution are lower in high uncertainty state. This challenges the conventional view that issuance is negative distribution.

Conventionally, people think that issuance and distribution are two opposite accounts. Since one is inflow to cash and the other is outflow, they seem to be the negative action of the other. Use of net issuance in the literature is consistent with this view. Even though the presence of financial frictions creates a liquidity inaction region between the two actions, they might still seem to be negatively correlated.

Shocks to productivity and shocks to cash balance seem to confirm this negative correlation between issuance and distribution. For example, in Panel 2b, a firm with higher productivity moves upward and becomes more likely to issue but less likely to distribute. Shocks to cash are likewise. A firm with higher cash balance is more likely to distribute but less likely to issue.

This view is violated in the setting of uncertainty shocks. As in Figure 3b, issuance and distribution are postponed in high uncertainty state. Figure 4b also confirms this result. These results suggest a positive correlation of aggregate issuance and distribution around uncertainty shocks, which are also confirmed by data. This positive correlation emphasizes the divorce of issuance and distribution and overturns the conventional belief. It is a result of financial frictions, and made evident by uncertainty shocks.

5.6 Stationary Distribution of Capital and Cash

A final set of results present the stationary distribution of the two state variables, earnings-capital and cash-capital ratios in Figure 6 and 7. They show that the stationary distribution of the state variables can match to those in previous literature or the histogram of sample.

Figure 6 presents the stationary distribution of the logarithm of the earnings-capital ratio (log z). Blue solid and red dash lines denote the densities for the corresponding variables in states L and R.

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38 For example, Eisfeldt and Muir (2014) and Belo, Lin, and Yang (2014) employ net issuance measure to infer aggregate issuance costs. Chen, Wang, and Zhou (2014) use net issuance to describe the equity financing activities under uncertainty shocks.

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H, respectively. The shape of the stationary distribution is similar to the normal distribution. Even though the logarithm of $Z$ follows a random walk, a firm will optimally invest when the productivity is high and disinvest when the productivity is low, trapping the earnings-capital ratio within a narrow band. For this reason, the capital to business condition itself is still well-defined and stationary. And as expected, in the high uncertainty state, the distribution has larger variance and fatter tail. This stationary distribution of productivity has similar shapes to the distribution of establishment-level productivity shown in Bloom et al. (2014).

![Image of stationary distribution of productivity](image)

**Figure 6: Stationary Distribution of Productivity**

Panel 7a of Figure 7 plots the stationary distribution of the cash to capital ratio. In both states, the densities of cash decreases with the cash ratio. The two humps are due to non-linearity of investment and financing options, especially when the investment boundaries intersect the issuance or distribution boundaries. In the high uncertainty state, the distribution of cash-capital ratios has less mass at zero and more on the large values. These results are consistent with the empirical histogram of cash-capital ratio in sample, as presented in Panel 7b. In Bolton, Chen, and Wang (2011), the densities of cash ratio cluster around the distribution boundary, which is very different from the empirical distribution. The improvement on this stationary distribution comes from the fact that firms in my estimation has lower average productivities and are often trapped in low productivities due to persistence.
5.7 The Real Impact of Aggregate Uncertainty Shocks and Liquidities

This subsection investigates how aggregate uncertainty shocks and liquidities affect real investment and output growth. Table 8 reports the results.

In Panel (a), I investigate the investment-to-capital ratios $i$, across different aggregate uncertainty states and over different levels of liquidities. From the first three columns, conditional on the same uncertainty state and cash-to-capital ratio, we can see that, investment is increasing with marginal product of capital, proxied by logarithm of earnings to capital ratio. The investment is high when marginal product of capital is low, and becomes zero when marginal product of capital is low. At extremely low marginal products of capital, which is not reported here, investment can be negative. This means that a firm conducts fire-sale to reduce its capital stock. Across uncertainty states, the investment ratios are lower in state H, because of the real options to wait, the precautionary savings motive, and the risk premia. With similar marginal values of product, the investment ratios across different uncertainty states are not significantly different except near the edge of the investment boundary, like $\log \pi = 0.356$ and $w = 0.05$, and $\log \pi = -0.061$ and $w = 0.25$. This suggests that aggregate uncertainty shocks affect firm investment mainly through the external margin, which is consistent with Bloom (2009).

The effects of cash holdings on real investment are not monotonic. For example, in the highest marginal product to capital levels, the investment ratios at a cash-to-capital ratio of 0.15 are higher
than those in the low or high cash ratios. This non-monotonicity is due to the real options to wait in investment and issuance. Take state L for example and \( \log {\pi} = 0.356 \). When cash ratios are really low, like 0.05, the firm would like to issue new equities and invest immediately. At this moment, the exercises of the investment and issuance options synchronize. However, when cash is slightly higher, the firm is near the issuance boundary but not worth to issue now because of the financial frictions. So the firm chooses not to issue but has to invest less in this case.

In Panel (b) of Table 8. I report the average investment and output growth across all potential states of marginal product of capital. One can infer that the average investment increases with liquidities and decreases with uncertainty states. The difference of investment between two states is widening as liquidities increase. When liquidities increase, the sensitivity of investment to marginal product of capital is higher. Therefore, the difference between the investment ratios in the two uncertainty states is higher in larger cash ratio.

The next row quantifies the average of output growth conditional on different liquidities and uncertainty states. The output growth is computed in the following way. From the model, we have

\[ \mathbb{E} \left( \frac{\Pi'}{\Pi} \right) = \exp (\mu_s) (1 + i - \delta)^a, \]

The first component is the mean growth rate of productivity. This component is assumed to be the same in this exercise for expositional reasons. The second component is related to investment. This component can be interpreted as the reallocation component of the growth rate because it is due to the difference between investment rates. From Table 8, we can infer that the reallocation component of the output growth is large, suggesting that aggregate uncertainties have large real impacts and such impacts vary with the position of liquidities. The higher the liquidity is, the larger the impacts of aggregate uncertainties are on the average output growth.

6 How Different Firms Respond to Uncertainty Shocks

This section describes differential response to uncertainty shocks for heterogeneous firms. Firm heterogeneity is investigated through two state variables in the model, productivity and cash. In either dimension, I contrast the model predictions and the empirical results. Overall, the empirical

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39 But when the cash-to-capital ratios are high enough, the investment ratios at high cash-to-capital ratios become higher than the investment ratios here.

40 The marginal value of cash measures how much a firm’s value changes with an extra dollar of cash inside the firm. The marginal value of cash determines the sensitivity of investment to marginal \( q \) and is generally increasing with cash, as suggested in Bolton, Chen, and Wang (2011). Chen (2014) has empirically estimated the marginal value of cash using public firm data from US.
Table 8: The Real Impact of Aggregate Uncertainty Shocks and Liquidities

\(w\) is cash-to-capital ratio \(W/K\). \(\pi\) is earnings-to-capital ratio, \(\Pi/K\), which captures marginal product of capital. \(\log \pi\) is normalized by the median cash-flow-to-capital ratio at state \(L\). So \(\log \pi = -0.061\) means that the cash-flow-to-capital ratio is about -0.06 percent lower than the median cash-flow-to-capital ratio at state \(L\).

<table>
<thead>
<tr>
<th>(w)</th>
<th>(L)</th>
<th>(H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\log \pi)</td>
<td>0.356</td>
<td>0.2215</td>
</tr>
<tr>
<td>(-0.061)</td>
<td>0 0 0</td>
<td>0 0 0.1830</td>
</tr>
</tbody>
</table>

(a) Investment Ratio \(i\)

(b) Aggregate over \(\pi\)

average(\(i\))  | 0.0256  | 0.0272  | 0.0586  | 0.0251  | 0.0255  | 0.0285  |
average(output growth) | -0.0040 | -0.0028 | 0.0207 | -0.0044 | -0.0041 | -0.0018 |

evidence confirms the model predictions. Especially, the results echo to the model implications that uncertainty shocks affect different firms differently and issuance and distribution are different margins of firm financing behaviors.

6.1 Heterogeneous Productivities

This subsection investigates the impact of uncertainties on firms with different productivities. The cross-sectional implications emerge naturally from the policy functions of my model. For example, Figure 4a predicts that high productivity firms tend to reduce investment and issuance more when uncertainty moves up, but their distributions are less affected. Therefore, I juxtaposes the model regression results and the panel data regressions with respect to the proxies of the state variables. In this subsection, I use last quarter sales to capital ratio as a proxy for productivity.\(^\text{41}\)

Three points need to be clarified before moving on. First, fixed effect regressions are good comparison to the structural estimation results. Fixed effect regressions control for time-invariant firm fixed effects and structural estimation has to assume that firms are the same except their different histories of state variables. Therefore, both share the idea that they are estimating the effects of independent variables on the dependent variable within similar firms. Second, in the fixed effect regressions, even though I include control variables as much as possible,\(^\text{42}\) the regression is

\(^{41}\)I also try alternative proxies for productivities like cash flow. It yields similar results.

\(^{42}\)Control variables in this section are the same as in Section 2. They include lag of cash to non-cash asset ratio, lag of sales to capital ratio, log book size of assets, firm age, lag of book leverage, lag quarterly real GDP growth, dummy of time after 2003 May, lag average Q, time trend, and seasonal dummies.
still not able to conditional on the exact values of other state variables. For example, Figure 9 is conditional on a specific value of the cash state. But real data are not able to do that. In the regression, cash variables and other control variables are included in a linear form. So there might be discrepancy between the model and data regressions. Finally, sales in data may not be a perfect proxy for productivity in the model. As a result, model and data regression results can differ.

Table 9 reports the model and regression results. In Panel (A), I compute the beta of productivity with respect to investment and financing variables across two uncertainty states. The differences in these beta are reported here, denoting the model estimation of the differential response of firms to uncertainty shocks. For investment, the interaction term is negative, suggesting that high productivity firms reduce their investment more than low productivity firms when uncertainty increases. Similarly, negative coefficients on issuance and its probability mean uncertainty shocks have larger impact on high productivity firms. Positive coefficients on distribution and its probability imply that distribution in low productivity firms is more affected by uncertainty shocks. These model regression coefficients are consistent with Figure 4a in Section 5.

Panel (B) and (C) report coefficients of the interaction between uncertainty and last quarter sales in fixed effect regressions. I use both the discrete states and the continuous proxy for uncertainty, logarithm of last period VIX to proxy uncertainty. In both panels, the interaction terms have negative signs. These coefficients are not statistically significant different from zero. However, one is not able to reject the model predictions that the differential impact is negative on investment. In this sense, the data are still aligned with model predictions.

The interaction terms have negative coefficients on issuance and its probabilities, and positive coefficients on distribution and its probabilities in both panels. These results are very economically significant and most of them are statistically significant, especially the financing probability variables. Furthermore, not only that their coefficient signs are consistent with the model predictions, but the ranking of their magnitudes also conforms the model. In the model, the absolute value of the coefficients order, from high to low, as: distribution probability, issuance probability, issuance, distribution, and investment. The empirical coefficients follow almost the same order.

Overall, the empirical results are consistent with the model predictions. They suggest different firms have different responses to uncertainty shocks on those financing activities. These implications corroborate the claim that issuance and distribution are two different margins.

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43For example, in reality, sales also depend on labor input. And labor input also has adjustment costs and is therefore a state variable in a broader model. Such omitted state variable can produce discrepancy between model and data regression results.
Table 9: Cross-Sectional Implications: Productivity

This table juxtaposes the model prediction and the data about the impact of uncertainties on firms with different productivities. In data, sales are used to proxy productivity. Data FE means fixed effect regressions with a full set of control. Control variables include lag of cash to non-cash asset ratio, lag of sales to capital ratio, log book size of assets, firm age, lag of book leverage, lag quarterly real GDP growth, dummy of time after 2003 May, lag average Q, time trend, and seasonal dummies. All regressions are fixed effect regressions with two-way clustered standard errors over firm and quarter level. Standard errors are in round parentheses below the coefficient estimates. All coefficient estimates and standard errors are multiplied by 100. ***, **, and * denote significance level at 1%, 5%, and 10% levels, respectively.

<table>
<thead>
<tr>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(A) Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B) Data FE: Two States</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H\times$Sale$[t-1]$</td>
<td>-0.058</td>
<td>-0.984**</td>
<td>-0.912***</td>
<td>0.042</td>
<td>3.004***</td>
</tr>
<tr>
<td></td>
<td>(1.290)</td>
<td>(0.451)</td>
<td>(0.280)</td>
<td>(0.035)</td>
<td>(1.037)</td>
</tr>
<tr>
<td>(C) Data FE: Continuous VIX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>logVIX$[t-1]\times$Sale$[t-1]$</td>
<td>-2.280</td>
<td>-0.302</td>
<td>-2.541*</td>
<td>0.061</td>
<td>10.620**</td>
</tr>
<tr>
<td></td>
<td>(1.835)</td>
<td>(4.154)</td>
<td>(1.516)</td>
<td>(0.233)</td>
<td>(5.025)</td>
</tr>
</tbody>
</table>

6.2 Heterogeneous Cash Balance

This subsection investigates the impacts of uncertainty shocks on firms with different cash ratios. Overall, the cross-sectional results over heterogeneous cash balance are weaker than those across heterogeneous productivities. Table 10 reports the results.

First, in Panel (A), model regressions predict that firms with higher cash balance will reduce investment more but financing activities when facing a positive uncertainty shock. In the first column, investment has negative sensitivity to uncertainty when cash is high. The intuition is as follows. When a firm is near its issuance boundary, investment in lower cash firms will react stronger to uncertainty shocks, as suggested in Figure 5a; when a firm is near its distribution, investment in higher cash firms will have higher sensitivity to uncertainty, as in Figure 5c. As a result, the sign of the interaction term coefficient depends on the stationary distribution. The model results show that a major part of density is between the investment and distribution boundary, so the effects near the distribution boundary dominates. That is, higher cash firms will have higher investment to uncertainty sensitivity. This explains the negative sign of the interaction term in the first column in Panel (A).

The differential impact on issuance can be immediately inferred from Figure 5b. The case for
distribution is more complicated and depends on the underlying motive of cash savings. In the low uncertainty state, cash savings are mainly for investment in the near future; while in the high uncertainty state, cash savings will not be used to fund the investment immediately but for those in the further future. When cash are more likely to fund immediate investment, distribution will be lower. As a result, the sensitivity of distribution to cash savings is lower in the low uncertainty state.\textsuperscript{44}

This explains why distribution and its probabilities have positive coefficients on the interaction terms.

Table 10: Cross-Sectional Implications: Cash

This table juxtaposes the model prediction and the data about the impact of uncertainties on firms with different cash to capital ratios. Data FE means fixed effect regressions with a full set of control as in Table 9. Standard errors are in round parentheses below the coefficient estimates. All coefficient estimates and standard errors are multiplied by 100. ***, **, and * denote significance level at 1%, 5%, and 10% levels, respectively.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H \times \text{Cash}$</td>
<td>-2.897</td>
<td>1.604</td>
<td>2.825</td>
<td>4.357</td>
<td>4.044</td>
</tr>
<tr>
<td>(B) Data FE: Two States</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H \times \text{Cash}_{[t-1]}$</td>
<td>0.091</td>
<td>-0.347</td>
<td>0.012</td>
<td>0.038**</td>
<td>1.126***</td>
</tr>
<tr>
<td></td>
<td>(0.153)</td>
<td>(0.531)</td>
<td>(0.117)</td>
<td>(0.015)</td>
<td>(0.283)</td>
</tr>
<tr>
<td>(C) Data FE: Continuous VIX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\log \text{VIX}<em>{[t-1]} \times \text{Cash}</em>{[t-1]}$</td>
<td>-0.392</td>
<td>-1.044**</td>
<td>-0.136</td>
<td>0.0304</td>
<td>1.467***</td>
</tr>
<tr>
<td></td>
<td>(0.279)</td>
<td>(0.500)</td>
<td>(0.131)</td>
<td>(0.0206)</td>
<td>(0.417)</td>
</tr>
</tbody>
</table>

Panel (B) and (C) report the fixed effect regression results. For investment, the interaction term is slightly positive when cash interacts with high uncertainty state dummy and more substantially negative when cash interacts with continuous log-VIX. But both coefficients are not significant.

For issuance and its probabilities, the results are even worse. The interaction terms have negative coefficients, different from model evidence. But most of them are again not statistically significant. Such failure is probably because the data regressions cannot perfectly conditional on another state variable, productivity. This is further because either sales to capital ratio is an imperfect proxy for productivities or linear terms are not enough to capture the nonlinear effects of productivities. As a result, cash serves as a complimentary proxy for productivity. Since the interaction between

\textsuperscript{44}In fact, the beta of cash to distribution is -0.016 in the low uncertainty state and 0.034 in the high uncertainty state in my model. For distribution probabilities, the beta of cash are -0.698 and -0.624 for low and high uncertainty states, respectively.
uncertainty and productivity has negative sign on issuance as in Table 9, the interaction between uncertainty and cash also has negative sign here.

Finally, the results for distribution and its probability are consistent with the model. Both are positive and economically significant. For distribution probabilities, they are statistically significant at 1%. This corroborates the model results.

7 Counterfactual Experiments

In order to evaluate the importance of the model ingredients and understand the mechanism, I conduct counterfactual experiments. These counterfactual experiments shut down the parameters related to the model ingredients one at a time and contrast these restricted models to the full model.

I here conduct several sets of counterfactual experiments. The first one compares the dynamic model to a static uncertainty model. The second experiment considers a world without financing costs. In the third set of experiments, I remove the state-dependent features on volatility and financing costs, respectively, in order to determine their quantitative importance. Finally, the experiments evaluate the importance of market risk and variance risk. Table 12-14 present the counterfactual experiment results. Panel (A) of these tables list the moments of the benchmark model for the purpose of comparison.

7.1 Why Dynamic Instead of Static Uncertainty?

This counterfactual experiment compares the dynamic model to one with static uncertainty. It answers the question that why dynamic uncertainty model is necessary to explain the reality. Furthermore, this experiment finds that about 6% of cash holdings in the low uncertainty state act as buffer against potential high uncertainty shocks.

In this counterfactual experiment, I assume that the state of the economy does not switch. If an economy is in low uncertainty state, it will stay in this low uncertainty state forever; similarly, an economy with high uncertainty will stay in this high uncertainty state. I keep the parameters the same in either single uncertainty state economy but forbid them to switch to the other state. I then solve the optimal firm policies in these two economies with different uncertainty level, and compare it to the benchmark model. Benchmark of such static uncertainty models includes Riddick and Whited (2009) and Bolton, Chen, and Wang (2011). Overall, my results suggest that static uncertainty models omit the probability of switching state and fail to explain what happen under uncertainty shocks.

Panel (B) of Table 11 reports the results. I first compare the low uncertainty states in static
Table 11: Counterfactual Experiments - Dynamic versus Static States

Benchmark model is the dynamic model I estimated in Section 5. “No Dyn. States” is the case where an economy have constant low or high uncertainty level. L denotes low uncertainty state and H denotes high uncertainty state. All moments in this table have been multiplied by 100 already.

<table>
<thead>
<tr>
<th></th>
<th>(A) Benchmark</th>
<th>(B) No Dyn. States</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Investment</td>
<td>4.73</td>
<td>3.80</td>
</tr>
<tr>
<td>Issuance</td>
<td>0.91</td>
<td>0.41</td>
</tr>
<tr>
<td>Issuance Prob.</td>
<td>1.76</td>
<td>0.99</td>
</tr>
<tr>
<td>Payout</td>
<td>6.28</td>
<td>6.09</td>
</tr>
<tr>
<td>Payout Prob.</td>
<td>42.69</td>
<td>37.52</td>
</tr>
<tr>
<td>Cash Ratio</td>
<td>16.53</td>
<td>17.72</td>
</tr>
</tbody>
</table>

uncertainty economy and in the dynamic model. Because firm in the static economy bears no risk to a bad state with high uncertainty, its expected productivity is high. So a firm invests more and issues more. Also, because its expected uncertainty is low, it will hoard less cash and distribute more to its equity holders. The extra cash ratio hoards in the state L in the benchmark model relative to the static one is about 2.5% of capital, or about 15% of cash. This can be considered as an excess precautionary savings due to the uncertainty shocks.

The difference between the static and dynamic model is more evident when uncertainty is high. In the high uncertainty state with static uncertainty, capital investment is not attractive. Because the economy has low productivity and high uncertainty. Even worse, it will never switch back to a profitable state. Therefore, a firm has no incentive to invest and save in the static H state. It has zero investment and cash holdings, and it pays dividends to the equity holders with probability 1.

This experiment illustrates the key difference between dynamic versus static uncertainty models. In dynamic model, a firm expects what can happen in the future and rationally prepares for potential good or bad shocks. A static model ignores such possibility and therefore fails to explain firm behaviors. In this case, rational expectation plays a key role in dynamic model and produces consistent results to the reality, as suggested by Lucas (1976).

7.2 A World without Financial Frictions

In this subsection, I conduct a counterfactual experiment in a world without financing costs. This helps one to understand why a firm needs to hoard cash and how financial frictions insert a liquidity inaction region between payout and issuance. Without financial frictions, positive uncertainty shocks
are not possible to depress payout and issuance simultaneously.

In this experiment, I retain all estimated parameter values from the estimated one, except that all financing cost to zero. In mathematics notation, it is

$$\lambda_{0}^{NoFriction} = \lambda_{1,s}^{NoFriction} = 0, \ s \in \{L, H\}.$$ 

I then solve this restricted model and compare it to the benchmark model. The results are presented in both Figure 8 and Panel (B) of Table 12.

Figure 8 plots the investment boundaries. Three points emerge when one compares it to Figure 3a. First, investment and disinvestment boundaries in both states are independent on cash ratios, shown as flat lines in the figure. Because a firm can now issue equities to raise cash without additional cost, it does not need to hoard cash inside and investment does not depend on internal cash savings. Second, the productivity thresholds for investment are higher than the asymptotic limit in the benchmark model due to the carry-cost of cash. Third, a positive uncertainty shock shifts the investment boundary up and shifts the disinvestment boundary down, enlarging the inaction region on capital. This is purely due to real option effects on investment. Fixed costs of investment make investment a real option and option value increases when uncertainty is high. As a result, a firm postpones investment and disinvestment.

Panel (B) of Table 12 presents the moment values and provides more information of this experiment. Four important observations stand out. First of all, investment in high uncertainty state is still higher than in low uncertainty state, but much higher than that in the benchmark model. This
Table 12: Counterfactual Experiments - If Financing is Costless

Benchmark model is the dynamic model I estimated in Section 5. No Fin. Cost denotes the case where I assume financing costs are zero when a firm raises outside equity. All moments in this table have been multiplied by 100 already.

<table>
<thead>
<tr>
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<th>(A) Benchmark</th>
<th></th>
<th>(B) No Fin. Cost</th>
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<tr>
<td></td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Investment</td>
<td>4.73</td>
<td>3.80</td>
<td>5.03</td>
<td>1.65</td>
</tr>
<tr>
<td>Issuance</td>
<td>0.91</td>
<td>0.41</td>
<td>8.20</td>
<td>2.64</td>
</tr>
<tr>
<td>Issuance Prob.</td>
<td>1.76</td>
<td>0.99</td>
<td>12.46</td>
<td>4.09</td>
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<tr>
<td>Payout</td>
<td>6.28</td>
<td>6.09</td>
<td>10.74</td>
<td>11.62</td>
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<tr>
<td>Payout Prob.</td>
<td>42.69</td>
<td>37.52</td>
<td>87.54</td>
<td>95.91</td>
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<tr>
<td>Cash Ratio</td>
<td>16.53</td>
<td>17.72</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

suggests that financial friction matters for real investment, and it is particularly important when uncertainty is high. Second, the sum of payout and issuance probability is always 1 in this economy. So there is no liquidity inaction region. Third, a firm has zero cash ratio in the frictionless economy because they can freely raise cash outside and inside cash earns lower return than the risk-free rate. Finally, the low uncertainty state investment in this economy without financing costs is lower than that in the benchmark economy. This is because the firm does not need to hoard cash and pays much more money to the shareholders.

7.3 Uncertainty versus Financing Cost Shocks

The following two counterfactual experiments investigate the importance of specific shocks. This paper investigates the effects of uncertainty shocks. But financing cost shocks seem to be positively correlated with uncertainty shocks and it can also depress investment, issuance, and payout simultaneously, as suggested in Bolton, Chen, and Wang (2013). Therefore, the comovement can be due to uncertainty shocks, financing cost shocks, or both. This subsection removes one shock at a time and compares them to the benchmark model.

The counterfactual experiments proceed as follow. For the no volatility shock case, I assume that the level of volatility in the high uncertainty state is the same as the one in the low uncertainty state,\(^{45}\) i.e.,

\[
\sigma_{j,H}^{\text{NoVolShock}} = \sigma_{j,L}^{\text{NoVolShock}} = \sigma_{j,L},
\]

\(^{45}\)In this sense, the name “high uncertainty state” is abused here. But I just want to keep the names consistent for the ease of comparison.
Table 13: Counterfactual Experiments - Uncertainty versus Financing Cost Shocks

Benchmark model is the dynamic model I estimated in Section 5. No Vol. Shock is the case where the cash flow volatilities stay at the low level, keeping other parameters unchanged. No Fin. Shock is the case a firm has constant financing costs as that in the low uncertainty state, other parameters unchanged. L denotes low uncertainty state and H denotes high uncertainty state. All moments in this table have been multiplied by 100 already.

<table>
<thead>
<tr>
<th></th>
<th>(A) Benchmark</th>
<th>(B) No Vol. Shock</th>
<th>(C) No Fin. Shock</th>
<th>(D) No Drift Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Investment</td>
<td>4.73</td>
<td>3.80</td>
<td>4.97</td>
<td>4.19</td>
</tr>
<tr>
<td>Issuance</td>
<td>0.91</td>
<td>0.41</td>
<td>0.95</td>
<td>0.30</td>
</tr>
<tr>
<td>Issuance Prob.</td>
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<td>0.99</td>
<td>1.83</td>
<td>0.52</td>
</tr>
<tr>
<td>Payout</td>
<td>6.28</td>
<td>6.09</td>
<td>4.39</td>
<td>5.33</td>
</tr>
<tr>
<td>Payout Prob.</td>
<td>42.69</td>
<td>37.52</td>
<td>34.47</td>
<td>42.74</td>
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<tr>
<td>Cash Ratio</td>
<td>16.53</td>
<td>17.72</td>
<td>17.91</td>
<td>17.67</td>
</tr>
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</table>

where $j$ denotes either aggregate or idiosyncratic shocks. At the same time, I am keeping other parameters unchanged.

Panel (B) of Table 13 reports the results of the counterfactual experiment without volatility shocks. Three points emerges from the comparison with the Benchmark case in Panel (A). First, investment in this counterfactual case is uniformly higher. And the discrepancy between two states is much narrower in this counterfactual economy. This suggests that without time-varying volatility, the model fails to take into account the real option effects from investment and thus unable to explain the large drop in investment. Second, with volatility level fixed, payout is higher but issuance is lower. So it fails to reproduce the simultaneous reduction in payout and issuance in the data. Third, in this counterfactual case, payout is higher, issuance and cash ratios are lower than the benchmark case in both low and high uncertainty states. This suggests that in a world with volatility shocks, a firm has additional incentive to save more liquidity than the case without volatility shock, to insure itself against the potential high volatility. This accounts for about 10% of the average cash holdings in both states.

The next experiment assumes constant financing costs in both uncertainty states. In this case, I force the high uncertainty state financing costs to be the same as the low uncertainty state one while keeping other parameters fixed. The new specification is

$$\Delta \lambda_0^{No\text{FinShock}} = \Delta \lambda_1^{No\text{FinShock}} = 0 .$$

Panel (C) of Table 13 reports the results of the experiment without financing cost shocks. By
juxtaposing these results with the benchmark and the no volatility shock cases, I find that both average payout and issuance increase. Graphs shows that a firm still postpones payout and issuance decisions, conditional on the same level of productivity and cash ratios. But this effect is subsumed by more disperse payout of firm states, which makes a firm more likely to reach extreme states on both ends of payout and issuance.

The next noticeable result is about investment. Without financing cost shocks, investment still drops in high uncertainty state. But the magnitude of this drop is larger than that in the no volatility shock case. This suggests that volatility shocks have relatively larger impact than financing cost shocks.

In sum, this subsection shows that (1) combination of volatility and financing cost shocks are indispensable to explain the simultaneous reduction in payout and issuance following positive uncertainty shocks; (2) volatility shocks have more significant impacts on real investment, while financing cost shocks have more important impacts on financing activities; (3) firms have extra precautionary saving motive against the uncertainty shocks, accounting for about 10% of average cash holdings.

7.4 How Important Are Risk Premia?

I then investigate the impacts of risk premia in three counterfactual experiments. In the first experiment, I assume market risk is not priced, naming it “no market risk”. This is achieved by assuming that the price parameter associated to the market risk is zero, or

\[ \eta_{\text{NoMarketRisk}} = 0. \]

In the second experiment, I assume the risk prices of state switching are zero, i.e.,

\[ \Gamma_{s,s'}^{\text{NoSwitchingRisk}} = 0, \quad s, s' \in \{L, H\}. \]

Finally, I look at the case where both risks are not priced.

Table 14 presents the counterfactual experiments which remove the risk premia. Overall, one can see the risk premia have less substantial impacts on firm investment but relatively large impact on financing, especially on cash holdings. If market risk is not priced, cash ratios will be higher than the benchmark case because capital has higher risk-adjusted returns and firms are willing to hold more cash to finance potential investment opportunities. If the switching risk is not priced, cash ratios will be much higher than the benchmark case. Because a firm now perceives lower probability of the bad state, the H state, to happen. As a result, it hoards more cash in both states in order to invest more. Finally, in the case where both risks are not priced, cash ratios are also lower,
suggesting that the effects from the risk price of state switching dominates those from market risk premium.

Table 14: Counterfactual Experiments: Roles of Risk Premia

Benchmark model is the dynamic model I estimated in Section 5. No Market Risk case assumes that the risk price for market cash flow risk is zero. No Switching Risk is the case where the risk price of switching states is zero. No Both Risk case assumes that both risks are not priced. All moments in this table have been multiplied by 100 already.

<table>
<thead>
<tr>
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<th>(A) Benchmark</th>
<th>(B) No Market Risk</th>
<th>(C) No Switching Risk</th>
<th>(D) No Both Risks</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>H</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Investment</td>
<td>4.73</td>
<td>3.80</td>
<td>4.70</td>
<td>4.02</td>
</tr>
<tr>
<td>Issuance</td>
<td>0.91</td>
<td>0.41</td>
<td>0.66</td>
<td>0.49</td>
</tr>
<tr>
<td>Issuance Prob.</td>
<td>1.76</td>
<td>0.99</td>
<td>1.60</td>
<td>1.32</td>
</tr>
<tr>
<td>Payout</td>
<td>6.28</td>
<td>6.09</td>
<td>4.94</td>
<td>8.39</td>
</tr>
<tr>
<td>Payout Prob.</td>
<td>42.69</td>
<td>37.52</td>
<td>39.16</td>
<td>46.73</td>
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<tr>
<td>Cash Ratio</td>
<td>16.53</td>
<td>17.72</td>
<td>17.17</td>
<td>14.72</td>
</tr>
</tbody>
</table>

8 Conclusion

This paper develops a parsimonious dynamic model to investigate the corporate financing and investment behaviors under uncertainty shocks. Four novel results are established. (1) A positive uncertainty shock predicts a substantial lower amount in issuance, distribution, and investment in a following quarter. (2) Such comovement can be explained by a model with financial frictions and investment irreversibility, which generate real option effects and delay both financing and real investment during high uncertainty periods. Time varying risk premia are quantitatively important to match the large impacts. (3) The impact of uncertainty shocks depends on individual firm’s productivity and liquidity. A more liquid or less productive firm is less affected and less likely to depress distribution and investment in high uncertainty episode, but more likely to delay issuance as it is more financially flexible. (4) Comparative statics in constant volatility models cannot explain the observed simultaneous reduction in issuance and distribution.

This paper adopts a reduced form approach on the financing costs because it focuses on the endogenous financing and investment behaviors under uncertainty shocks. The model is not able to derive whether higher financing costs are a result of higher uncertainty or the opposite, or both are the outcome of another source, like asymmetric information. In a more sophisticated model, the level and change of financing costs can endogenously emerge as an equilibrium outcome and
therefore is able to explain the source of the positive correlation between uncertainty shocks and financing costs. I leave this question for future research.
References


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Appendix A: Empirical Results

A1 Definition of Key Variables in Model and Data

This appendix subsection lists the definition of key variables in model and their construction in data in Table A.1.

Table A.1: Definition of Key Variables in Model and Data

This table juxtaposes the definition of key variables in the model and in data. Strings inside round parentheses indicates the acronym of the corresponding accounts in Compustat or whether it is from Security Data Corporation (SDC) Platinum database. All variables are adjusted by inflation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>$W$</td>
<td>Cash and equivalents (CHE)</td>
</tr>
<tr>
<td>Capital</td>
<td>$K$</td>
<td>Total assets (AT) − cash</td>
</tr>
<tr>
<td>Investment</td>
<td>$I$</td>
<td>Capital expenditure (CAPX) − sale of property, plant and equipment (SPPE), + Research and Development expenditures (XRD) + acquisitions (AQC)</td>
</tr>
<tr>
<td>Distribution</td>
<td>$-E1_{(E&lt;0)}$</td>
<td>Dividends payout (DVC) + Repurchase (from SDC)</td>
</tr>
<tr>
<td>Issuance</td>
<td>$E1_{(E&gt;0)}$</td>
<td>Secondary equity offerings (from SDC)</td>
</tr>
<tr>
<td>Net issuance</td>
<td>$E$</td>
<td>Issuance − Distribution</td>
</tr>
<tr>
<td>Cash flow</td>
<td>$CF_{OP}$</td>
<td>Operating income before depreciation (OIBDP) + XRD</td>
</tr>
</tbody>
</table>

A2 Bayesian Estimation of Regime Switching

This appendix subsection describes the Bayesian estimation of regime switching in VIX and reports the results. I first illustrate the model to be estimated. Then I describe the procedure of Forward-Filtering-Backward-Smoothing (FFBS) algorithm, and briefly discuss the estimation results at the end.

In my estimation, I assume that VIX follows a stationary Markov switching process with two regimes. Within each regime, the logarithm of VIX is normally distributed:

$$\log v_{s,t} = \mu^v_s + \sigma^v_s \epsilon^v_t$$

where $v_{s,t}$ is VIX at state $s$ at time $t$, $s$ is low or high uncertainty states \{LU, HU\}. $\mu^v_s$ and $\sigma^v_s$ are the conditional mean and volatility of log-VIX. $\epsilon^v_t$ is a time-$t$ standard normal shock. The Markov transition probability matrix is the same as in the model

$$P \equiv \mathbb{P}(s'|s) = \begin{bmatrix} p_{LU} & 1 - p_{LU} \\ 1 - p_{HU} & p_{HU} \end{bmatrix}.$$
Given a time series of realization of VIX, \( \{v_t\}_{t} \), I have six parameters to estimate, \( \{p_s, \mu_s^v, \sigma_s^v\}_{s \in \{LU, HU\}} \), and also the latent states \( \{s_t\}_{t} \) of the realized VIX.

The estimation of the regime switching is implemented using Bayesian estimation. I define the realized observations of VIX as \( Y \equiv \{v_t\}_{t} \), given some prior on the parameters distribution of \( \theta^v \equiv \left\{ (p_s, \mu_s^v, \sigma_s^v)_{s \in \{LU, HU\}} \right\} \), I want to estimate the posterior distribution \( p(\theta^v|Y) \). To estimate the system, I construct prior as follows: a VIX is considered to be in high uncertainty state if it is higher than its 90th percentile in the whole sample, and in low uncertainty state otherwise. I then use Forward-Filtering-Backward-Smoothing (FFBS) algorithm to estimate the system.\(^{46}\)

The estimation results of parameters are reported in Table A.2. According to Table A.2, the monthly transition probability matrix is

\[
P_{\text{Mon}} = \begin{bmatrix} 0.9576 & 0.0424 \\ 0.3109 & 0.6891 \end{bmatrix}.
\]

This implies that the quarterly transition probability matrix is the monthly one raised to the third,

\[
P_{\text{Qtr}} = (P_{\text{Mon}})^3 = \begin{bmatrix} 0.9125 & 0.0875 \\ 0.6420 & 0.3580 \end{bmatrix}.
\]

These are the numbers I use as the transition probability matrix in physical probability measure in the structural estimation. The steady state distribution for the low and high uncertainty state implied in this transition matrix is

\[
[\pi_{LU}, \pi_{HU}] = [0.8800, 0.1200].
\]

\(^{46}\)For more details, see Titterington, Smith, and Makov (1985).
Figure A.1 and Table A.3 report the estimated latent states of each month. The left panel reports high uncertainty states in my prior, i.e., if it is higher than the 90th percentile. The right panel of the figure reports the posterior estimation of the high uncertainty state. It is considered as a high uncertainty state here if the estimated probability of being in high uncertainty state exceeds 50%. Comparing these two latent states, I find that some isolated spikes are considered as high uncertainty states in the prior but not in the posterior. Two examples are the Asian in November 1997 and Flash Crash of stock market in May 2010. They are not considered as high uncertainty states because the probability of them being in which state depends on their neighborhood states. The fact that their neighborhood are not in the same state reduces their likelihood. For the same reason, some months with relative lower VIX in the middle of high VIX months are considered as in high uncertainty states. Examples of this kind include the months from November 2011 through January 2003, between Corporate Scandal and Gulf War II events.

These figures report the Bayesian estimation results of latent states in monthly VIX, from 1994 January through 2014 September. Prior of this estimation is that a VIX is considered to be in high uncertainty state if it is higher than its 90th percentile in the whole sample, and in low uncertainty state otherwise. Posterior is estimated using Forward-Filtering-Backward-Smoothing (FFBS) algorithm.

Figure A.1: Prior vs. Posterior States of Uncertainties

Finally, note that in both my empirical and model specification, I match the last period end uncertainty state with current period end financial variables. I do it in this way because the uncertainty measure is forward-looking, decisions made by a firm conditional on last period end uncertainty level can only be observed in this period end. For example, November 2011 is considered
Table A.3: High Uncertainty States in Prior and Posterior

Periods during recession are *italicized*.

<table>
<thead>
<tr>
<th>Year</th>
<th>Month</th>
<th>VIX</th>
<th>Prior HU</th>
<th>Posterior HU</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>11</td>
<td>32.206</td>
<td>1</td>
<td>0</td>
<td>Asian Crisis</td>
</tr>
<tr>
<td>1998</td>
<td>8</td>
<td>31.588</td>
<td>1</td>
<td>1</td>
<td>Russian Debt Crisis</td>
</tr>
<tr>
<td>1998</td>
<td>9</td>
<td>38.205</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>10</td>
<td>36.608</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>9</td>
<td>35.065</td>
<td>1</td>
<td>1</td>
<td>9/11</td>
</tr>
<tr>
<td>2001</td>
<td>10</td>
<td>32.721</td>
<td>1</td>
<td>1</td>
<td>Enron Scandal</td>
</tr>
<tr>
<td>2002</td>
<td>7</td>
<td>34.050</td>
<td>1</td>
<td>1</td>
<td>Corporate Scandal</td>
</tr>
<tr>
<td>2002</td>
<td>8</td>
<td>33.743</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>9</td>
<td>37.648</td>
<td>1</td>
<td>1</td>
<td></td>
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<tr>
<td>2002</td>
<td>10</td>
<td>35.243</td>
<td>1</td>
<td>1</td>
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<td>2002</td>
<td>11</td>
<td>28.175</td>
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<tr>
<td>2002</td>
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</tr>
<tr>
<td>2003</td>
<td>1</td>
<td>27.424</td>
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<td></td>
</tr>
<tr>
<td>2003</td>
<td>2</td>
<td>32.218</td>
<td>1</td>
<td>1</td>
<td>Gulf War II</td>
</tr>
<tr>
<td>2003</td>
<td>3</td>
<td>30.634</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>9</td>
<td>30.239</td>
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<td></td>
</tr>
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<td>2008</td>
<td>10</td>
<td>61.177</td>
<td>1</td>
<td>1</td>
<td>Lehman Collapse</td>
</tr>
<tr>
<td>2008</td>
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<td>1</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>12</td>
<td>52.405</td>
<td>1</td>
<td>1</td>
<td></td>
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</tr>
<tr>
<td>2009</td>
<td>2</td>
<td>45.571</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>3</td>
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<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>2009</td>
<td>4</td>
<td>38.064</td>
<td>1</td>
<td>1</td>
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</tr>
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<td>2009</td>
<td>5</td>
<td>31.978</td>
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<td>2009</td>
<td>6</td>
<td>29.140</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>5</td>
<td>31.930</td>
<td>1</td>
<td>0</td>
<td>Flash Crash</td>
</tr>
<tr>
<td>2011</td>
<td>8</td>
<td>35.029</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>9</td>
<td>36.530</td>
<td>1</td>
<td>1</td>
<td>Euro debt crisis</td>
</tr>
<tr>
<td>2011</td>
<td>10</td>
<td>32.829</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>11</td>
<td>31.942</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

58
as a high uncertainty month. This will be matched with the fiscal quarter including the whole month of December 2011, which are quarterly statements disclosed in the fiscal quarter end, i.e., December 2011, January and February 2012. Quarterly statements at the end of November 2011 is not considered a match to November 2011 because a firm observing high uncertainty at the end of November did not have time to adjust its decisions yet. This is also consistent with Gulen and Ion (2015).

### A3 Comovement under Uncertainty Shocks: Evidence from Continuous VIX

In this subsection I present the empirical evidence of the comovement of firm policies under uncertainty shocks using continuous VIX. The results are very similar to the two state results as in

Table A.4 presents the fixed effect regression results with continuous VIX. The first column suggests that an inter-quartile increase of uncertainty, which is about 0.5, reduces equity issuance by 0.17% of non-cash assets. Compared to 1.16% of mean issuance, the impact of uncertainty is more than 10% on issuance, which is substantial. The second column confirms this effect, by looking at the probability of equity issuance. An inter-quartile increase in uncertainty reduces firm issuance probability by 0.15%, both economically and statistically significant.

Next two columns investigate the impact of uncertainty to amount and probability of equity distribution, including both dividend payout and repurchase. A 0.5 unit increase in log-VIX reduces distribution by 0.02% and distribution probability by 0.63%, both are economically and statistically significant when compared to their mean levels.

Column (5) reports the result for net issuance, which is equity issuance net of distribution. Although both issuance and distribution shrink, issuance reduce even more than distribution. As a result, the overall effects of uncertainty shocks is negative and significant. However, these results prompt a caveat on the usage of net issuance to summarize two different financing activities.\footnote{Recent usage of net issuance includes Eisfeldt and Muir (2014) and Belo, Lin, and Yang (2014). Both papers try to use net issuance to infer aggregate financing costs or activities.} Issuance and distribution are two financing decisions at different margins. A firm reduces issuance may not suggest that it is more likely to distribute in an environment with time-varying uncertainties. One has seen aggregate empirical evidence by juxtapose regression results (1) through (5): aggregate issuance and distribution can shrink on positive uncertainty shocks simultaneously. I am going to present a model to discuss this point more.

Column (6) describes the response of cash holdings, cash-to-asset ratio, to uncertainty shocks. An interquartile increase in log-VIX predicts 0.17% increase in next quarter cash holdings. This is
Table A.4: Panel Data Evidence on Continuous VIX

log[VIX(t-1)] is the logarithm of the S&P 500 Volatility Index (VIX) in last three months. Net issuance is equity issuance net of distribution. Control variables include lag of cash to non-cash asset ratio, lag of sales to capital ratio, log book size of assets, firm age, lag of book leverage, lag quarterly real GDP growth, dummy of time after 2003 May, lag average Q, time trend, and seasonal dummies. All regressions are fixed effect regressions with two-way clustered standard errors over firm and quarter level. Standard errors are in round parentheses below the coefficient estimates. All coefficient estimates and standard errors are multiplied by 100. ***, **, and * denote significance level at 1%, 5%, and 10% levels, respectively.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[coef×100] log[VIX(t-1)]</td>
<td>-0.334***</td>
<td>-0.296***</td>
<td>-0.036***</td>
<td>-1.259***</td>
<td>-0.297***</td>
<td>0.341***</td>
<td>-0.389***</td>
</tr>
<tr>
<td></td>
<td>(0.109)</td>
<td>(0.100)</td>
<td>(0.012)</td>
<td>(0.415)</td>
<td>(0.110)</td>
<td>(0.101)</td>
<td>(0.069)</td>
</tr>
<tr>
<td>Control</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R-square</td>
<td>0.004</td>
<td>0.004</td>
<td>0.018</td>
<td>0.023</td>
<td>0.005</td>
<td>0.368</td>
<td>0.135</td>
</tr>
<tr>
<td># Obs</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
<td>228,175</td>
</tr>
<tr>
<td># Firms</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
</tr>
</tbody>
</table>

consistent with Gao and Grinstein (2014), which shows that increase in realized aggregate volatility induces firms to save more.

The last column presents the results for investment. A 0.5 increase in log VIX reduces real investment by 0.19%. This is more than 3% decrease of investment from its mean level of 5.7%. This finding is consistent with Bloom (2009) and Bloom et al. (2014).

A4 Repurchase and Debt Financing

This subsection presents evidence that higher uncertainty also predicts lower activities in equity repurchase, new debt issuance, and debt repurchase. Equity repurchase results confirm that my model predictions also apply to repurchase alone. And results on debt issuance and repurchase suggest that there could also be some debt issuance costs so the real options to wait argument also applies to debt. Results are presented in Table A.5.
Table A.5: Repurchase and Debt Financing under Uncertainty Shocks

(a) Conditional Moments

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>L</th>
<th>H</th>
<th>( t )-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repurchase</td>
<td>0.392%</td>
<td>0.390%</td>
<td>0.340%</td>
<td>-5.01***</td>
</tr>
<tr>
<td>Debt issuance</td>
<td>2.871%</td>
<td>2.914%</td>
<td>2.526%</td>
<td>-8.40***</td>
</tr>
<tr>
<td>Debt repurchase</td>
<td>2.073%</td>
<td>2.069%</td>
<td>2.107%</td>
<td>1.33</td>
</tr>
</tbody>
</table>

(b) Fixed Effect Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[coef\times100]</td>
<td>Repurchase</td>
<td>Debt Iss.</td>
<td>Debt Repur.</td>
</tr>
<tr>
<td>High H</td>
<td>-0.014</td>
<td>-0.409***</td>
<td>-0.067***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.051)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>Control</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R-square</td>
<td>0.010</td>
<td>0.000</td>
<td>0.026</td>
</tr>
<tr>
<td># Obs</td>
<td>230,801</td>
<td>230,801</td>
<td>230,801</td>
</tr>
<tr>
<td># Firms</td>
<td>8,258</td>
<td>8,258</td>
<td>8,258</td>
</tr>
</tbody>
</table>

A5 Variance Risk Premium

Figure A.2 plots the forecasted variance by VIX and the variance premium, the difference between squared VIX and the forecasted variance from 1994 January through 2010 August. Data are provided by Bekaert and Hoerova (2014).

Variance premium is positive most of the time. It is also highly correlated with VIX, with a correlation coefficient of 0.72. In my model specification, this implies that in high uncertainty states, the discount rates should be higher than in low uncertainty state. In the specification of stochastic discount factor, it is equivalent to say that the risk price of switching from low to high uncertainty state is negative. In terms of risk-adjusted probabilities of switching, it says that the risk-adjusted probabilities switching from low to high uncertainty states is higher than the true probabilities.\(^{48}\)

\(^{48}\)See Appendix B1 for more details about risk-adjusted probabilities.
This figure plots the forecasted variance (blue area) and variance premium (red area) for S&P 500 from 1994 January through 2010 August. The sum of the two terms are the square of VIX index. Data are provided by Bekaert and Hoerova (2014). Shaded areas are the NBER recession periods.

Figure A.2: Forecasted Variance and Variance Premium

A6 Relationship between Uncertainty and Financing Cost Shocks

An important result in the structural estimation is that financing costs are increasing when uncertainty is high. Besides the structural approach, this relationship between uncertainty and financing costs can also be explicitly tested. This appendix subsection provides the evidence on the positive relationship between financing costs and aggregate uncertainty. Here I define the key measures of financing costs, present the empirical evidences, and provide interpretation of those results.

My empirical definition of financing costs differ from the previous literature. The costs of Seasoned Equity Offerings (SEO) are typically divided into two categories, direct and indirect costs. As in Gao and Ritter (2010), the direct costs usually refer to gross spread, which is the total fees the issuer paid to the underwriters, and the indirect costs are the underpricing or discount of new issues. However, these costs do not capture the underpricing suffered by the existing shareholders. For this reason, I define my measure of financing costs, from the existing shareholders standpoint,
as

\[ \text{Total costs} \equiv \text{gross spread} - \text{announcement return} \times \text{total equity value before issuance}. \]

To empirically implement this measure of total costs, I obtain its component from different sources. Gross spread and the issuance principal are from Securities Data Company (SDC) Platinum database. I use the cumulative abnormal returns (CAR) from 3 days before through 1 day after announcement as the benchmark announcement return. Using other CARs from alternative event window may change the magnitudes of the results but will not change the statistical significance and the qualitative results. Corresponding, I use the total equity values 3 days before announcement as the equity value before issuance. All measures are denominated by last quarter end total assets, both to remove the size effects and to be consistent with the theoretical model.

Also, for each type of costs, I separate issuing firms into two size groups. Altinkilic and Hansen (2000) find that gross spread behaviors differ in issuing firm qualities, especially firm size. 49 Using the median firm total assets for the full sample (including non-issuing firms) in the last quarter as a cutoff, I define a firm to be large if its last quarter size is above the median size and small otherwise. Although the median firm size is defined based on both issuing and non-issuing firms, the issuance sample is divided relatively even. Using the median firm size for only issuance firms as a cutoff will not alter the results much.

Table A.6 describes how aggregate uncertainties affect the financing costs of SEO. Panel A.6a summarizes the key variables. Several important points can be inferred from the table. (1) Conditional from issuance, the average issuance amount is more than a half of the book assets. (2) Total costs of SEO is substantial, which is about 16% of the book assets. 50 (3) Gross spread is about 3% of the book assets and announcement returns are ranging from 3-5%. (4) Gross spread is dwarfed by the indirect costs from existing shareholder perspective. This can be inferred from comparing gross spread (3%) to the total costs (16%). (5) Small firms have much larger financing costs than large firms, consistent with Altinkilic and Hansen (2000).

Panel A.6b of Table A.6 presents the regression results. I first present the results for total costs, and then divide them into direct and indirect components. Lastly, I report the placebo tests for CAR. Regression (1) and (2) in Panel (b) of Table A.6 report the fixed and linear costs of issuance, and their response to uncertainty shocks. One can see that both the fixed and linear financing costs are economically significant. Most of them are statistically significant except the fixed financing

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49 Altinkilic and Hansen (2000) also find that idiosyncratic volatilities of stock returns are another important determinant of issuance costs. While this paper does not address the idiosyncratic volatility effects, their findings indirectly corroborate the effects of aggregate uncertainty, which has been shown to be positively correlated with idiosyncratic volatilities in Bloom (2009).

50 Firms in my sample have average Tobin’s Q of 1.8. This gives the total financing costs relative to market value of firms are about 8.89%.
Table A.6: How Aggregate Uncertainties Affect the Costs of Seasoned Equity Offerings

Principal, total costs, and gross spread, are denominated by last quarter end total assets. Principal and gross spread is from SDC Platinum database. Cumulative abnormal returns (CAR) compute the abnormal returns (excluding dividends) on issuance date from CAPM model, which uses value-weighted market return as a single factor. The estimation period spans from 246 days to 46 days before the issuance date. Firms are separated into Large and Small depending on whether its last quarter total assets are above the median size in the last quarter. VIX is the logarithm of average index in the past three months before announcement date. HU is equal to one if VIX in last quarter is within its top 5th percentile. VIX, HU, and the interaction terms are demeaned so that the constant term reflects the fixed cost of issuance. Placebo tests use the announcement returns 10 days before or after the actual SEO announcement date. All ratio variables are winsorized at its 1th and 99th percentiles. Heteroskedasticity robust standard errors are reported in round parenthesis. Both coefficients and standard errors are multiplied by 100 in Panel B. ***, **, * denote 1%, 5%, and 10% statistical significance, respectively.

(a) Summary Statistics for Issuance Sample

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>mean</th>
<th>std.dev</th>
<th>median</th>
<th>mean-Large</th>
<th>mean-Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal</td>
<td>5,469</td>
<td>0.5669</td>
<td>1.0427</td>
<td>0.2752</td>
<td>0.2640</td>
<td>0.9120</td>
</tr>
<tr>
<td>Total costs</td>
<td>4,829</td>
<td>0.1644</td>
<td>0.6184</td>
<td>0.0463</td>
<td>0.0642</td>
<td>0.2910</td>
</tr>
<tr>
<td>Gross spread</td>
<td>4,954</td>
<td>0.0306</td>
<td>0.0551</td>
<td>0.0150</td>
<td>0.0120</td>
<td>0.0519</td>
</tr>
<tr>
<td>CAR(-3,+1)</td>
<td>5,330</td>
<td>-0.0387</td>
<td>0.1187</td>
<td>-0.0328</td>
<td>-0.0309</td>
<td>-0.0471</td>
</tr>
<tr>
<td>CAR(-1,+1)</td>
<td>5,330</td>
<td>-0.0291</td>
<td>0.1010</td>
<td>-0.0268</td>
<td>-0.0242</td>
<td>-0.0343</td>
</tr>
<tr>
<td>CAR(-5,+5)</td>
<td>5,330</td>
<td>-0.0418</td>
<td>0.1566</td>
<td>-0.0370</td>
<td>-0.0326</td>
<td>-0.0519</td>
</tr>
<tr>
<td>CAR(-10,+10)</td>
<td>5,330</td>
<td>-0.0462</td>
<td>0.2075</td>
<td>-0.0400</td>
<td>-0.0341</td>
<td>-0.0593</td>
</tr>
</tbody>
</table>

(b) Regression results

<table>
<thead>
<tr>
<th></th>
<th>Total Costs</th>
<th>Gross Spread</th>
<th>Neg. CAR(-3,+1)</th>
<th>Placebo Neg.CAR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Large</td>
<td>(2) Small</td>
<td>(3) Large</td>
<td>(4) Small</td>
</tr>
<tr>
<td>Principal</td>
<td>18.402***</td>
<td>20.511***</td>
<td>4.331***</td>
<td>5.410***</td>
</tr>
<tr>
<td></td>
<td>(5.996)</td>
<td>(3.775)</td>
<td>(0.100)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>VIX</td>
<td>13.609**</td>
<td>19.009***</td>
<td>0.350***</td>
<td>-0.090*</td>
</tr>
<tr>
<td></td>
<td>(5.814)</td>
<td>(3.970)</td>
<td>(0.129)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>VIX*Prin.</td>
<td>27.059*</td>
<td>11.056</td>
<td>0.895**</td>
<td>-0.478***</td>
</tr>
<tr>
<td></td>
<td>(15.460)</td>
<td>(13.18)</td>
<td>(0.361)</td>
<td>(0.159)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.617</td>
<td>10.465***</td>
<td>-0.015</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(1.363)</td>
<td>(2.629)</td>
<td>(0.022)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Obs.</td>
<td>2,560</td>
<td>2,269</td>
<td>2,648</td>
<td>2,302</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.080</td>
<td>0.064</td>
<td>0.941</td>
<td>0.962</td>
</tr>
</tbody>
</table>
costs for small firms. Small firms have slightly lower linear financing costs but must larger linear costs. In response to uncertainty shocks, the financing cost inflate sharply. One unit increase in log VIX boosts the fixed costs by 14% and 19% of book assets for large and small firms, respectively, while the linear costs are increased by 27% and 11% of book assets.

The following four regressions investigate the response of the separate components of the total costs. Regression (3) and (4) report the effects of uncertainty shocks on the direct issuance costs, gross spread. While the uncertainty shocks have positive effects on financing costs of large firms, it has negative impacts on small firms. This can be due to the fact that in high uncertainty state, smallest firms where prohibited from issuance due to extremely high costs of financing. And the relatively larger firms (still in the small firm group) are the only issuers and they have relatively low issuance costs. This results in a relative lower financing costs for the whole small firms group.

Regression (5) and (6) study the indirect financing costs. Two points are worth noticed. First, constant terms for the indirect costs are economically and statistically significant, but not the linear term with respect to the issuance principal. This is because indirect costs are a measure of both fixed and linear financing costs, because it affects both existing (fixed and linear costs) and new equity values (linear costs). Second, uncertainty shocks have substantial effects on the announcement returns. One unit increase in log VIX inflates the negative returns by 4% and 3% for large and small firms, respectively.

One may be concerned that the CAR results are driven by the fact the aggregate stock returns are negatively correlated with VIX index. For this reason, I conduct placebo tests. The placebo tests artificially move the announcement date 10 days before or after the issuance date, and see whether VIX still have strong impact on the announcement returns. As once can see from regression (7) and (8) in Panel (b) of Table A.6, the VIX effects are not significant. This is because the CAR method already removes the effects from aggregate stock returns. Unreported tests also separate the firms into large and small and re-conduct the placebo tests, the results are still not significant.

So far, I have shown substantial evidence on how aggregate uncertainties increase financing costs. But I haven’t explained why. Several studies provide potential explanations. First of all, higher uncertainty aggravates moral hazard (Feng (2014)) and asymmetric information problems, making a firm more difficult to access to outside financing. These will increase financing costs. Second, higher uncertainty can be a result of increasing asymmetric information (Senga (2014)), which also increase financing costs. Third, Verrier and Valencia (2013) show that heightened uncertainty can reduce supply of credit due to precautionary savings motive of bancks. All those explain why

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51 For the asymmetric information problem in Myers and Majluf (1984) framework, when both good and bad types of firms have very volatile performance, making the market more difficult to distinguish them based on operating performance. And as a result, the negative response to equity issuance can be larger.
aggregate uncertainty shocks and financing costs are positively correlated. But exploring which one is more plausible is beyond the scope of this paper.

Appendix B: Model and Estimation

B1 Simplification of the Problem in Risk-Neutral Measure

The model is can be further simplified under risk-neutral measure, as in most corporate finance models. The dynamic of $Z$ under risk-neutral measure is

$$\log Z' = \log Z + \mu_s - \beta_s \eta \sigma_{a,s} - \frac{1}{2}{\sigma_s}^2 + \sigma_s \varepsilon' .$$

(12)

In this equation, the productivity growth is discounted by a risk-adjustment term $\beta_s \eta \sigma_{a,s}$. Here $\beta$ is the response of firm productivity to aggregate productivity. Assuming that shocks to aggregate productivity has volatility $\sigma_{a,s}$, $\beta_s$ can be defined as

$$\beta_s \equiv \frac{\text{cov} (\sigma_s \varepsilon'_a, \sigma_{a,s} \varepsilon'_a)}{\sigma_{a,s}^2} = \frac{\rho_s \sigma_s}{\sigma_{a,s}} .$$

Here $\varepsilon'_a$ is the normal random shock to aggregate productivity. $\rho_s$ is the correlation between firm and aggregate productivities. This formula is the same as that in Bolton, Chen, and Wang (2011). For simplicity, I assume that $\rho_s$ and $\sigma_{a,s}$ are constant. So $\beta_s \eta \sigma_{a,s}$ can be simplifies to $\rho \eta \sigma_s$.

Under the risk-neutral probability measure $Q$, the transition probabilities are:

$$Q \equiv \mathbb{Q}(s'|s) = \begin{bmatrix} q_L & 1 - q_L \\ 1 - q_H & q_H \end{bmatrix} .$$

(13)

with

$$p_{s,s'} = q_{s,s'} \exp \left( \Gamma^{jk} \right) .$$

(14)

If the regime shift is priced, then $q_L$ and $q_H$ can be different from $p_L$ and $p_H$. In particular, if agents are risk-averse to the high uncertainty state $H$, one would expect agents perceive the state $H$ has higher probability to happen in the risk-neutral measure, i.e., $q_L < p_L$ and $q_H > p_H$. These transition probabilities are therefore also called risk-adjusted probabilities. The differences between those probabilities, $\exp \left( \Gamma^{jk} \right)$, reflect the prices of risk on the regime shift.
Under risk-neutral measure, the firm’s recursive problem is

\[ V(K, W, Z, s) = \max_{\{I,F,D\}} \left\{ D - F - 1_{\{F>0\}} (\lambda_0 s K + \lambda_1 s F) + \frac{1}{1 + r} \left[ E^Q_{s' | s} V(K', W', Z', s') \right] \right\} . \]  

(15)

Since \( r > 0 \), by Contraction Mapping Theory, this problem has a unique solution.

Finally, consider the problem after denominating \( K \). The dynamics of normalized revenue \( z \equiv \Pi / K \) is

\[ \log z' = \log z - (1 - \alpha) \log (1 - \delta + i) + \left( \mu_s - \beta \eta \sigma_{a,s} - \frac{1}{2} \sigma_s^2 \right) + \sigma_s \tilde{e}', \]

As a result, under risk-neutral measure, the firm’s problem can be simplified to

\[ v(w, z, s) = \max_{\{i,d,f\}} \left\{ d - f - 1_{\{f>0\}} (\lambda_0 s + \lambda_1 s) + \frac{1 - \delta + i}{1 + r} \left[ E^Q_{(z', s') | (z, s)} v(w', z', s') \right] \right\} \]

s.t.

\[ \log z' = \log z - (1 - \alpha) \log (1 - \delta + i) + \left( \mu_s - \beta \eta \sigma_{a,s} - \frac{1}{2} \sigma_s^2 \right) + \sigma_s \tilde{e}', \]

\[ (1 - \delta + i) w' = (1 + r W) w + z - \left( i + 1_{i \neq 0} \xi_0 + \frac{\xi_2}{2} i^2 \right) + f, \]

\[ Q(s' | s) = Q \]

\[ w \geq 0 . \]

I then solve and estimate this simplified problem. After solving this problem, I convert the probabilities from risk-neutral measure to physical measure using Equation 14.

### B2 Solution and Structural Estimation Procedure

The model is solved and structurally estimated using a version of SMM, called Numerical Generalized Method of Moments (NGMM). Here I describe the procedure of the solution and estimation.\(^{52}\)

I first approximate the continuous state variables by discrete values. The value of logarithm of \( z \), productivity-to-capital ratio, is discretized into 81 grids around -2, the mode of the stationary distribution. It is made sure to include the whole capital inaction region, as suggested by Bloom (2009). The value of \( w \), cash-to-capital ratio, is discretized into 21 states, from 0 to 5. I assign equal and finer grids from 0 to 0.8, and looser grids from 1 to 5. Because there are more density

\(^{52}\)For more details of this method and the general framework, please refer to \( ? \).
and sensitivity in low cash states than in the high cash states.

Then I guess a set of parameters, denoted by \( \theta \) and solve the model. The model is solved using Policy Function Iteration, which is an improved version of Value Function Iteration, as in Judd (1998). This solution gives the optimal policy functions, value functions, and transition probabilities of state variables.

Using the policy functions and the transition probabilities of state variables from the solution, I numerically compute the stationary distribution of state variables and the moments. This is done by noticing that a policy mapping matrix, a matrix mapping current states to next period states, has non-negative values in all its elements and a sum of 1 in each row.\(^{53}\) In this case, the policy mapping matrix can be interpreted as a Markovian transition matrix of the state variables. The composite Markovian transition matrix of the policy mapping matrix and random state transition matrix exists. Then the stationary distribution of state variables can be computed as the steady-state probabilities of the composite Markovian transition matrix. Further more, with this stationary distribution of state variables \( \pi_{(w,z,s)} \), moments as functions of the state variables and their distribution can be numerically computed. For example, if one wants to compute the expected value of a function on the state variable space \( x(w,z,s) \), it can be simply computed as

\[
E_{GMM}[x(w,z,s)] = \sum_{(w,z,s)} \pi_{(w,z,s)} x(w,z,s).
\]  

(17)

This is the key difference that my NGMM approach differs from SMM, it avoids simulating artificial data and thus the random errors from the simulated data.

The model moments are denoted by \( m(\theta) \), computed using the approach as in Equation 17. Suppose the data moments are \( \hat{M} \) and their variance-covariance matrix is \( \hat{S} \). \( \hat{S} \) is estimated using the influence function approach as in Erickson and Whited (2002) and Warusawitharana and Whited (2014). The distance between the model and data moments is then \( g(\theta) = m(\theta) - \hat{M} \). The estimator of \( \hat{\theta} \) solves the following GMM problem with a weighting matrix \( \hat{W} \),

\[
\hat{\theta} = \arg \min_{\theta} g(\theta)^\top \hat{W} g(\theta).
\]

The weighting matrix \( \hat{W} \) is required to be positive definite and converge to a deterministic positive

\(^{53}\)Here is a concrete example, if a policy function maps the \( i \)th element of capital \( K \) to the \( j \)th element of \( K' \), then the policy mapping matrix has its elements on the \( i \)th row being all zero except the \( i \)th-row-\( j \)th column element being 1. If a policy function maps the \( i \)th element of capital \( K \) to a value of \( K' \) in between the \( j \)th and \( (j+1) \)th elements, i.e., \( \beta K'_j + (1 - \beta) K'_{j+1} \), then the policy mapping matrix has its elements on the \( i \)th row being all zero, except the \( i \)th-row-\( j \)th column element being \( \beta \) and the \( i \)th-row-\( (j+1) \)th column element being \( 1 - \beta \). As long as the discretized state space includes all inaction region in the model, all elements in the policy mapping matrix should be non-negative and no greater than 1.
definite matrix $W$. In this paper, I choose $W$ to be a diagonal matrix with its elements being the square of unconditional mean of the corresponding moments. This choice of weighting matrix has an economic meaning that the corresponding objective function is minimizing the percentage deviation of the model moment from the data moments.\(^{54}\)

Inference of the estimation is the same as the usual GMM. By Central Limit Theorem, $\hat{\theta}$ is asymptotically distributed as following

$$
\sqrt{N} \left( \hat{\theta} - \theta_0 \right) \xrightarrow{d} N \left( 0, \left( G^\top \hat{W} G \right)^{-1} G^\top \hat{W} S \hat{W}^\top G \left( G^\top \hat{W} G \right)^{-1} \right).
$$

where $G$ is the derivative of the moment condition with respect to $\theta$ evaluating at $\theta_0$, i.e., $G \equiv \frac{\partial g(\theta)}{\partial \theta} \big|_{\theta = \theta_0}$. $S$ is the probability limit of $\hat{S}$. As both $\theta_0$ and $S$ are not observable, the asymptotic variance of $\hat{\theta}$ can be estimated as ($G \equiv \frac{\partial g(\theta)}{\partial \theta} \big|_{\theta = \hat{\theta}}$),

$$
\text{AsyVar} \left( \hat{\theta} \right) = \frac{1}{N} \left( \hat{G}^\top \hat{W} \hat{G} \right)^{-1} \hat{G}^\top \hat{W} \hat{S} \hat{W}^\top \hat{G} \left( \hat{G}^\top \hat{W} \hat{G} \right)^{-1}.
$$

An important issue of the structural estimation is that, it estimates the parameters of an average firm, not the average of the parameters across firms. The structural estimation can only account for the heterogeneity stems from the prescribe state variables, productivity-to-capital ratio, cash-to-capital ratio, and uncertainty states. It is not able to account for other firm heterogeneity. For this reason, I adopt the approach in Warusawitharana and Whited (2014) by removing the firm fixed effects to compute the variance-covariance matrix of sample moments $\hat{S}$. In order to accommodate other dimension of heterogeneity like size, I divide the sample into small and large firms and compare their results to my benchmark results. They are very similar.

\(^{54}\)Three popular choices of weighting matrix are used in the literature, identity, optimal, and information function approach one as in Warusawitharana and Whited (2014). The first one is inappropriate here because it will estimate the moments with small values with large variance relative to the moment magnitudes. In my case with two regimes with asymmetric numbers of observations, optimal weighting matrix and the information function approach one will estimate the conditional moments in the low uncertainty states much more accurate than those in high uncertainty states, as the latter has relatively less observations and higher variance.