Collateral Quality Uncertainty and the Business Cycle *

Jing Zhou †

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Abstract

It is widely recognized that the 2007–2009 financial crisis is preceded by accumulation of low-quality asset-backed loans in credit markets. However, the relationship between collateral quality uncertainty and the business cycle is greatly under-explored in the literature. It is intriguing and necessary to ask the following question: What are the qualitative and quantitative impacts of fluctuations in collateral quality on the real economy? Using a dynamic stochastic general equilibrium model, this paper shows that uncertain collateral quality can drive boom-bust credit cycles. When information about quality is costly, the credit cycles features a regime switching effect, as emphasized in Gorton and Ordoñez (2014). The occurrence of the regime switches in credit markets generates asymmetric responses to collateral quality shocks, which in turn amplifies the business cycles, contributes additional volatility as well as generates interest rate spikes. By calibrating the model using US data, I find that even small fluctuations in collateral quality can well explain the stylized patterns of real business cycles, especially with regard to house prices and rents. Moreover, the results indicates that the collateral quality is of particular relevance to the 2007–2009 financial crisis.

Keywords: information acquisition sensitivity, regime switch, collateral constraint, asset pricing, business cycles, financial frictions, house prices, heterogeneous firms.

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†Correspondence: Department of Economics, the Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. Tel: (852) 6699-0057. Email: jzhoubb@connect.ust.hk Website: http://jing-zhou.weebly.com/
1 Introduction

“A continuing barrier to private investment in financial institutions is the large quantity of troubled, hard-to-value assets that remain on institutions’ balance sheets. The presence of these assets significantly increases uncertainty about the underlying value of these institutions and may inhibit both new private investment and new lending.”

— Ben S. Bernanke, January 2009

Uncertain collateral quality, among other factors, set the stage for the 2007–2009 financial crisis in the US. Over 70 percent of asset-backed securities\textsuperscript{1} were mortgage-related including subprime mortgage-backed securities that were pooled with prime mortgage-backed securities for distribution in financial markets. Up to Q3 2007, securitization increased quickly and became prevalent for a wide variety of credit types. The issuance of US non-agency asset-backed securities surpassed that of straight corporate debt from 2002 to 2007. In the meanwhile, the share of subprime mortgages in the market increases from less than 10 percent of mortgages to more than 20 percent from 1995 through 2007,\textsuperscript{2} which fueled a prominent credit boom in the US: “The funding available through the shadow banking system grew sharply in the 2000s, exceeding the traditional banking system in the years before the crisis.”\textsuperscript{3} Motivated by these observations, in this paper, I attempt to answer the following question: What are the qualitative and quantitative impacts of fluctuations in collateral quality on the business cycle?

The information opaqueness associated with the collateral quality uncertainty has proven to be one of the triggers of the crisis. Kurlat\textsuperscript{(2013)} and Bigio\textsuperscript{(2015)} demonstrate that adverse selection caused by information asymmetry affects the equilibrium asset quality in financial markets, hence affects the cost of funding and liquidity. However, Pagano and Volpin\textsuperscript{(2012)} point that what makes the information acquisition prohibitively

\textsuperscript{1}Calculated from Gorton and Metrick\textsuperscript{(2012)}
\textsuperscript{2}Source: Financial Crisis Inquiry Commission Report
\textsuperscript{3}Source: Federal Reserve Flow of Funds Report
difficult is not the information availability from data providers, but the expensive cost of subscription, the sophisticated mind and considerable skills required to analyze it. Therefore investors have to rely on less and coarse information published by rating agencies. Just as an article in the Financial Times reported, “Much of regulators’ efforts have been focused on pushing issuers to provide more information on individual loans … However, …. this is not a top priority and they were in fact more concerned with developing methods to analyze and compare cash flow data cross different deals” (Hughes (2010)).

Given both the complexity and the “originate-and-distribute” nature of the deals, lack of clear and precise information about the asset quality is serious to both borrowers and lenders in credit markets. Therefore, instead of introducing asymmetric information, I treat the collateral quality uncertainty exogenous and symmetric to both parties of loans, and further ask what role the fluctuations in this uncertainty plays in business cycles, which is largely under-explored in the literature. To be more specific, in this paper, I assume that the collateral can either have positive intrinsic value or zero intrinsic value. Before any possible information disclosure, it is not publicly observable. The collateral quality is the probability that the collateral has positive intrinsic value ex ante.

The main findings of this paper are: (i) Considering the collateral quality uncertainty and the information cost thereof into types of debt contracts can endogenously generate a regime-switching effect between lenders being motivated to acquire the quality information or not. This adds volatility to the real economy and causes interest rate spikes. (ii) The fluctuations in the collateral quality, together with the accompanying regime switches, also lead to significantly amplified responses of real variables. (iii) They can also explain the patterns of the business cycle with regard to housing prices and the house price-rent ratio, the latter of which can be hardly explained by shocks that are common in the literature on housing prices. (iv) I show that the fluctuations in collateral quality are of particular

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4 Another example of the great opacity of the asset quality during this period is: An emergency measure launched by the US government during the crisis, the “Troubled Asset Relief Program,” aimed at removing these complex assets from the balance sheets of the banking system to decrease uncertainty. However, this task soon proved extremely difficult and this initial goal was abandoned.
relevance to the recent financial crisis rather than other recent boom-bust business cycles.

The model in this paper is an infinite horizon dynamic stochastic general equilibrium model. Firms own all the houses. Households rent houses from firms and derive utility from housing services, providing houses with intrinsic value and a positive price. However, houses, the only type of assets that can serve as collateral in the credit market, are of varying quality. High-quality houses provide positive utility to households, while low-quality houses provide zero utility. Naturally, a high-quality house qualifies as collateral while a low-quality house does not. The true quality can be discovered by incurring an information acquisition cost. All houses look identical ex ante and they have some probability of being high or low quality. This probability is the above-mentioned quality of houses and the stochastic process characterizing it is the “quality shock.”

At the beginning of each period, firms face idiosyncratic investment-specific shocks. To finance their investments, firms use houses as collateral to borrow from households. For that, there are two types of debt contracts for loans secured by houses: Under one type of contracts, the creditor pays the information cost; loans secured by high-quality houses are financed, while loans secured by low-quality houses are not. Under the other type of contracts, no investigation is conducted, no information cost is paid, and loans are made according to the perceived market value of the collateral.

On the one hand, when there is a high probability that the houses are of high quality, information acquisition is avoided and all loans are supported by the perceived value of collateral. Investments can be made whenever the corresponding investment efficiencies are sufficiently favorable. Capital and output grow relatively quickly. On the other hand, when probable quality is low, information costs are paid, and only loans with high-quality collateral are supported. Some favorable investment opportunities are missed. Capital accumulation and output growth are relatively slow. A regime switch occurs when the probable quality fluctuates across the cutoff value at which lenders who decide whether to investigate are indifferent between the two types.
The regime switch occurs due to a combination of imperfect quality and information acquisition costs. The presence of an information acquisition cost weakens the incentives of lenders to lend prudently when the perceived quality is sufficiently high and the credit market is competitive. As a result, when the quality is higher than the cutoff value, capital is cheaper and more investments occur because both the number of the investing firms (the extensive margin) and the ex ante amount of investment per firm (the intensive margin) are higher. And vice versa. By contrast, in models without regime switching but otherwise identical, the extensive margin and intensive margin do not move in the same direction.

Aside from the regime switching, the responses of variables are also amplified by the fluctuations in collateral quality because: (i) The effective quantities of collateral are affected; (ii) The prices of the collateral goods (house prices in this model) are also affected since dividend payments (house rental income in this model) in the future are affected. These two effects, together with the regime switches thereof, create nonlinearly magnified impacts on the amount of credit in the economy and hence other variables.

Interestingly, an interest rate spike appears around a regime switch. During a downward switch, the interest rate rises quickly as funding becomes more costly. This is consistent with the observed behavior of interest rates during the financial crisis.

Another important implication of this model is that the fluctuations in quality shocks can generate high volatility in house prices and in the house price-rent ratio, as well as a positive correlation between the house price-rent ratio and output. These patterns are consistent with the data shown in Figures 1 and 2. The highly volatile house price-rent ratio observed in US data is especially puzzling if we interpret house prices as the present values of future rent streams. These are striking patterns in business cycles and housing markets, and they are difficult to explain through the shocks commonly considered in the literature. The intuition behind a quality shock is the following: First, house prices are volatile as they are directly affected by the probability that houses are high quality in

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5 The rent for owner-occupied home is imputed.
the future. By contrast, house rents capture the marginal utility of household to housing services which are much smoother. Second, house prices consist of two components. One component is derived from the streams of future house rents, and the other component is derived from the collateral value for relaxing borrowing constraints. I call the latter the “liquidity premium” which is more volatile than the former due to the regime switches. At the same time, a positive quality shock leads to higher output and the higher house price-rent ratio, indicating a positive correlation between the latter two. Despite this simple setup, other shocks that are common in the literature, such as financial shocks, housing preference shocks and equity shocks, are not able to generate these patterns individually or collectively.

Calibrated using two shocks, a technology shock and a collateral quality shock, the model can fit above-mentioned patterns, as well as other traditional real business cycle statistics in US data. Without the collateral quality shock, the volatility of all variables decreases. In particular, the investment volatility decreases by 20 percent. Meanwhile, the volatility of house prices and the house price-rent ratio decreases to an implausibly low level.

How large is the size of the collateral quality shock needed to generate business cycle moments consistent with the data? The calibration shows that, without resorting to other shocks, the standard deviation of the quality shock needed is as small as about one-third of the typical standard deviation of a technology shock. The time persistence and volatility of this shock are also similar to those implied by the delinquency rates of commercial loans secured by real estate from 1994 to 2008 in the US (see Figure 3). One advantage of such a small shock is that it allows me to solve the model using a modified perturbation method without losing satisfactory precision, even though the model features discontinuity.

Finally, I examine whether and to what extent the quality shock is relevant to the recent financial crisis. I properly choose the shock sequences to match the data from

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6Source: Board of Governors of the Federal Reserve System.
the financial crisis and simulate the economy. The results show that the quality shock, in addition to the technology shock, makes a significant contribution to the decrease in output, investment and working hours observed during the recession. Further, the fluctuations in prices of housing markets can hardly be explained when excluding the quality shock. For comparison, I also simulate the economy to match the outcomes of two other periods featuring peaks and troughs: 1990 through 1991 and 2000 through 2001, and find that the quality shock plays a very limited role during these periods. This suggests that the collateral quality shock is of particular relevance to the financial crisis of 2007 – 2009 but not to previous boom and bust cycles over the last two and one-half decades.

**Literature review**  The study most similar to this paper is Gorton and Ordoñez (2014) which also discusses two types of debt, information-sensitive and information-insensitive contracts, in a one-period static framework and an overlapping generation framework. The authors consider the endogenous emergence of a credit boom before a credit crisis featuring a regime switch. Although it shares some features, this study is distinct in two important ways: First, the model developed here is a dynamic general stochastic equilibrium framework, which allows for introducing the variables of typical interest in business cycle analysis and a more careful quantitative examination of the importance of this friction. Second, this model explicitly incorporates the housing market, and the prices are determined endogenously. The implications derived from doing this enhance our understanding of some crucial patterns in housing markets. Thus, this work can be viewed as a further exploration that is complementary to theirs by considering how and to what extent this friction is important to the real economy.

This paper also contributes to the studies of information opaqueness in financial markets. On the one hand, Hanson and Sunderam (2013) show that issuing too many safe assets during good times results in information scarcity and exacerbates market collapses during bad times. Dang, Gorton, and Holmström (2012) and Pagano and Volpin (2012) argue that some degree of information opaqueness may be desirable when taking the in-
formation costs in normal times into account. On the other hand, the results presented in this paper imply that given the potentially great consequences for the real economy, regulations on information disclosure in financial markets should be seriously considered.

The friction explored here adds another source to the intensively discussed literature on financial frictions and their real effects. Two seminal works in this area are Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999). These two papers and a large number of succeeding works emphasize positive feedback between financial frictions and real economic activities. Various types of financial frictions are inspected: Jermann and Quadrini (2012) demonstrate the importance of financial shocks; Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) analyze equity shocks; Christiano, Motto, and Rostagno (2014) and Arellano, Bai, and Kehoe (2012) examine uncertainty shocks, etc. This study describes a new dimension of financial frictions – the collateral quality uncertainty. The collateral quality affects the amount of credit through both the effective quantities and the price of collateral goods.

In addition to the first-order propagation mechanisms featured in the literature on financial frictions, second-order disturbances are also introduced in this study. This offers another view of how regime switches are potentially embedded in small disturbances. On this front, He and Krishnamurthy (2014) discuss a systemic risk wherein there is tipping point phenomenon. When leverage varies, the economy can transit from normal states to systemic risk states. The trigger in their work is leverage while in this work the trigger is the collateral quality. Other works, such as Brunnermeier and Sannikov (2014), Mendoza (2010), also feature transitions of an economy between two types of states, usually normal state and crisis state.

Also, this paper is related to the literature that aims to understand fluctuations in house price data. Iacoviello (2005) and Iacoviello and Neri (2010) attribute these volatile movements mainly to housing preferences and monetary policies. In their works, houses

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7See Quadrini (2011) for a survey.
are also used as collateral to finance expenditures. Liu, Wang, and Zha (2013) argue that competition between the household sector and the business sector coupled with amplification of credit constraints with houses as collateral, would powerfully magnify a housing preference shock and give rise to excessive volatility in land prices. In contrast to house prices, house rents are considered less often in the literature. Shocks that are common in the literature on house prices are often unable to explain the excessive volatility of house prices relative to house rents or the positive correlation between the house price-rent ratio and output. Miao, Wang, and Zha (2014) propose a new shock, a liquidity premium shock, which may bridge this gap. This study provides support for their arguments using another shock.

The remainder of this paper proceeds as follows. Section 2 describes a basic model wherein the two types of lending are introduced into a Kiyotaki and Moore (1997) setup. I solve the model and derive the equilibrium system in Section 3 and illustrate the main mechanisms in Section 4. Further, Section 5 discusses three relevant issues: (i) making loans intertemporal and the implications of the interest rate during a regime switch; (ii) how the qualitative implications of this shock are distinguished from other shocks; and (iii) the inclusion of working capital into the collateral constraint. After including consumption habit formation and aggregate capital adjustment costs in the basic model, in Section 6 I calibrate the model using US data from 1975 to 2010 and examine the relevance of the quality shock for the recent financial crisis. Section 7 concludes.

2 Basic Model

In this section, I develop a simple model to introduce the collateral quality and costly information acquisition into a setting similar to Kiyotaki and Moore (1997) and study how the fluctuations in collateral quality generates business cycles.

Consider a discrete time, infinite horizon economy populated by households and firms that are infinitely lived and of measure one. There are two types of goods: consumption
goods (the numeraire goods, which also serve as capital goods in the basic model) and houses. Firms produce numeraire goods, hiring the labor supplied by households and combining it with capital. Firms also own and trade houses. In the credit market, firms collateralize houses and borrow from households to finance their investment needs. The firms are owned by households and pay them dividends. Households consume and pay rent.

2.1 Household

A representative household maximizes the lifetime utility function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(C_t) + \varphi_H (\eta_t H_{t-1}) - \varphi_N \frac{N_t^{1+\nu}}{1+\nu} \right]$$  \hspace{1cm} (1)$$

where $E_0$ is the expectation operator, $\beta \in (0, 1)$ is the discount factor, $C_t$ is the consumption in period $t$, $\eta_t H_{t-1}$ denotes the amount of high-quality houses rented, and $N_t$ are the hours of work committed by the household. $\varphi_H$ captures the household’s preference for housing services and $\varphi_N$ captures its preference for leisure. $1/\nu$ is the Frisch elasticity of labor supply.

The key friction in this model is that the houses are not always high quality. If one unit of housing is high quality, it provides positive utility $\varphi_H$ to its tenants. If it is low quality, it provides zero utility and is hence useless. A house is high quality with ex ante probability $\eta_t$. At the beginning of each period, a fraction $\eta_t$ of the total stock of houses $H_{t-1}$ in the economy is high quality. And only high-quality houses are charged rent, which I explain in more detail in Section 2.3.

$R_t$ denotes the equilibrium house rent; $W_t$, the real wage. Assume that households lend to firms at the beginning of a period and receive repayment at the end of that period so

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8Throughout this paper I abstract from explicitly modeling financial intermediaries. Alternatively, one can conceptualize competitive financial intermediaries that absorb deposits from households and lend them to firms.
that the gross interest rate is simply $1$. Thus, the budget constraint of a representative household is

$$C_t + \eta_t R_t H_{t-1} = D_t + W_t N_t + M_t$$  \hspace{1cm} (2)$$

where $D_t$ denotes the lump-sum dividends received from firms (described below in Section 2.5), $W_t N_t$ is the labor income and $M_t$ is the income from acquiring information about the collateral quality for creditors in the credit market. I will explain it in Section 2.4. As all obligations occur within one period and are fully secured by collateral, they do not affect the budget constraint.

Assume that the Lagrangian multiplier associated with (2) is $\Lambda_t$. Maximizing the objective function (1) subject to the budget constraint (2) yields the following first-order conditions for consumption (3), labor supply (4), and housing services (5):

$$\frac{1}{C_t} = \Lambda_t,$$ \hspace{1cm} (3)

$$\varphi N_t \nu = \Lambda_t W_t,$$ \hspace{1cm} (4)

$$\varphi H E_t \eta + 1 = R_t E_t \Lambda_{t+1} \eta_{t+1}.$$ \hspace{1cm} (5)

$\Lambda_t$ is the marginal utility of consumption, as implied by equation (3). Equation (4) states that the wage rate is equal to the marginal rate of substitution between leisure and consumption. Equation (5) indicates that the house rent is equal to the marginal rate of substitution between housing services and consumption.

The related literature may associate housing service utility with a type of diminishing returns. Here, I assume it to be linear. This is because the quality of one unit of housing is not continuous. Diminishing returns will complicate the problem without fundamentally changing the results.

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9In the baseline model, for the ease of presentation, the borrowing and lending are assumed to be intratemporal. Making debts intertemporal does not alter the key results. See Section 5.1.
2.2 Firms

A firm indexed by \( i \) uses a Cobb-Douglas constant returns-to-scale production technology that employs capital and labor as inputs. The firm \( i \) produces the numeraire goods \( Y_t(i) \) according to

\[
Y_t(i) = A_t K_{t-1}(i)^\alpha N_t(i)^{1-\alpha}
\]  

(6)

where \( A_t \) is the technology, \( K_{t-1}(i) \) is the capital stock determined in \( t - 1 \), and \( N_t(i) \) is the labor input. \( \alpha \) and \( 1 - \alpha \) are the output elasticities of capital and labor, respectively.

Firms are heterogeneous in their investment-specific efficiencies each period. At the beginning of period \( t \), firm \( i \) draws a random investment-specific shock \( \varepsilon_t(i) \). Capital accumulates according to

\[
K_t(i) = (1 - \delta) K_{t-1}(i) + \varepsilon_t(i) I_t(i)
\]

where \( I_t(i) \) is the investment. Let \( F(\varepsilon) \) denote the cumulative density function of \( \varepsilon_t(i) \). Assume that \( \varepsilon \) is drawn independently and identically across firms and over time.

The obligations of firms must be secured by the collateral they offer. Assume that the numeraire good can be easily diverted or hidden so that lenders cannot seize them in the case of default. The only source of collateral in this economy is houses, which are owned and traded by firms. Households do not have incentives to own houses because by owning houses, firms can draw value not only from rental income, but also from the liquidity benefit by collateralizing them to lenders when they draw favorable investment-specific shocks. However, households can only enjoy the former benefit. Thus households prefer renting houses to purchasing them.\(^{10}\)

The timing of the model in period \( t \) is summarized below and illustrated by the following figure:

\(^{10}\)Refer to Proposition 3 in Miao, Wang, and Zha (2014) for a proof.
1. Firms and households observe technology shock $A_t$ and collateral quality shock $\eta_t$. Firms observe their own investment-specific shocks $\varepsilon_t(i)$. 

2. Firms sign debt contracts with households. 

3. Firms make investments $I_t(i)$ and produce $Y_t(i)$ by using $K_{t-1}(i)$ and hiring $N_t(i)$. Firms repay their obligations. 

4. Transactions in the rental and housing market take place. Before the housing market opens, the low-quality houses are destroyed. 

2.3 Collateral Quality 

As the low-quality houses are cleared before they are traded in the housing market, the stock of housing before the realization of a new quality shock next period includes only high-quality houses. That is, the opaqueness of information can last at most one period. This is a simplifying assumption, as in Kurlat (2013). Therefore, there is no learning occurring during the dynamics. Such ex ante homogeneity in collateral reflects the fact that, with ratings, loans backed by low-quality collateral are packaged with loans backed
by high-quality collateral into a vehicle, the process of which creates opacity of collateral quality.

In addition, assume that there are new houses entering the housing market such that total supply of houses for purchase and rent is fixed, i.e., $H_t = 1$.

### 2.4 Debt contract

Given the timing, firms must rely on external financing for investments. They borrow from households in a competitive credit market by collateralizing at most $H_{t-1}(i)$ units of houses, i.e., the houses they hold at the beginning of period $t$. The collateralized houses are seized by the lender and resold on the housing market if the face value of the debt is not repaid. The borrower can re-enter the credit market without penalty.

The true quality of collateral can be investigated by incurring an information acquisition cost, which is similar to the monitoring cost in Townsend (1979). The cost is a small fraction of the market value of the collateral. It may also be assumed as a constant cost (e.g. Gorton and Ordoñez (2014)) or denominated in wages (e.g. Benhabib, Liu, and Wang (2014)). However, the proportionality assumption makes calculation convenient and prevent other sources of nonlinearity so that we can see the nonlinear effects generated by the regime switch more clearly.

There are two types of debt contracts: *information-sensitive debt* and *information-insensitive debt*, similar to Gorton and Ordoñez (2014). However, the focuses are slightly different: their work focuses on the borrower’s optimal choice of the size of the loan, while the focus here is simply the lenders’ incentive to launch information acquisition. In this model, borrowers always prefer the information-insensitive type to save the information acquisition cost, while lenders may not always agree. I analyze two types of debt contracts in turn below.
**Information-sensitive Debt**  Under this contract, lenders learn the true quality of collateralized houses by paying acquisition costs, and the debt conditions are conditional on the resulting information. Thanks to the homogeneity of degree one of this model, I simply consider one unit of housing as collateral in this section. As the debt is intratemporal and the market is competitive, the lenders break even between lending or not

\[
\eta_t(P_t - B_t^{IS}) = \gamma \eta_t P_t \quad \text{or} \quad P_t - B_t^{IS} = \gamma P_t
\]  

where \( P_t \) is the house price (the high-quality house price; recall that low-quality houses exit the market before trading). \( B_t^{IS} \) is the size of the loan under this type, and \( \gamma \) reflects the information acquisition cost.

The left-hand side is the benefit of lending. If the collateral is high quality and the contract is signed, the borrower repays \( P_t \) anyway with a no-arbitrage argument. So the benefit for a lender is \( P_t - B_t^{IS} \) conditional on the collateral being good. To acquire the information, the lender pays a fraction \( \gamma \) of the market value of the collateral, \( \eta_t P_t \), regardless of the results, which is the cost of lending.

If the collateral is good, the size of the loan is \((1 - \gamma)P_t\); if bad, the borrower may receive nothing.

**Information-insensitive Debt**  Under this contract, lenders do not learn the true quality of the collateral simply lending according to its perceived value. Again, given that lenders break even

\[
\eta_t P_t - B_t^{II} = 0
\]  

where \( B_t^{II} \) is the size of the loan under this contract. Again, the left-hand side is the benefit of lending, i.e., \( \eta_t P_t - B_t^{II} \) regardless of the true quality. And the right-hand side the cost of lending, which is just 0.
However, this type of debt would not be implemented unless the lenders have no incentive to deviate. The lenders may deviate if they find it more profitable to verify the quality and secretly pay the cost while only lending to those with good houses. In other words, this type of contract is implemented only if the expected gain of deviation is no larger than the cost of doing so

\[ \eta_t(P_t - B_t^{II}) < \gamma \eta_t P_t. \]  

(9)

The right-hand side is the expected cost of secretly verifying quality. The left-hand side is the expected gain from behaving as if the lenders honor the information-insensitive contracts when the collateral is good. Therefore, the following proposition is straightforward.

**Proposition 1** Information-insensitive contracts are implementable only if

\[ \eta_t \geq 1 - \gamma. \]  

(10)

This is, when \( \eta_t < 1 - \gamma \), the only implementable regime in equilibrium is information sensitive. When \( \eta_t \geq 1 - \gamma \), both regimes are implementable, and the lenders are indifferent between the two. However, as borrowers always prefer the information-insensitive type to the information-sensitive type ex ante and the credit market is competitive, the information-insensitive regime would be chosen in equilibrium whenever it is implementable. Therefore, the simple condition (10) governs which regime is active.

\[ \text{This is because the ex ante resale value of a unit of housing (} \eta_t P_t \text{) is the same under the two types of contracts, but the ex ante size of a loan is higher under the information-insensitive type than under the information-sensitive type. (} \eta_t P_t \text{ vs. } (1 - \gamma) \eta_t P_t) \]
2.5 The Firm’s Problem

In this section, I define the problem faced by firm $i$. The firm’s labor hiring decision is static

$$\max_{N_t(i)} Y_t(i) - W_t N_t(i)$$

(11)

which gives

$$N_t(i) = \left[ \frac{(1 - \alpha) A_t}{W_t} \right]^{\frac{1}{\alpha}} K_{t-1}(i).$$

(12)

The maximized production revenue net of the labor cost is $R^K_t = \alpha A_t^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}} K_{t-1}(i)$ where $R^K_t$ can be interpreted as the rate of return to capital.

Then, firm $i$’s dynamic problem can be written as

$$\max_{I_t(i), H_t(i)} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} D_t(i)$$

(13)

subject to

$$D_t(i) = R^K_t K_{t-1}(i) - I_t(i) - (1 - 1_t) \cdot \frac{\gamma}{1-\gamma} I_t(i)$$

$$+ P_t [\eta_t H_{t-1}(i) - H_t(i)] + P_t H_t^\alpha + \eta_t R_t H_{t-1}(i),$$

(14)

$$0 \leq I_t(i) + (1 - 1_t) \cdot \frac{\gamma}{1-\gamma} I_t(i) \leq \eta_t P_t H_{t-1}(i),$$

(15)

$$K_t(i) = (1 - \delta) K_{t-1}(i) + \varepsilon_t(i) I_t(i),$$

(16)

$$1_t = \begin{cases} 1 & \text{if } \eta_t \geq 1 - \gamma, \\ 0 & \text{if } \eta_t < 1 - \gamma. \end{cases}$$

(17)

Equation (14) is the (ex ante) dividends paid by firm $i$ to its owners. The first term is the revenue from product sale net of the wage payment; the second term is the investment that
firm $i$ makes this period; the third term is the information acquisition cost paid depending on the regime implemented; the fourth term is the expenditure on house purchasing; the fifth term is the value of new houses; and the last term is the rental income.

The inequality of equation (15) on the left is the investment irreversibility constraint. Investment is irreversible so there is no negative investment. The inequality on the right is the collateral constraint. The size of investment is bounded by firm holdings of collateral, as well as by the type of the debt contract. Constraint (17) is a restatement of condition (10).

In this model, I do not restrict firms from issuing equity. If firms also face an equity financing constraint, their maximum investment in each period would be bounded by both the collateral and the equity financing constraints, depending on which binds first. That would complicate the model solution and contaminate the nonlinear effect of the quality shock.

Additionally, I do not include working capital in the liquidity needs in the basic model either. According to Mendoza (2010), Jermann and Quadrini (2012), and Bigio (2015) introducing working capital or payroll into the borrowing constraint would cause even stronger impacts on hours worked, output, and hence real economy. I discuss that in Section 5.3. In short, this model is kept intentionally clean and simple to allow a clear inspection of the key mechanisms.

### 2.6 Competitive Equilibrium

A competitive equilibrium consists of sequences of individual quantities $\{I_t(i), K_t(i), N_t(i), Y_t(i), H_t(i)\}$, aggregate quantities $\{C_t, I_t, K_t, N_t, Y_t, H_t\}$ and prices $\{\Lambda_t, W_t, R^K_t, R_t, P_t\}$ such that (i) household optimize; (ii) workers and firms optimize; and (iii) the markets for
labor, houses, house rentals, and numeraire goods all clear, i.e.,

\[ N_t = \int_0^1 N_t(i)di, \quad H_t = \int_0^1 H_t(i)di = 1, \quad Y_t = C_t + I_t \]

where \( Y_t = \int_0^1 Y_t(i)di \) and \( I_t = \int_0^1 I_t(i)di \).

3 Model Solution

In this section, I first solve for the policy of an individual firm. Then, I consider the aggregation and characterize the competitive equilibrium.

3.1 The Firm’s Policy

The Bellman equation of a firm’s dynamic problem is

\[
V(\varepsilon_t(i), K_{t-1}(i), H_{t-1}(i); A_t, \eta_t) = \max_{I_{t}(i), H_{t}(i)} D_{t}(i) + \beta E_{t} \frac{\Lambda_{t+1}}{\Lambda_{t}} V(\varepsilon_{t+1}(i), K_{t}(i), H_{t}(i); A_{t+1}, \eta_{t+1}) \tag{21}
\]

subject to (14), (15), (16) and (17). As the problem is homogeneous of degree one, the value function can be conjectured to take the form

\[
V(\varepsilon_t(i), K_{t-1}(i), H_{t-1}(i); A_t, \eta_t) = v_{t}(\varepsilon_t(i), A_t, \eta_t)K_{t-1}(i) + p_t(\varepsilon_t(i), A_t, \eta_t)H_{t-1}(i) + P_{t}H_{t}^{n}. \tag{21}
\]

Consider the investment decision first. Given the idiosyncratic shock independently and identically distributed across firms and periods, the marginal value of capital next period, represented by \( E_{t} \frac{\Lambda_{t+1}}{\Lambda_{t}} \int v_{t+1}(\varepsilon, A_{t+1}, \eta_{t+1})f(\varepsilon)d\varepsilon \), is consistent across firms. However, the cost of accumulating one unit of capital at the margin is heterogeneous across firms due to the investment-specific shock. As a result, when a firm draws a shock that is higher than a certain cutoff level, it prefers to invest as much as possible, whereas when a firm draws a shock that is lower than the cutoff level, it prefers to make no investment. Firms with
an investment efficiency at the cutoff value are indifferent between investing or not.

I now turn to the house purchasing decision of firms. The houses have two functions for firms in this economy: First, they serve as a store of value. Second, they can help relaxing the firms’ borrowing constraints and meet their liquidity needs. The benefits of these two roles constitute the market value of houses. The house prices equates the housing supply and demand such that firms are indifferent between purchasing one more unit of housing or not at the margin.

The interpretation of the firm’s policy is described above; it is formalized in following proposition.

**Proposition 2** (i) A firm $i$’s investment decision is

$$I_t(i) = \begin{cases} (1 - \gamma) \eta_t P_t H_{t-1}(i) + I_t \cdot \gamma \eta_t P_t H_{t-1}(i) & \text{if } \epsilon_t(i) \geq \epsilon_t^*, \\ 0 & \text{if } \epsilon_t(i) < \epsilon_t^*. \end{cases} \quad (22)$$

And the firm $i$ is indifferent among any choices of $H_t(i)$ satisfying $0 \leq H_t(i) \leq H_t$.

(ii) The cutoff of investment efficiency $\epsilon_t^*$, Tobin’q $q_t$, and house prices $P_t$ satisfy

$$q_t \epsilon_t^* = 1 + (1 - I_t) \cdot \frac{\gamma}{1 - \gamma}, \quad (23)$$

$$q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ R_{t+1}^K + (1 - \delta) q_{t+1} \right], \quad (24)$$

$$P_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \eta_{t+1} \left[ R_{t+1} + P_{t+1} + (1 - \gamma + I_{t+1} \cdot \gamma) P_{t+1} q_{t+1} \int_{\epsilon \geq \epsilon_{t+1}^*} (\epsilon - \epsilon_{t+1}^*) f(\epsilon) d\epsilon \right], \quad (25)$$

and the transversality conditions hold

$$\lim_{j \to +\infty} E_t \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} q_{t+j} K_{t+j}(i) = 0 \text{ and } \lim_{j \to +\infty} E_t \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} P_{t+j} H_{t+j}(i) = 0. \quad (26)$$

Equation (22) is the investment decision as discussed above. Equation (23) implies that
for firms that are indifferent between investing or not, the benefit of making one more unit of investment is \( q_t \varepsilon_t^* \), as the left-hand side represents, while the cost of making one unit of investment is \( \frac{1}{1-\gamma} \) if information-sensitive debt is signed and simply 1 otherwise, as the right-hand side represents.

Equation (24) is for Tobin’s q. The left-hand side is the marginal cost of installing one more unit of capital, while the right-hand side is the marginal benefit of doing so. The marginal benefit consists of the capital income from production and the capital value net of depreciation.

Equation (25) is the asset pricing equation for houses. The left-hand side is the marginal cost of purchasing one more unit of housing in market; it is simply the house prices. The right-hand side is the ex ante marginal benefit of holding one more unit of housing. Conditional on the house being high quality (indicated by \( \eta_t \)), it can yield rental income in addition to the resale value, \( R_{t+1} + P_{t+1} \). The last term \([ (1-\gamma) + 1_{t+1} \cdot \gamma ] P_{t+1} q_{t+1} \int_{\varepsilon \geq \varepsilon_{t+1}^*} (\varepsilon - \varepsilon_{t+1}^*) f(\varepsilon) d\varepsilon \) is the expression for the liquidity premium. It can be understood as follows: \([ (1-\gamma) + 1_{t+1} \cdot \gamma ] P_{t+1} q_{t+1} \) is the value of the investment by collateralizing one unit of housing in the credit market in period \( t+1 \). However, it would only be meaningful when a firm draws a sufficiently favorable investment-specific shock. Given the size of the loan supported by one unit of collateral, the higher the firm’s investment-specific shock, the larger the benefit. The integral term \( \int_{\varepsilon \geq \varepsilon_{t+1}^*} (\varepsilon - \varepsilon_{t+1}^*) f(\varepsilon) d\varepsilon \) just translates the expected benefits from the investment to the expected benefits from the capital stock accumulated. The transversality conditions are usual according to Ekeland and Scheinkman (1986).

3.2 Equilibrium System

After aggregating capital, investment, output, labor and houses, the equilibrium system is characterized by the following proposition.

Proposition 3 The equilibrium system is given by the following eleven equations: (23),
\[ \varphi N_t^{w} = \frac{W_t}{C_t}, \quad (27) \]
\[ \varphi H E_t \eta_{t+1} = R_t E_t \frac{\eta_{t+1}}{C_{t+1}}, \quad (28) \]
\[ I_t = [(1 - \gamma) + I_t \cdot \gamma] \eta_t P_t \left( \int_{\varepsilon > \varepsilon^*_t} f(\varepsilon) d\varepsilon \right), \quad (29) \]
\[ K_t = (1 - \delta) K_{t-1} + [(1 - \gamma) + I_t \cdot \gamma] \eta_t P_t \left( \int_{\varepsilon > \varepsilon^*_t} \varepsilon f(\varepsilon) d\varepsilon \right), \quad (30) \]
\[ W_t N_t = (1 - \alpha) Y_t, \quad (31) \]
\[ Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \quad (32) \]
\[ Y_t = C_t + I_t, \quad (33) \]
\[ R_t^K = \alpha A_t^{\frac{1}{\alpha}} \left( \frac{1 - \alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}}, \quad (34) \]

for eleven corresponding variables \{\(C_t, I_t, Y_t, K_t, N_t, \varepsilon^*_t, W_t, q_t, R^K_t, R_t, P_t\}\}. The usual transversality conditions hold.

Combining equations (3), (4) and (5) immediately yield equations (27) and (28). Equations (29) and (30) are derived by aggregating investment and capital accumulation across all firms. Equation (31) is the labor market clearing condition equating the total labor supply (the left-hand side) and the total labor demand (the right-hand side). Equation (32) is the aggregate output of all firms and equation (33) is the resource constraint of the economy. Detailed derivations are provided in Appendix A.2.

### 4 The Mechanisms: Credit Booms and Collapses

This section illustrates the impacts of a shock to the collateral quality on the real economy. The parameters are the same as in the calibration, except that there is no consumption habit formation or capital adjustment cost.
4.1 Impulse Responses to the Quality Shock

The presence of regime switching does not allow the application of traditional perturbation methods to solve the model. To overcome this difficulty, I apply the tools developed by Guerrieri and Iacoviello (2015) which modifies the perturbation methods to solve dynamics models with occasionally binding constraints. This method works much more efficiently than highly nonlinear global solution methods and is satisfactory for models in which shocks are reasonably small. Finally, the generated results are readily comparable with those of models without regime switching.

The quality shock is bounded. It is generated in the following way. Define an auxiliary stochastic variable $\tilde{\eta}_t$ that follows a standard AR(1) stochastic process $\tilde{\eta}_t = \rho \tilde{\eta}_{t-1} + (1 - \rho \tilde{\eta}) \tilde{\eta}_0 + \sigma \tilde{\eta} \tilde{\epsilon}_t$. Then transform $\tilde{\eta}_t$ to $\eta_t$ using a logistic function $\eta_t = \bar{\eta} + (\eta - \bar{\eta}) \frac{\exp(\tilde{\eta}_t)}{1 + \exp(\tilde{\eta}_t)}$. In this way, $\eta_t$ is bounded by $[\eta, \bar{\eta}] \subset [0, 1]$. The upper bound $\bar{\eta}$ and lower bound $\eta$ characterize the range within which collateral quality fluctuates. I examine the responses to a positive disturbance in the quality shock, the size of which is sufficiently large to trigger a regime switch.

Assume that the nonstochastic steady state regime is information sensitive. The impulse responses to the quality shock are shown by Figures 4 and 5. In each panel, there are two lines: The solid lines are the impulse responses generated by the basic model, and the dashed lines are the impulse responses generated by a reference model wherein information sensitivity (the regime in the steady state) is always on. The comparison of the baseline impulse responses and the reference impulse responses can be interpreted in three ways: (i) The solid lines are the responses with regime switching turned on, while the dashed lines are responses with regime switching turned off. (ii) The solid lines are the responses to a large positive shock, while the dashed lines are responses to a small positive shock. The shock is “large” if condition (10) is satisfied on impact and “small” otherwise.

\[ F(\varepsilon) = 1 - (\frac{1}{\chi})^\chi, \] where the lower bound of the shock is simply 1 and the shape parameter governing the heterogeneity is $\chi$. 

12 The cumulative density function of the investment-specific shock is $F(\varepsilon) = 1 - (\frac{1}{\chi})^\chi$, where the lower bound of the shock is simply 1 and the shape parameter governing the heterogeneity is $\chi$. 

22
(iii) The solid lines are the responses to a large positive shock, while the dashed lines are responses to an equal-sized negative shock except that they should be flipped down. When the shock is negative, it would never make condition (10) satisfied regardless of how large it is.

It is remarkable that the two groups of response lines differ from each other substantially. We study the intuition now. Suppose there is a positive disturbance in the collateral quality. If the size of the disturbance is small enough that condition (10) is not met, the responses of the real variables should behave like the reference impulse responses, except that the magnitude would be re-scaled proportionally to the size of the disturbance. In contrast, if the size of the disturbance is large enough that condition (10) is met on impact, the information-insensitive regime is triggered and lasts for a while. After that the disturbance washes out, the collateral quality decreases gradually and returns to its steady state value. Eventually, the information-sensitive regime would be implemented again. The switch can be visualized clearly in the panel titled “Quality” in Figure 5. In that panel, the solid line is the collateral quality over time, and the dotted line represents the cutoff value at which the condition (10) is exactly met.

When there is a positive collateral quality shock, house prices rise immediately. Obviously, the house rent component increases due to the presence of more high-quality houses (although the rent declines in the initial periods). Moreover, the liquidity premium increases much more vigorously, as larger loans can be supported by the same quantity of collateral. This contributes to a strong response in the house price-rent ratio. The intuition for this result is explained further in Section 4.3.

When supported by a smaller quantity of high-quality collateral, investment becomes cheaper. Thus, Tobin’s q declines, and the rate of return to capital rises initially. Investment is stimulated. Capital, hours worked and output all increase.

The key difference between the two groups of lines lies in the extensive margin illustrated by “Cutoff investment efficiency” in Figure 5. The cutoff efficiency decreases with the
regime switch (though this is not visually obvious), while the cutoff efficiency increases without the regime switch. This contrast arises from the switch in equation (23). If there is no switch, \( q_t\varepsilon^*_t \) is a constant. The fact that \( q_t \) decreases must indicate that \( \varepsilon^*_t \) increases. However, if \( q_t\varepsilon^*_t = \frac{1}{1-\gamma} \) switches to \( q_t\varepsilon^*_t = 1 \) in the impact period, the cutoff efficiency \( \varepsilon^*_t \) may decrease rather than increase. In other words, with regime switching, a sufficient positive shock may cause both the intensive and extensive margins to relax. This explains why the liquidity premium upsurges so remarkably with regime switching compared to the increase observed without it—both of the two margins contribute to an increase in the demand for collateral.

With this difference, the investment in the information-insensitive regime is considerably larger than that in the default regime. Consequently, output and capital stock increase while consumption decreases. At the same time, a higher stock of capital calls for higher labor inputs. In the calibration, labor is assumed to be indivisible, which implies that wage co-moves with consumption. This is why wage initially decreases. As a substitute to consumption, housing service demand decreases, so house rents also initially fall.

Output increases over time. Accumulated capital drives down the rate of return to capital but calls for more labor inputs. Thus, hours worked remain high at this time. The wealth effect gradually becomes dominant such that consumption, wages and house rents increase above their steady state values.

By the switching period, the quality decreases below the cutoff value, and the information-sensitive regime is active again. After the switch, the investment becomes more expensive, the rate of return to capital falls and Tobin’s \( q \) rises immediately. The extensive margin of the investors decreases immediately as the equation \( q_t\varepsilon^*_t = 1 \) changes from \( q_t\varepsilon^*_t = \frac{1}{1-\gamma} \) back to \( q_t\varepsilon^*_t = \frac{1}{1-\gamma} \). All these factors result in a sudden decrease in aggregate investment and, hence, a sudden decrease in aggregate output and a decreasing trend in capital stock.

As investment is suppressed, consumption, wage and house rents increase immediately after the regime switch. The implied discontinuity is interesting. Given an abrupt decrease
in the rate of return to capital, it is optimal to allocate consumption in a discontinuous way. This further implies a spike in the “implicit” intertemporal interest rate (which is measured by $1/ \left[ \beta E_t \left( \frac{C_t}{C_{t+1}} \right) \right]$) during the switch. This is consistent with what we observed during the financial crisis. When the financial crisis occurred, interest rates of various types of bonds skyrocketed. See the detailed discussion in Section 3.1.

House prices also exhibit an abrupt change with the regime switch. As analyzed above, house rents increase at the switch. However, the liquidity premium exhibits a steep decrease just before the switch. Overall, house prices exhibit a discontinuity similar to that of the house rents component as it weighs more in the house price. This also explains the pattern observed for the house price-rent ratio in the model.

4.2 Magnification Effect

Comparing the impulse responses of the basic model and the reference model, we can see that the regime switch affects the real economy in three dimensions: (i) It causes stronger responses. Without the regime switch, the responses are much smoother and smaller. As noted in Bigio (2015), strong responses of output are rare in models wherein borrowing constraints only distort investment. (ii) It causes asymmetric responses – the responses of an economy to a “large” shock may be disproportionately larger than to a “small” one, and the magnitude of the responses of an economy to a positive shock may be disproportionately larger than to an equal-sized negative shock; (iii) It creates additional volatility to the economy. All these characteristics are evident in the disparity between the solid lines and the dashed lines.

Among the above-mentioned points, (i) deserves more discussion. The total credit sustained by the total collateral in the economy is represented by $(1 - \gamma + 1_t \cdot \gamma) \eta_t P_t H_t$ where $H_t = 1$. Inspecting equation (25), it is clear that when $\eta_t$ varies and is time-persistent, $P_t$ responds positively. So the total value of qualified collateral changes not

\footnote{For example, the interest rate increases on 10-year US Government Treasury as well as AAA and BBB corporate bonds during the financial crisis.}
only due to the quantity change of qualified collateral \((\eta_t H_t)\), but also due to the price change of collateral goods \((P_t)\). On top of that, the regime switch also contributes to the amplified responses via aggregate investment: It affects the loan size per unit of collateral (represented by \((1 - \gamma + 1 \cdot \gamma)\)) as well as the extensive margin of the investors (represented by \(\int_{\varepsilon > \varepsilon^*_t} f(\varepsilon) d\varepsilon\) in equation (29)). By contrast, if regime switching is shut down, the shrink in the extensive margin would significantly offset the increase of house prices. All these channels contribute to magnify the responses of the variables to a disturbance in the collateral quality shock. They are summarized by the following log-linearized version of equation (29) where a variable with a hat denotes the percentage deviation from its steady-state value:

\[
\hat{I}_t = (1 - \gamma + 1 \cdot \gamma) + \hat{\eta}_t + \hat{P}_t + \hat{H}_t + \left(\int_{\varepsilon > \varepsilon^*_t} f(\varepsilon) d\varepsilon\right).
\]

### 4.3 Implication of Collateral Quality Shock on Housing Price and Housing Price-Rent Ratio

Finally, we compare the magnitudes of responses of output and house prices. An improvement in collateral quality raises house prices, then investment and hence output. Thus the response of house prices is larger than that of output. Due to the strong house price response, the house price-rent ratio also responds more forcefully than output. Moreover, the house price-rent ratio shows a positive correlation with output following this shock. Recall that in previous section we establish that technology shock only is not able to explain the large volatility of house prices and the price-rent ratio. In contrast, the quality shock could explain the three salient patterns. The intuition is explained below:
House prices have two components: the house rent component and the liquidity premium component. The rent component is denoted by $P_{rt}$ and defined as $P_{rt} = \beta E_t \Lambda_{t+1} \eta_{t+1} (R_t + P_{rt+1})$. It is simply the conventional pricing equation for houses, the present discounted value of the sum of future rent streams. The liquidity premium component is denoted by $P_{lt}$ and defined as $P_{lt} = P_t - P_{rt}$. This is the additional value of the role that houses play in the credit market. In this model, both components are positive conditional on the fact the house is high quality.

$$P_t = \beta E_t \Lambda_{t+1} \eta_{t+1} \left[ R_{t+1} + P_{lt+1} + (1 - \gamma + 1_{t+1} \cdot \gamma) P_{lt+1} q_{lt+1} \int_{\varepsilon \geq \varepsilon_{t+1}^* (\varepsilon - \varepsilon_{t+1}^*) f(\varepsilon) d\varepsilon \right].$$

House prices are much volatile than rents for two reasons: (i) On the one hand, house prices are directly affected by the probability of collateral being high quality, as shown by equation (25). This is consistent with the real-world commonsense that in the subprime mortgage crisis, large numbers of houses lost value and even went underwater, which drove down house prices. On the other hand, house rents reflect the marginal utility of households with respect to housing services, which is much smoother. (ii) The liquidity premium component is far more volatile than the house rent component, leading to higher volatility of house prices relative to house rents.

It is noteworthy that comparisons between house prices and output and between the house price-rent ratio and output do not depend on whether the regime switch is on or off. It is the fluctuation in collateral quality rather than the regime switch that is responsible for these qualitative implications. The presence of regime switching may further strengthen these channels.
5 Further Exploration

We further explore the quality shock in this section in the following ways: (i) making the loan intertemporal instead of intratemporal and checking the response of the interest rate; (ii) examining whether the quality shock is important for explaining house prices after taking into other shocks affecting house prices into account; (iii) including payroll in the collateral constraint.

5.1 Interest Rate Spikes

The financial crisis featured interest rate spikes on corporate bonds. During normal times, firms with Aaa (triple A) ratings are considered the safest, and they can borrow at a rate close to the rate on government bonds. Firms with Baa (triple B) can borrow at a higher rate but the premium is typically small. In September 2008, there were sharp increases in the rates of Aaa and Baa corporate bonds, while the rates on government bonds remained very low. (Refer to the left one in Figure 12.) Suddenly, the sudden surges of interest rates made borrowing difficult for most firms.

In the basic model, I show that regime switching can facilitate appearence of interest rate spikes. In this section, I replace the intratemporal debts in the basic model with intertemporal debts and examine the behavior of the interest rate.

To introduce intertemporal debts, firms pay their obligations back not at the end of each period but at the beginning of next period before all shocks are realized. Thus, borrowers are charged a predetermined interest rate.

Under this setup, the budget constraint of a representative household (2) becomes

\[ C_t + \eta_t R_t H_{t-1} = D_t + W_t N_t + M_t + R_{bt-1} B_{t-1} - B_t, \]  

(35)

where \( B_t \) is the total size of the loans that a household lends to firms, and \( R_{bt-1} \) is the
interest rate. It is determined by the first-order condition with respect to $B_t$

$$1 = \beta R_{bt} E_t \frac{\Lambda_{t+1}}{\Lambda_t}. \quad (36)$$

Meanwhile, the equilibrium conditions for the two types of debt contracts change to

$$R_{bt} B_t^{IS}(i) = (1 - \gamma) P_t, \quad (37)$$
$$R_{bt} B_t^{II}(i) = \eta_t P_t, \quad (38)$$

which simply say what the firm $i$ needs to repay is the principle and interest. They are determined in the same way as in the basic model. Finally, firm $i$ uses loans to finance its investment $I_t(i): B_t(i) = I_t(i) + (1 - 1_t) \cdot \frac{\gamma^2}{1 - \gamma} I_t(i)$.

The economic response with intertemporal debts is smoother than that with intratemporal debts around the regime switch (not plotted). This is because just before the switch occurs, the interest rate has already increased, which reduces the sizes of loans. See the dash-dotted line in Figure 6. The interest rate spike appears because, right after the regime switching, borrowing becomes costlier due to the information acquisition cost. The behavior of the interest rate is explored further in the following experiments: (i) There is only one large shock in the initial period, as in Figure 5; (ii) In addition to the initial shock, there is a positive shock right before the original switching period such that the quality does not fall below the cutoff value as in the basic model; (iii) In addition to the initial shock, there is a negative shock right before the original switching period such that the quality falls below the cutoff value immediately, i.e., the regime switch occurs one period ahead.

The results are shown in Figure 6. The blue dash-dotted line, the solid line with star markers, and the dashed line with diamond markers represent the impulse responses corresponding to the cases (i), (ii) and (iii), respectively. In case (i), the switch occurs in periods 22 and 23. The spike in the interest rate occurs just in period 22. The interest
rate has already risen several periods before, subsequently decreasing towards the posterior trend.

In case (ii), the interest rate also rises several periods before the switch. However, a spike does not appear. Instead, the rate decreases towards the previous trend. However, the rate spikes in later periods where the quality deteriorates enough to trigger the regime switch. Interestingly, the interest rate moves not only when a regime switch occurs, it but does move when quality is closer to the cutoff value. Even when the regime switch is not actually triggered, the interest rate also responds to some extent. This is similar to the anticipation effect described in [He and Krishnamurthy (2014)]. In their work, house prices fall before entering the capital constrained region. As the economy moves closer to the binding condition of the constraint, the likelihood of falling into the constrained region increases, which affects asset prices immediately. This is a unique feature associated with regime switching. In the reference model without regime switching, the responses of the interest rate would be flat (not plotted).

In case (iii), the interest rate rises in the same way as in the previous two cases, and it decreases directly toward the trend after the switch. There is no other spike since the quality is kept below the cutoff value afterwards. In this case, the unexpected deterioration of the quality prevents an interest rate spike.

However, as lenders in the credit market bear no risk of loss in this model, spreads between rates cannot be accounted for.

5.2 Distinguishment From Other Shocks

In Section 4, I have shown that the collateral quality shock outperforms the technology shock in accounting for the high volatility of house prices and the price-rent ratio. In this section, I further examine the importance of the quality shock when considering other shocks that may also affect house prices. Specifically, I compare the qualitative implications of the quality shock with those of technology shocks and three other shocks commonly...
used to explain house price movements and the financial crisis in the literature: financial constraint shocks, housing demand shocks and equity shocks.

The model setup is the same as before, but uses these new shocks. Technology shocks are believed to be the typical driving force of business cycles. Assume that the technology shock follows a standard AR(1) stochastic process: \( \ln(A_t) = \rho A \ln(A_{t-1}) + \sigma A \epsilon_t^A \). Meanwhile, the period utility of the household derived from housing services becomes \( \varphi_H \eta_t H_{t-1} \), where \( \varphi_H \) reflects the demand for housing services. It follows an AR(1) stochastic process: \( \ln(\varphi_H) = \rho_H \ln(\varphi_{H, t-1}) + (1 - \rho_H) \ln(\varphi_H) + \sigma_H \epsilon_H^t \). The financial constraint is added such that \( 0 \leq I_t(i) \leq [(1 - \gamma) + 1_t \cdot \gamma] \theta_t \eta_t \rho_t H_{t-1}(i) \). The parameter \( \theta_t \) stands for the financial constraint. Similar to the quality shock, it is also bounded, so it is generated in a way similar to \( \eta_t \). The equilibrium system of this modified model is given in Appendix A.3.

**Technology Shock**  The impulse responses of the economy to a positive disturbance of the technology are given by the left column of Figures 7. With an improved production technology, the rate of return to capital increases on impact. Tobin’s q is higher, so it is profitable for more firms to invest, that is, the cutoff efficiency is lower. As a result, higher demand for investment pushes up aggregate investment as well as demand for collateral.

House prices are increased by a positive technology shock. This change comes from two sources: one is the higher demand for collateral boosted by higher demand for investment. The other is the wealth effect leading to higher demand for housing services.

The liquidity premium is more responsive to a technology shock than is the house rent component, as the technology shock affects collateral demand directly. However, given the calibration, the house rent component weighs more than the liquidity premium in house prices in the steady state. Overall, the responsiveness of house prices is roughly one-to-one correspondence with the technology shock, while output exhibits greater than one-to-one correspondence. Apparently, the technology shock solely is not enough to account for the greater volatility of house prices than of output observed in the data. More importantly, there is little fluctuation in the house price-rent ratio, although movements occur in the
right direction. This result is inconsistent with the data indicating that the house price-rent ratio is even more volatile than output. Therefore, the technology shock can hardly be regarded as an explanation for the notably higher volatility of house prices and the house price-rent ratio.

**Housing Demand Shock**

The right column of Figure 7 provides the impulses responses to a positive housing demand shock. This shock generates a larger response of house prices relative to output, but the dynamic response of rents is substantially higher than that of house prices, leading the house price-rent ratio to negatively respond to the shock. The intuition for this is that the increase in house prices is mainly contributed by the increase in rents, while the liquidity premium moves little. Thus, the movements of house prices could hardly be larger than the movements of house rents. Also, house prices and the price-rent ratio are negatively correlated with this shock only. Therefore, a housing demand shock should not be the dominant factor explaining house prices after taking house rents into account.

In the literature, a housing demand shock is regarded as one of the most important sources of house price fluctuations. Iacoviello (2005) estimates housing demand shocks and find them utterly volatile. Iacoviello and Neri (2010) analyzes the relative contributions of various shocks related to housing markets and find that housing demand accounts for around 25 percent of house price volatility. Liu, Wang, and Zha (2013) notes that a rise in housing demand from the household sector would give rise to a sizable response of house prices. This is because, housing demand in the household sector raises house prices, which may relax the borrowing constraints of producers in the business sector. Both expansion of production and higher household wealth would aggravate increasing in house prices.

However, rent is not considered in these studies. Rent is directly linked to housing demand shocks. In contrast, the liquidity premium is less affected. As a result, if housing demand shocks were the main driving force of house price movements, it would be difficult for us to understand the following conflict: the data show a positive correlation between
house prices and the house price-rent ratio, while the model yields an evident negative one.

In fact, Miao, Wang, and Zha (2014) address this problem by noting that, to generate a house price materially more volatile than its house rent component, a liquidity premium shock is required. They define the liquidity premium in their analytical model as the difference between house prices and discounted present value of future rent streams, similarly to the definition used in this model. They show that a shock to this difference can reconcile the movement gap between house prices and rents. By referring to the collateral quality shock, this model corroborates their arguments.

Financial Shock I now turn to the financial constraint shock. In practice, the parameter $\theta_t$ is also related to “haircut” in the repo market – the difference between the amount borrowed and the value of the underlying asset. The haircut is usually close to 0 in normal times, but in bad times, it could increase significantly, causing substantial costs for borrowers.\footnote{See Copeland, Martin, and Walker (2011).}

With the financial constraint shock, the house pricing equation (25) becomes

$$P_t = \beta E_t \Lambda_{t+1} \eta_{t+1} \left[ R_{t+1} + P_{t+1} + \theta_{t+1} (1 - \gamma + 1_{t+1} \cdot \gamma) P_{t+1} q_{t+1} \int_{\epsilon \geq \varepsilon_{t+1}} (\varepsilon - \varepsilon_{t+1}) f(\varepsilon) d\varepsilon \right]. \tag{39}$$

Comparing equation (39) with equation (25), it is easy to see that the financial constraint affects the liquidity premium directly. It is naturally expected that this component responds much more strongly than the house rent component to a financial constraint shock.

The impulse responses are given by Figure 8. The financial constraint shock has a stronger impact on investment via the credit channel and, hence, on output. House prices show a weak response, similar to that of house rents. Therefore, the financial constraint shock has a stronger impact on output than on house prices. Meanwhile, the house price-rent ratio responds reluctantly to this shock. This is why this shock solely cannot be
dominant in accounting for the observed patterns in house prices and rents.

**Equity Shock** Several analyses of the recent financial crisis assign an important role to an equity shock (see, e.g., Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Bigio (2015)). This is a disturbance that alters the net worth of entrepreneurs. It is also regarded as a disturbance in the quality of capital that is used in production. Following these works, I introduce the equity shock $\kappa_t$ into the capital accumulation process as follows

$$ K_t(i) = \kappa_t [(1 - \delta)K_{t-1}(i) + \varepsilon_t(i)I_t(i)], $$ (40)

where the stochastic evolution process of the equity shock $\kappa_t$ is AR(1): $\ln(\kappa_t) = \rho_{\kappa} \ln(\kappa_{t-1}) + \sigma_{\kappa} \epsilon^\kappa_t$. I set $\rho_{\kappa} = 0.95$ and $\sigma_{\kappa} = 0.01$.

The impulse responses to an equity shock are given by Figure 8. Indeed, this shock can generate sizable output and house price movements. A positive disturbance can make the effective capital in production rise, which increases output. At the same time, it can function as a positive aggregate investment-specific shock, which increases demand for collateral. However, this shock still suffers the limitation that the house price-rent ratio is less volatile than output and negatively correlated with it, which is inconsistent with the data.

### 5.3 Collateral Constraint on Working Capital

In the basic model, external financing only feeds investments. However, Mendoza (2010) and Bigio (2015) note that borrowing constraints on working capital, especially payroll, is important to generate large and immediate real responses to shocks in financial markets. In this section, I consider that payroll is also subject to credit constraint. Specifically, $W_tN_t(i) + I_t(i) \leq B_t(i)$. The size of a loan is still determined in the same way as in the
basic model. The only difference in the setup is that equation (29) becomes

$$0 \leq I_t(i) \leq [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t H_{t-1}(i) - W_t N_t(i).$$  \hspace{1cm} (41)$$

Figure 9 shows the impulse responses of this economy to a positive shock to collateral quality. The figure also includes the impulse responses in the basic model. The key differences between the two cases are that the responsiveness with payroll is generally stronger than without, especially for output, investment, house prices and house rents, as the inclusion of payroll makes the collateral constraints tighter. As noted in the literature, including working capital into the borrowing constraint produces relatively larger responses in the real economy.

Furthermore, if the investment and working capital were jointly chosen subject to the collateral constraint, further resource misallocation that would hurt aggregate productivity might occur. This may be another source of output response.

6 Quantitative Examination

Having understood the qualitative impacts of the collateral quality shock on business cycles, I ask the following question: How quantitatively important is the quality shock to explaining the stylized facts of real business cycle, including those regarding house prices and house rents? To answer this question, I calibrate the model using US data from 1975 to 2010 to match the real business cycle moments first, and then conduct an experiment to examine whether the quality shock is particularly important to the 2007–2009 financial crisis.

\footnote{For the timing, the labor decisions are made ahead of the realization of the investment-specific shocks.}

\footnote{The amount of the capital and house purchase are indeterminate due to the homogeneity of degree one. To make the upper bound of the investment always positive, we could just assume that $\frac{H_t(i)}{K_t(i)}$ is constant across firms.}
6.1 The Full Model

To calibrate the model, I add two frictions that are typical in the business cycle literature to the basic model: consumption smoothing incentives and convex investment adjustment costs.

First, the utility function of the household now includes consumption habit formation, i.e., the period utility derived from consumption becomes \( \ln(C_t - \varphi C_{t-1}) \), where \( \varphi \) captures the strength of the motive for consumption smoothing.

Second, consumption goods and capital goods are differentiated. For the capital goods, there are capital good producers, as in Bernanke, Gertler, and Gilchrist (1999), Gertler and Kiyotaki (2010) and many others. Capital good producers make new investment goods using inputs of numeraire goods subject to adjustment costs. They sell new investment goods to firms at a price \( Q_t \). The objective of a capital good producer is to choose \( \{I_t\} \) to solve

\[
\max_{I_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left\{ Q_t I_t - \left[ 1 + \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \right\}
\]

where \( \Omega \) is the adjustment cost parameter. The optimal level of investment goods satisfies the first-order condition

\[
Q_t = 1 + \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 + \Omega \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \Omega \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2.
\]

The aggregate investment, aggregate capital and cutoff efficiency of investment are derived accordingly. The modified equilibrium system is presented in Appendix A.3.

6.2 Data

As discussed in Section 4.1, the discontinuity of the equilibrium system makes it difficult to apply conventional quantitative methodologies – such as Bayesian estimation – to con-
front model with the observed data series. In this context, I perform calibration as a quantitative examination to the model.

I consider the moments of six quarterly the US time series: real per capita output, real per capita consumption, real per capita investment (in consumption units), per capita hours worked (as a fraction of total time endowment), house prices, and the house price-rent ratio. The sample spans from Q1 1975 to Q4 2010. I follow Liu, Wang, and Zha (2013) to construct the first four variables. More details can be found in their Appendix A.

The last two data series, house prices and the house price-rent ratio are of particular interest. I check various sources for these data. For house price data, I check four data series: the Federal Housing Finance Agency (FHFA) Home Price Index, the FHFA all-transaction house price index, the FHFA repeat-sales house price index and the Case-Shiller-Weiss index. To obtain the real house price index, I use consumption deflators from two sources: one is the private consumption deflator from the national account statistics and the other is the price index for consumption constructed by Davis and Heathcote (2007).

For the house price-rent ratio data, I inspect three data series: (i) FHFA purchasing-only price index for house prices and the micro data from the Decennial Census of Housing for house rents. The rents paid for rental units are used to estimate rents for owner-occupied units. The rental data are gross and do not account for income taxes or depreciation. (ii) the Macromarkets LLC national house price index (formerly Case-Shiller-Weiss) for house prices and the same house rents as above. The data sources of (i) and (ii) are supported by

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17See Guerrieri and Iacoviello (2015) for a new method to conduct Bayesian estimation of a nonlinear model.

18The FHFA home price index is constructed in the same way as Liu, Wang, and Zha (2013). The FHFA Home Price Index from Q1 1975 to Q1 1991 is seasonally adjusted and spliced to be consistent with the Purchase Only FHFA Home Price Index from Q1 1991 to the present. See the definition of “LiqLandPricesSAFHFASplice” in their Appendix A for more details.

19Please refer to the data source and definition to Appendix in Miao, Wang, and Zhou (2015).

20Please refer to the data sources for the FHFA repeat-sales house price index and Case-Shiller-Weiss index to http://www.lincolninst.edu/subcenters/land-values/price-and-quantity.asp.

21Please refer to the data and sources to Appendix in Miao, Wang, and Zhou (2015).
Davis, Lehnert, and Martin (2008). (iii) the FHFA all-transaction index for house prices and the house rent data from OECD Main Economic Indicators database for house rents.

I detrend all the logarithmic time series using the Hodrick-Prescott (HP) filter by setting the multiplier to 1600. The cyclical parts of each series are the differences between the original series and the HP trend series. Then I compute several moments of the cyclical parts of each variable: the standard deviation, the first-order autocorrelation, and the correlations between each variable and the output.

The moments for house prices and the house price-rent ratio are reported in Table 1 and 2, respectively. The three key patterns discussed above are robust: (i) house price volatility is larger than output volatility; (ii) the house price-rent ratio volatility is larger than output volatility; and (iii) the house price-rent ratio is positively correlated with both output and house prices. Data series of various sources generate moments close to each other except that the Case-Shiller-Weiss index delivers notably larger standard deviations. The choice of the deflator does not affect the patterns, which confirms the robustness of the patterns.

For calibration, I choose the FHFA data series instead of the Case-Shiller-Weiss data series. I use the FHFA repeat-sales data and deflator constructed by Davis and Heathcote (2007) and summarize the moments in column 1 in Table 4. It is also worth mentioning that I use house prices rather than land prices which would be even much more volatile. This choice may raise questions about the assumption that the house supply is fixed. From 2003 to 2006, the core period of the housing boom, sales of houses in the US did not change much. The small change in the quantity of houses is also discussed in Iacoviello and Neri (2010).

22 To compare with Table 1, the land price in Liu, Wang, and Zha (2013) computed from FHFA house price gives a land price volatility as much as 2.67 times that of output volatility, the land price in Davis, Lehnert, and Martin (2008) computed from the FHFA home price gives a volatility 2.78 times the output volatility, and the Case-Shiller-Weiss home price gives a land price volatility 5.28 times the output volatility.

23 The annual data from 2003 to 2006 are: 1086, 1203, 1283, 1051 thousands of units. Source: US Census Bureau.
6.3 Calibration

I group the parameters into two categories: one consults well-established facts or standard values in the literature. The other meets the moments of our particular interest, which are in bold in column 2 of Table 4. All the parameter values are reported in Table 3 and explained in detail below.

The fixed parameters are chosen as follows. The subjective discount factor, $\beta$, is 0.993, which matches the annual risk-free rate in the US. Labor is assumed to be indivisible to make the volatility of hours worked closer to the data, as suggested in Hansen (1985), that is, $\nu = 0$. The parameter for the labor disutility $\varphi_N$ makes the steady state hours worked 1/3 of the total time. The parameter for the housing service preference is set to 0.26 to make the ratio of new house investment to output, $PH^n/Y$, 5%, as reported in Gomme and Rupert (2007). The output elasticity of labor $1 - \alpha$ is 0.6, which is standard in the literature. The capital depreciation rate is 0.03. For the distribution of the idiosyncratic investment-specific shock, the minimum value is set to 1 and the parameter governing the shape is chosen to match the investment to output ratio, roughly 0.25 in US data. I set the lower bound of the financial shock and the upper bound at 0.71 and 0.95, respectively, in line with the data. The mean of the financial constraint shock is simply the arithmetic mean of the upper and lower bound. The unconditional mean of the technology shock is 1, the parameter for time persistence is 0.95, and the standard deviation of the innovation term is 0.01. These are standard values in real business cycle literature.

For the second category, the consumption habit formation parameter $\varphi_C$ is set to 0.23, and the aggregate adjustment cost parameter $\Omega$ is set to 0.42 to conform with the time persistences of consumption and investment. The parameters $\{\tilde{\eta}, \eta, \gamma, \rho^{\tilde{\eta}}, \sigma^{\tilde{\eta}}\}$ are the ones we want to pay special attention to. I elaborate on them below.

The delinquency rates of loans backed by real estate in the US during the 1994-2008 provide hints for the upper and lower bound of the quality shock. More specifically, $\tilde{\eta}$

\footnote{Note that this is the period when the subprime mortgage loan were vigorous. During this period,}
is set to 1 and $\eta$ is set to 0.85. The mean of the quality shock $\eta$ is just the arithmetic mean of the two. For the stochastic process, I can only choose $\rho_\tilde{\eta}$ and $\sigma_\tilde{\eta}$ for the time persistence and standard deviation. I choose them to make the model-generated time persistence and standard deviation closer to the moments related to the house price and the price-rent ratio data. As for the information acquisition cost $\gamma$, there is no consensus in the literature. According to Carlstrom and Fuerst (1997), the monitoring cost could range from 4 percent (a bankruptcy cost estimate by Warner (1977)) to 20 percent (the sum of direct and indirect bankruptcy costs by Altman (1984)) of total firm assets. If liquidation costs are taken into account, the cost would be even higher. Carlstrom and Fuerst (1997) sets this parameter to 25%. In this model, I choose 7.2%, which roughly equals the delinquency rate of the loans backed by real estate just before the onset of the financial crisis in 2008.

6.4 Results and Discussion

The moments for business cycles from both the data and the model are presented in Table 4. Column 1 presents the moments delivered by the US data; column 2, the moments by the model; and column 3, the results without the quality shock. The moments in bold in column 2 are my targets.

The model can adequately replicate the data in terms of these moments. Given the assigned parameters, adding just one more shock to the typical productivity shock is enough to do a good job explaining the stylized business cycle statistics, especially the standard deviation of output, investment, house prices and the house price-rent ratio, as well as the positive correlation between both house prices and the house price-rent ratio and output. With the highlighted key patterns explained satisfactorily, the moments not matched intentionally are still reasonable. Moreover, among the five free variables $\{\overline{\eta}, \eta, \gamma, \rho_\tilde{\eta}, \sigma_\tilde{\eta}\}$ that direct the stochastic process of the quality shock, three of them ($\{\overline{\eta}, \eta, \gamma\}$) are deter-

unqualified collateral was admitted into circulation in credit markets and but the deliquency rates have not been disturbed by the financial crisis yet.
mined by the observed data, as elaborated above. Only two, \(\{\rho_\eta, \sigma_\eta\}\), are the “real” free parameters to match the four moments – \(\text{std}(p_t)/\text{std}(y_t)\), \(\text{std}(p_t/r_t)/\text{std}(y_t)\), \(\text{corr}(p_t, y_t)\), and \(\text{corr}(p_t/r_t, y_t)\) – simultaneously. They are able to do so because of the mechanisms discussed in Section 4.2 and 4.3. In this aspect, such parsimonious calibration indicates that the model outperform models that can function equivalently but require more free parameters.

How reasonable are the parameters regarding the quality shock? With the parameterization, the first-order autocorrelation of the quality shock \(\eta_t\) is roughly 0.99, and the unconditional volatility is 0.011. By comparison, the first-order autocorrelation of the quality implied by the delinquency rate of the commercial loans secured by real estate from Q1 1994 to Q2 2008 shown in Figure 3 is 0.98, and the unconditional volatility is 0.012. Considering that the moments of the delinquency rate is not intentionally targeted, the model agrees with the real data very well. At the same time, the unconditional volatility of the technology shock is 0.028, which is triple the size of the quality shock. This result assures us that the quality shock is indeed small.

When I shut down the quality shock in the calibrated model, some highlighted patterns disappear. The results are reported in column 3 in Table 4. The output volatility does not drop by much, which serves an evidence of the small size I attribute to the quality shock. In contrast, the investment volatility drops by 20 percent, mainly due to the absence of the regime switches. This can be verified by the investment volatility shown in the two columns titled “IS only” and “II only” in Table 5. More importantly, the volatility of house prices and the price-rent ratio are implausibly lower than the data, and the output volatility. The volatility of the house price-rent ratio is especially low with the technology shock only, as inferred in Section 4. Meanwhile, the correlation between the house price-rent ratio and output is too high.

Table 5 presents the results from other variations of the model. Columns 1 and 2 are the same as those in the previous table. Columns 3 and 4 are the models with the technology
and the financial shock only and with the technology and the housing preference shock only, respectively. Columns 5 and 6 are models with no regime switching – the information-sensitive regime only and the information-insensitive regime only, respectively.

I properly adjust the standard deviation of the financial shock in column 3 and that of the housing preference shock in column 4 to make the output volatility consistent with the data. The limitations of these two variations are clearly reflected by the house price volatility and the price-rent ratio volatility. The financial shock and the technology shock cannot give reasonably high values for these two moments. The housing preference shock and the technology shock together might be able to yield much volatile house price and price-rent ratio values, but the negative correlation between the house price-rent ratio and output is obviously counterfactual. The intuitions are discussed in Section 5.

Finally, I demonstrate the contributions made by the regime switches in the last two columns of Table 5. When holding the parameters unchanged but eliminating the regime switches, the absolute volatility of all variables decreases, with investment decreasing the most. This supports the claim that the regime switches bring additional volatility into the economy.

6.5 Relevance to the 2007-2009 Financial Crisis

Is the collateral quality shock particularly relevant to the recent financial crisis in 2007-2009? The parameterized model allows me to conduct an experiment on that. I choose a sequence of the productivity shocks and a sequence of the quality shocks that closely track the paths of the variables in the data, especially the house price path, use the chosen shock sequences to simulate the model and examine the role of the quality shock. For the data series, I detrend the data as in Bigio (2015).

Figure 10 displays the results. The green lines with circle markers are the paths depicted by the US data, the red lines with square markers are the paths depicted by the model and the blue lines with diamond markers are the paths generated by the model with the quality shock.
shock shut down. As in Bigio (2015), the sharp decrease in productivity contributes to the crisis since we do not consider changes in utilization rates of input factors.

The quality shock also contributes to the crisis. This can be seen from the comparison between the two lines with and without the quality shock. The quality decreases sharply across the cutoff line, which triggers a regime switch. Output, investment and labor show deeper decreases. More importantly, the collapse of house prices and the price-rent ratio would not be accounted for when excluding the quality shock, as analyzed in Section 5.2 by a combination of other shocks.

To further examine the relevance of the quality shock to other boom-bust cycles, I conduct the same exercises using data for 1990 - 1992 and 2000 - 2002 periods, which are the two other recent business cycles. The results are given in Figure 11. In contrast to the 2007-2009 financial crisis, the house price and the house price-rent ratio data varied much less during these two episodes. Consequently, the lines generated by the model with and without quality shocks are very similar, implying an ignorable role of the quality shock in these boom-bust cycles. Therefore, the quality shock is particularly relevant to the recent financial crisis.

In addition, as analyzed in Section 5.1 there is an interest rate spike just before a regime switch. I simulate the model and compare the response of the interest rate in the model with that in the data. In Figure 12 I plot the Moody’s Aaa and Baa corporate bond yields on the left and the model implied interest rate on the right. Clearly, the model suggests an interest rate spike at the beginning of the crisis of a similar magnitude. Notice that the timing of the interest rate spike in the data is a bit later than in the model. NBER demarcates the start of the recession in 2007, nearly one year before the declared bankruptcy of Lehman Brothers. Part of this delay may be due to the information asymmetry in financial markets and the public. However, this is beyond the scope of this paper.
7 Conclusion

With the rise of security-based credit in modern financial markets, not all collateral is high quality and the true quality of collateral is not easily observable or assessable by unsophisticated investors and the public. To investigate the quality of collateral, an information acquisition cost is paid. This paper examines the qualitative and quantitative implications of this friction. Qualitatively, there are asymmetric responses to positive and negative shocks, as well as to small and large shocks. When the collateral quality is high, the information acquisition cost can be avoided and all collateral can be used to borrow against. When the collateral quality is low, the information cost is paid, and only collateral of high quality is accepted for loans. The switches between the two regimes – in which the information is either acquired or not – can amplify the responses of real variables and introduce additional volatility into the economy. Quantitatively, even small shocks of collateral quality can help explain the large volatility of house prices and the house price-rent ratio as well as the positive correlation between the house price-rent ratio and output. This model mitigates the limitations of other shocks that are commonly used in the literature to explain housing price movements.

Two extensions of the current work would be interesting: one is to endogenize the fluctuations of collateral quality and the accumulation of opaqueness before crises. Another is to associate the information regime changes with learning, which may account for slowly increasing booms and sudden collapses. These are challenging but important issues for future research.
References


8 Figures

Figure 1: The Logarithm of output, investment, house prices, and the house price-rent ratio in the US.
Figure 2: Business cycles of output, investment, house prices, and the house price-rent ratio in the US. The data series are cyclical parts of the logarithmic variables after Hodrick-Prescott filtered with the multiplier set at 1600.
Figure 3: Delinquency rate on loans secured by real estate, all commercial banks in the US. The red solid line is the delinquency rate is on all loans secured by real estate. The blue dashed line is on residential real estate loans, which include loans secured by one- to four-family properties, including home equity lines of credit. The green dash-dotted line is the delinquency rate on commercial real estate loans, which include construction and land development loans, loans secured by multifamily residences, and loans secured by nonfarm, nonresidential real estate.
Figure 4: Impulse responses to a positive collateral quality shock.
Figure 5: Impulse responses to a positive collateral quality shock (continued).
Figure 6: Dynamics of the interest rate with intertemporal loans.
Figure 7: Impulse responses to a positive technology shock (left column) and to a positive housing preference shock (right column).
Figure 8: Impulse responses to a positive financial constraint shock (left column) and to a positive equity shock (right column).
Figure 9: Impulse responses to a positive quality shock in the model with working capital subject to the collateral constraint.
Figure 10: Model fit with the financial crisis data.
Figure 11: Model fit with the 1990-1991 and 2000-2001 boom-bust cycle data.
Figure 12: Model fit of the interest rate with the financial crisis data
### Table 1: Business Cycle Moments of House Prices

<table>
<thead>
<tr>
<th>House price</th>
<th>Deflator 1</th>
<th>Deflator 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Standard deviation relative to output</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHFA purchase only</td>
<td>1.27</td>
<td>1.30</td>
</tr>
<tr>
<td>FHFA repeat-sales</td>
<td>1.34</td>
<td>1.36</td>
</tr>
<tr>
<td>FHFA all-transaction</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>Case-Shiller-Weiss</td>
<td>2.20</td>
<td>2.22</td>
</tr>
<tr>
<td><strong>B. First-order autocorrelation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHFA purchase only</td>
<td>0.91</td>
<td>0.93</td>
</tr>
<tr>
<td>FHFA repeat-sales</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>FHFA all-transaction</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>Case-Shiller-Weiss</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>C. Correlation with output</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHFA purchase only</td>
<td>0.51</td>
<td>0.44</td>
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<tr>
<td>FHFA repeat-sales</td>
<td>0.55</td>
<td>0.49</td>
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<tr>
<td>FHFA all-transaction</td>
<td>0.53</td>
<td>0.53</td>
</tr>
<tr>
<td>Case-Shiller-Weiss</td>
<td>0.50</td>
<td>0.46</td>
</tr>
</tbody>
</table>

1 “FHFA purchase only” is the Federal Housing Finance Agency (FHFA) Home Price Index.
2 “FHFA repeat-sales” is the FHFA repeat-sales house price index.
3 “FHFA all-transaction” is the FHFA all-transaction house price index.
4 “Case-Shiller-Weiss” is the Macromarkets LLC national house price index.
5 “Deflator 1” is the private consumption deflator from national account statistics.
Table 2: Business Cycle Moments of House Price-Rent Ratio

<table>
<thead>
<tr>
<th>House price-rent ratio</th>
<th>Deflator 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Standard deviation relative to output</strong></td>
<td></td>
</tr>
<tr>
<td>FHFA repeat-sales</td>
<td>1.33</td>
</tr>
<tr>
<td>FHFA all-transactions</td>
<td>1.38</td>
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<tr>
<td>Case-Shiller-Weiss</td>
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<td><strong>B. First-order autocorrelation</strong></td>
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<td>FHFA repeat-sales</td>
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<tr>
<td>FHFA all-transactions</td>
<td>0.92</td>
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<tr>
<td>Case-Shiller-Weiss</td>
<td>0.93</td>
</tr>
<tr>
<td><strong>C. Correlation with output</strong></td>
<td></td>
</tr>
<tr>
<td>FHFA repeat-sales</td>
<td>0.42</td>
</tr>
<tr>
<td>FHFA all-transactions</td>
<td>0.70</td>
</tr>
<tr>
<td>Case-Shiller-Weiss</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Table 3: Parameters for the Calibrated Model

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
<th>Description / Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.993</td>
<td>annual risk-free rate in the US</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0</td>
<td>indivisible labor</td>
</tr>
<tr>
<td>$\phi_N$</td>
<td>2.56</td>
<td>steady state labor at $1/3$</td>
</tr>
<tr>
<td>$\phi_H$</td>
<td>0.23</td>
<td>housing investment to GDP ratio</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
<td>standard in the literature</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>standard in the literature</td>
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<tr>
<td>$\varepsilon_{min}$</td>
<td>1</td>
<td>assigned</td>
</tr>
<tr>
<td>$\chi$</td>
<td>8</td>
<td>investment to GDP ratio</td>
</tr>
<tr>
<td>$\overline{H}$</td>
<td>1</td>
<td>normalization</td>
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<tr>
<td>$\theta$</td>
<td>0.71</td>
<td>lower bound of loan-to-value ratio in the US</td>
</tr>
<tr>
<td>$\overline{\theta}$</td>
<td>0.95</td>
<td>upper bound of loan-to-value ratio in the US</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.83</td>
<td>$0.5\theta + 0.5\overline{\theta}$</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.95</td>
<td>standard in the literature</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.01</td>
<td>standard in the literature</td>
</tr>
</tbody>
</table>

A. Assigned

B. Calibrated

| $\phi_C$ | 0.23 | time persistence of consumption in the US |
| $\Omega$ | 0.42 | time persistence of investment in the US |
| $\gamma$ | 0.072 | the delinquency rate on loans secured by real estate at the start of the crisis |
| $\eta_\lambda$ | 0.85 | the lower bound of the overall delinquency rate and foreclosure rate in the US from 1995 to 2007 |
| $\eta$ | 1 | the upper bound of the data above |
| $\eta$ | 0.92 | $0.5\eta + 0.5\overline{\eta}$ |
| $\rho_\eta$ | 0.99 | time persistence of house prices and the price-rent ratio |
| $\sigma_\eta$ | 0.035 | volatility of house prices and the price-rent ratio |
Table 4: Business Cycle Moments of the Model and Data

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>Tech. only</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Standard deviation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{std}(y_t)$</td>
<td>0.017</td>
<td><strong>0.017</strong></td>
<td>0.016</td>
</tr>
<tr>
<td>$\text{std}(c_t)$</td>
<td>0.0093</td>
<td>0.010</td>
<td>0.0096</td>
</tr>
<tr>
<td>$\text{std}(i_t)$</td>
<td>0.042</td>
<td>0.045</td>
<td>0.036</td>
</tr>
<tr>
<td>$\text{std}(n_t)$</td>
<td>0.018</td>
<td>0.011</td>
<td>0.0074</td>
</tr>
<tr>
<td>$\text{std}(p_t)$</td>
<td>0.023</td>
<td>0.025</td>
<td>0.014</td>
</tr>
<tr>
<td>$\text{std}(p_t/r_t)$</td>
<td>0.023</td>
<td>0.022</td>
<td>0.044</td>
</tr>
<tr>
<td><strong>B. Standard deviation relative to output</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{std}(c_t)/\text{std}(y_t)$</td>
<td>0.55</td>
<td>0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>$\text{std}(i_t)/\text{std}(y_t)$</td>
<td>2.47</td>
<td><strong>2.63</strong></td>
<td>2.19</td>
</tr>
<tr>
<td>$\text{std}(n_t)/\text{std}(y_t)$</td>
<td>1.05</td>
<td>0.64</td>
<td>0.45</td>
</tr>
<tr>
<td>$\text{std}(p_t)/\text{std}(y_t)$</td>
<td>1.36</td>
<td><strong>1.46</strong></td>
<td>0.84</td>
</tr>
<tr>
<td>$\text{std}(p_t/r_t)/\text{std}(y_t)$</td>
<td>1.33</td>
<td><strong>1.28</strong></td>
<td>0.27</td>
</tr>
<tr>
<td><strong>C. First-order autocorrelation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(y_t, y_{t-1})$</td>
<td>0.90</td>
<td>0.80</td>
<td>0.81</td>
</tr>
<tr>
<td>$\text{corr}(c_t, c_{t-1})$</td>
<td>0.90</td>
<td>0.73</td>
<td>0.72</td>
</tr>
<tr>
<td>$\text{corr}(i_t, i_{t-1})$</td>
<td>0.87</td>
<td>0.82</td>
<td>0.86</td>
</tr>
<tr>
<td>$\text{corr}(n_t, n_{t-1})$</td>
<td>0.92</td>
<td>0.78</td>
<td>0.84</td>
</tr>
<tr>
<td>$\text{corr}(p_t, p_{t-1})$</td>
<td>0.94</td>
<td>0.68</td>
<td>0.60</td>
</tr>
<tr>
<td>$\text{corr}(p_t/r_t, p_{t-1}/r_{t-1})$</td>
<td>0.96</td>
<td>0.69</td>
<td>0.49</td>
</tr>
<tr>
<td><strong>D. Correlation with output</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(c_t, y_t)$</td>
<td>0.93</td>
<td>0.81</td>
<td>0.98</td>
</tr>
<tr>
<td>$\text{corr}(i_t, y_t)$</td>
<td>0.97</td>
<td>0.93</td>
<td>0.99</td>
</tr>
<tr>
<td>$\text{corr}(n_t, y_t)$</td>
<td>0.81</td>
<td>0.76</td>
<td>0.84</td>
</tr>
<tr>
<td>$\text{corr}(p_t, y_t)$</td>
<td>0.49</td>
<td><strong>0.64</strong></td>
<td>0.94</td>
</tr>
<tr>
<td>$\text{corr}(p_t/r_t, y_t)$</td>
<td>0.42</td>
<td><strong>0.35</strong></td>
<td>0.80</td>
</tr>
</tbody>
</table>

1 The numbers in bold are of my particular interest and targeted.
2 $\text{std}(x_t)$ denotes the volatility of variable $x_t$.
3 $\text{corr}(x_t, x_{t-1})$ denotes the first order correlation of variable $x_t$.
4 $\text{corr}(x_t, y_t)$ denotes the correlation of variable $x_t$ with output.
Table 5: Business Cycle Moments of the Model and its Variations

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>Tech. &amp; Fin.</th>
<th>Tech. &amp; Pref.</th>
<th>IS only</th>
<th>II only</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Standard deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{std}(y_t)$</td>
<td>0.017</td>
<td><strong>0.017</strong></td>
<td>0.017</td>
<td>0.017</td>
<td>0.0164</td>
<td>0.0167</td>
</tr>
<tr>
<td><strong>B. Standard deviation relative to output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{std}(i_t)/\text{std}(y_t)$</td>
<td>2.47</td>
<td><strong>2.63</strong></td>
<td>2.50</td>
<td>2.50</td>
<td>2.21</td>
<td>2.16</td>
</tr>
<tr>
<td>$\text{std}(p_t)/\text{std}(y_t)$</td>
<td>1.36</td>
<td><strong>1.46</strong></td>
<td>0.82</td>
<td>5.21</td>
<td>1.52</td>
<td>1.49</td>
</tr>
<tr>
<td>$\text{std}(p_t/r_t)/\text{std}(y_t)$</td>
<td>1.33</td>
<td><strong>1.28</strong></td>
<td>0.30</td>
<td>4.66</td>
<td>1.27</td>
<td>1.25</td>
</tr>
<tr>
<td><strong>C. Correlation with output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(p_t,y_t)$</td>
<td>0.49</td>
<td><strong>0.64</strong></td>
<td>0.91</td>
<td>0.40</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>$\text{corr}(p_t/r_t,y_t)$</td>
<td>0.42</td>
<td><strong>0.35</strong></td>
<td>0.75</td>
<td>-0.22</td>
<td>0.29</td>
<td>0.29</td>
</tr>
</tbody>
</table>

1. The numbers in bold are of my particular interests and targeted.
2. “Model” denotes the results from the full model.
3. “Tech. & Fin.” denotes the results from the model with technology and financial shocks only.
4. “Tech. & Pref.” denotes the results from the model with technology and housing preference shocks only.
5. “IS only” denotes the results from the model with the information-sensitive regime always active.
6. “II only” denotes the results from the model with the information-insensitive regime always active.
A Appendices

A.1 Proof to Proposition 2

In this section, I solve a firm $i$’s policy in a guess-and-verify manner. We can conjecture that the value function takes the form

$$V(\varepsilon_t(i), K_{t-1}(i), H_{t-1}(i); A_t, \eta_t) = v_t(\varepsilon_t(i), A_t, \eta_t)K_{t-1}(i) + p_t(\varepsilon_t(i), A_t, \eta_t)H_{t-1}(i) + P_tH_t^\alpha. \quad (44)$$

Also conjecture that

$$P_t = \beta E_{t+1} \Lambda_{t+1} \int p_{t+1}(\varepsilon, A_{t+1}, \eta_{t+1})f(\varepsilon)d\varepsilon \quad (45)$$

where $P_t$ is the house price.

Denote

$$q_t = \beta E_{t+1} \Lambda_{t+1} \int v_{t+1}(\varepsilon, A_{t+1}, \eta_{t+1})f(\varepsilon)d\varepsilon. \quad (46)$$

Substitute the conjectured form into the Bellman equation and write it as

$$V(\varepsilon_t(i), K_{t-1}(i), H_{t-1}(i); A_t, \eta_t) = \max_{K_t(i), H_t(i)} R^K_t K_{t-1}(i) - \left[1 + (1 - 1_t) \cdot \frac{\gamma}{1 - \gamma}\right] \frac{K_t(i) - (1 - \delta)K_{t-1}(i)}{\varepsilon_t(i)}$$

$$+ P_t\eta_t H_{t-1}(i) + P_tH_t^\alpha + R_t\eta_t H_{t-1}(i) + q_tK_t(i) \quad (47)$$

subject to

$$(1 - \delta)K_{t-1}(i) \leq K_t(i) \leq (1 - \delta)K_{t-1}(i) + \varepsilon_t(i)[(1 - \gamma) + 1_t \cdot \gamma]\eta_tP_tH_{t-1}(i). \quad (48)$$

When $\varepsilon_t(i)q_t \geq 1 + (1 - 1_t) \cdot \frac{\gamma}{1 - \gamma}$, it is profitable to invest. The firm would invest as
much as possible, i.e.

\[ I_t(i) = [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t H_{t-1}(i), \quad (49) \]

\[ K_t(i) = (1 - \delta) K_{t-1}(i) + \varepsilon_t(i) [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t H_{t-1}(i). \quad (50) \]

Substituting the equations above into the value function, we obtain

\[
V(\varepsilon_t(i), K_{t-1}(i), H_{t-1}(i); A_t, \eta_t) \\
= R^K_t K_{t-1}(i) + P_t H^n_t + R_t \eta_t H_{t-1}(i) + (1 - \delta) q_t K_{t-1}(i) \\
+ \varepsilon_t(i) [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t H_{t-1}(i). \quad (51) \]

When \( \varepsilon_t(i) q_t < 1 + 1_t \cdot \frac{\gamma}{1-\gamma} \), it is not profitable to invest. The firm will invest zero, i.e.

\[ I_t(i) = 0, \quad (52) \]

\[ K_t(i) = (1 - \delta) K_{t-1}(i). \quad (53) \]

Substituting the equations above into the value function, we obtain

\[
V(\varepsilon_t(i), K_{t-1}(i), H_{t-1}(i); A_t, \eta_t) \\
= R^K_t K_{t-1}(i) + P_t \eta_t H_{t-1}(i) + P_t H^n_t + R_t \eta_t H_{t-1}(i) + (1 - \delta) q_t K_{t-1}(i). \quad (54) \]

Combining equation (51) and (54), matching the coefficients with (44) and substituting them back to equation (45) and (46) yield

\[
q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ R^{K}_{t+1} + (1 - \delta) q_{t+1} \right], \\
P_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \eta_{t+1} \left[ R_{t+1} + P_{t+1} + (1 - \gamma + 1_{t+1} \cdot \gamma) P_{t+1} q_{t+1} \int_{\varepsilon \geq \varepsilon^*_t} (\varepsilon - \varepsilon^*_t) f(\varepsilon) d\varepsilon \right].
\]

The statements in Proposition 2 are proved.
A.2 Proof to Proposition 3

Equations (27) and (28) can be directly obtained by substituting equation (3) into equations (4) and (5).

The aggregate investment in (29) is

\[ I_t = \int_0^1 I_t(i) di \]
\[ = \int_{\varepsilon_t(i) < \varepsilon_t^*} 0 \cdot di + \int_{\varepsilon_t(i) \geq \varepsilon_t^*} [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t H_{t-1}(i) di \]
\[ = [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t \int_{\varepsilon_t(i) \geq \varepsilon_t^*} H_{t-1}(i) di \]
\[ = [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t \left( \int_{\varepsilon \geq \varepsilon_t^*} f(\varepsilon) d\varepsilon \right) \]

where the last equality is due to the i.i.d. distribution of \( \varepsilon \).

The aggregate capital in (30) is

\[ K_t = (1 - \delta) K_{t-1} + \int_0^1 \varepsilon_t(i) I_t(i) di \]
\[ = (1 - \delta) K_{t-1} + [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t \int_{\varepsilon_t(i) \geq \varepsilon_t^*} \varepsilon_t(i) H_{t-1}(i) di \]
\[ = (1 - \delta) K_{t-1} + [(1 - \gamma) + 1_t \cdot \gamma] \eta_t P_t \int_{\varepsilon \geq \varepsilon_t^*} \varepsilon f(\varepsilon) d\varepsilon. \]

The aggregate output in (32) is

\[ Y_t = \int_0^1 Y_t(i) di \]
\[ = \int_0^1 A_t K_{t-1}^\alpha (i) \left( \frac{(1 - \alpha) A_t}{W_t} \right)^{\frac{1-\alpha}{\alpha}} K_{t-1}^{1-\alpha}(i) di \]
\[ = A_t \left( \frac{(1 - \alpha) A_t}{W_t} \right)^{\frac{1-\alpha}{\alpha}} K_{t-1} \]
\[ = A_t K_{t-1}^\alpha N_t^{1-\alpha}. \]
The labor market clearing condition is derived as in (31) because

\[ N_t = \int_0^1 N_t(i) di = \int_0^1 \left[ \frac{(1 - \alpha)A_t}{W_t} \right]^\frac{1}{\alpha} K_{t-1}(i) di = \left[ \frac{(1 - \alpha)A_t}{W_t} \right]^\frac{1}{\alpha} K_{t-1}. \]

By definition, the aggregate information acquisition cost \( M_t = \int_0^1 (1 - \gamma_t) \cdot \frac{\gamma}{1 - \gamma} I_t(i) di \).
Substituting that into the market clearing condition for numeraire goods gives

\[
C_t + \eta_t R_{t-1} H_{t-1} = \int_0^1 \left[ Y_t(i) - W_t N_t(i) - \left( 1 - \frac{\gamma_t}{1 - \gamma} + 1 \right) I_t(i) + P_t [\eta_t H_{t-1}(i) - H_t(i)] 
+ P_t H^n_t + \eta_t R_t H_{t-1}(i) \right] di + W_t N_t + M_t 
= Y_t - \left( 1 - \frac{\gamma_t}{1 - \gamma} + 1 \right) I_t + R_{t-1} \eta_t H_{t-1} - P_t H_t + P_t H^n_t + \eta_t R_t H_{t-1} + M_t 
= Y_t - I_t + \eta_t R_{t-1} H_{t-1}
\]
which leads to \( C_t + I_t = Y_t \) immediately.

The equations in Proposition 3 are proved.

**A.3 Equilibrium System of the Full Model**

The solution to the full model with a housing demand shock, financial constraint shock, consumption habit formation and convex capital adjustment cost is analogous to the solution to the basic model. The equilibrium system can be characterized as following proposition, which is analogous to Proposition 2. The equilibrium system is given by the following...
thirteen equations:

\[ q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ R_{t+1} + (1 - \delta) q_{t+1} \right], \quad (55) \]

\[ P_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \eta_{t+1} \left[ R_{t+1} + P_{t+1} + \theta_{t+1} + \int_{\varepsilon \geq \varepsilon^*_{t+1}} \left( \frac{\varepsilon - \varepsilon^*_{t+1}}{\varepsilon^*_{t+1}} - 1 \right) f(\varepsilon) d\varepsilon \right], \quad (56) \]

\[ \Lambda_t = \frac{1}{C_t - \varphi C_{C_t-1}} \left( \beta E_t \frac{\varphi_C}{C_{t+1} - \varphi C_t} \right), \quad (57) \]

\[ \varphi_{N t} = W_t \Lambda_t, \quad (58) \]

\[ \varphi_{H_t} = R_t E_t \Lambda_{t+1} \eta_{t+1}, \quad (59) \]

\[ I_t = \left[ (1 - \gamma) + \mathbf{1}_t \cdot \gamma \right] \frac{\theta_{t} \eta_{t} P_{t}}{Q_t} \int_{\varepsilon \geq \varepsilon^*} f(\varepsilon) d\varepsilon, \quad (60) \]

\[ K_t = (1 - \delta) K_{t-1} + \left[ (1 - \gamma) + \mathbf{1}_t \cdot \gamma \right] \frac{\theta_{t} \eta_{t} P_{t}}{Q_t} \int_{\varepsilon \geq \varepsilon^*} \varepsilon f(\varepsilon) d\varepsilon, \quad (61) \]

\[ q_t \varepsilon^*_{t} = \left( 1 + \mathbf{1}_t \cdot \gamma \right) \frac{\gamma}{1 - \gamma} Q_t, \quad (62) \]

\[ W_t N_t = (1 - \alpha) Y_t, \quad (63) \]

\[ Y_t = A_t \left( K_{t-1}^{\alpha} N_t^{1-\alpha} \right), \quad (64) \]

\[ Y_t = C_t + \left[ 1 + \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] I_t, \quad (65) \]

\[ R_{t}^{K} = \alpha A_t^{\frac{1}{2}} \left( \frac{1 - \alpha}{W_t} \right)^{\frac{1-\alpha}{2}}, \quad (66) \]

\[ Q_t = 1 + \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 + \Omega \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \left( 1 - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \Omega \left( \frac{I_{t+1}}{I_t} - 1 \right) \right)^2, \quad (67) \]

for the thirteen variables \( \{C_t, I_t, Y_t, K_t, N_t, \Lambda_t, \varepsilon^*_t, Q_t, q_t, R_{t}^{K}, R_t, P_t, W_t\} \). The usual transversality conditions hold.